

## Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Friday, February 20.

1. [10 pts.] Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 in the book (p. 210).
  - a)  $\text{Clouds} \Rightarrow \text{Rain}$
  - b)  $(\text{Clouds} \Rightarrow \text{Rain}) \Leftrightarrow ((\text{Clouds} \wedge \text{Hot}) \Rightarrow \text{Rain})$
  - c)  $\neg(\text{Rain} \Rightarrow \text{Clouds}) \Rightarrow (\text{Clouds} \Rightarrow \text{Rain})$
  - d)  $\neg(\text{Clouds} \vee \text{Rain} \vee \text{Hot}) \wedge \text{Rain} \wedge \neg\text{Hot}$
  - e)  $(\text{Clouds} \wedge \text{Rain}) \vee (\text{Rain} \Rightarrow \neg\text{Clouds})$

2. [25 pts.] Consider a knowledge base  $KB$  that contains the following propositional logic sentences:

$$\begin{aligned} Q \vee R \\ P \vee R \Rightarrow Q \\ P \Rightarrow \neg Q \end{aligned}$$

- a) Construct a truth table that shows the truth value of each sentence in  $KB$  and indicate the models in which the  $KB$  is true.
  - b) Does  $KB$  entail  $Q$ ? Use the definition of entailment to justify your answer.
  - c) Does  $KB$  entail  $P \Rightarrow R$ ? Extend the truth table and use the definition of entailment to justify your answer.
  - d) Does  $KB$  entail  $\neg Q \vee P$ ? Extend the truth table and use the definition of entailment to justify your answer.
3. [35 pts.] Building on the kinship domain (p. 254), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates *Daughter*, *Son*, *Wife*, *GrandChild*, *GreatGrandParent*, *Brother*, *Sister*, *Aunt*, *Uncle*, and *FirstCousin*. Here, a predicate of form  $Predicate(x,y)$  should be read in English as “ $x$  is the  $Predicate$  of  $y$ .” Only use these predicates and the predicates defined on p. 254-255 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.
  4. [20 pts.] Represent the following sentences in first order logic, assuming that the domain consists only of people. The only predicates you may use are  $loves(x,y)$ ,  $knows(x,y)$ , and  $avoids(x,y)$ , where a predicate of form  $Predicate(x,y)$  means that “ $x$   $Predicate$   $y$ .” Choose meaningful constants where appropriate.
    - a) Somebody knows and loves Tim.
    - b) Everybody who knows Sue avoids Sue.
    - c) There is somebody that everybody loves.
    - d) Nobody knows everybody.
    - e) There are some people who love nobody but themselves.

5. [10 pts.] Consider the minesweeper agent example we discussed in class. Recall that we use  $\text{NearbyMines}(s,n)$  to represent the relation between a square  $s$  and the number of mines adjacent to it ( $n$ ). We also use  $\text{Mine}(s)$  to indicate that square  $s$  has a mine, and  $\text{Adjacent}(s,t)$  to represent that squares  $s$  and  $t$  are adjacent to each other. Write an axiom that precisely describes the implications of  $\text{NearbyMines}(s,2)$  for any square  $s$ . You may assume that  $\text{Adjacent}(s,t)$  is correctly defined.