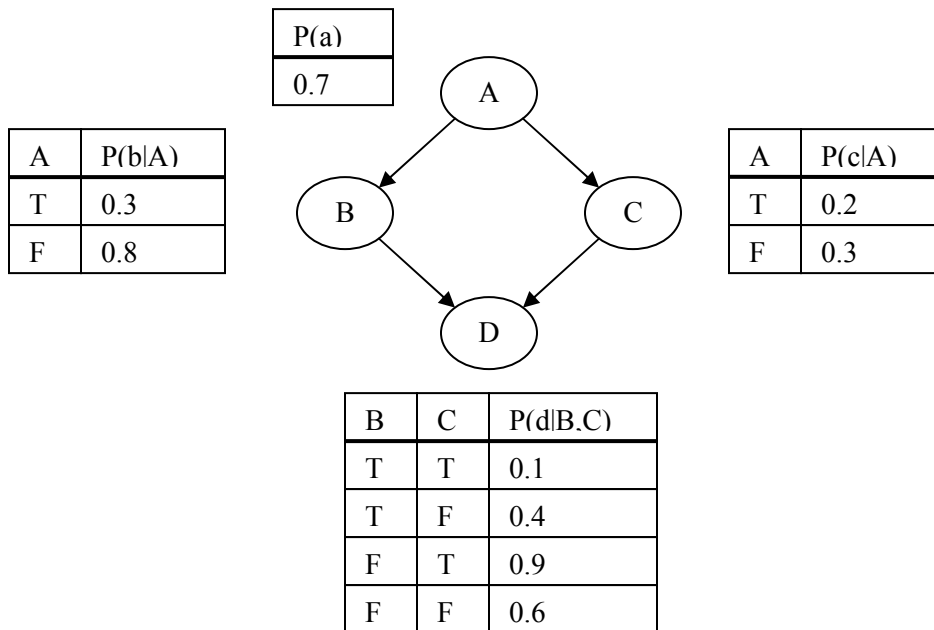


Homework #6: Chapters 14-16

The following exercises are due at the beginning of class on Monday, April 16. Note, this homework is continued on the reverse side of the paper.

1. [25 pts.] Consider the Bayesian network below, where A, B, C, and D are all Boolean random variables. Compute the following probabilities and probability distributions, using a <true,false> ordering for all Boolean variable probability distributions. You must give computed numeric answers and show all of your work.

- a) $P(a \wedge \neg b \wedge c \wedge d)$
 b) $\mathbf{P}(A \mid b \wedge \neg c \wedge \neg d)$
 c) $\mathbf{P}(B \mid c \wedge \neg d)$



2. [20 pts.] Do exercise 14.12 (a-d) from the book (p. 562).
3. [20 pts.] Consider a simple game where a sub is moving in a 3x3 grid. Being on the surface, you cannot see the sub, but have a sonar that detects a ping at a specific location on each time step. At each time step, the sub moves to an adjacent square. The sub is equally likely to move to each legal (i.e., in bounds of the grid) square to the north, east, south, or west of its current location; and each legal diagonal square is half as likely as a move in one of the four compass points. Note, the sub only has 8 legal moves when it is in the center (location (2,2)), and could have as few as three legal moves when it is in a corner. The sub must move to a new square at each time step. The ping location for the sonar will be one square north, east, south, or west of the sub's true location with a 10% probability for each such legal adjacent square. Any other squares where the sub is not located will be the ping location with a 5% probability. Otherwise, the ping location will be the true location of the sub. Give a Bayesian

network structure (similar to the one in Figure 15.2, p. 569) for this Markov process, including values for all relevant conditional probability tables. Use two discrete variables Sub_t and $Sonar_t$ to represent each state. The domain of both of these variables are the 9 possible squares; identified by their (x,y) coordinates as $(1,1)$, $(1,2)$, $(1,3)$, $(2,1)$, $(2,2)$, $(2,3)$, $(3,1)$, $(3,2)$ and $(3,3)$.

4. [10 pts.] Show that any second-order Markov process can be rewritten as a first-order Markov process with an augmented set of state variables. Can this always be done *parsimoniously*, i.e., without increasing the number of parameters needed to specify the transition model?
5. [25 pts.] Consider a student who has the choice to buy or not buy a textbook for a course. Assume that we have three Boolean random variables: B indicates whether the student chooses to buy the book, M indicates whether the student has mastered the material in the book, and P indicates whether the student passes the course. Since the course has an open-book final, P is **not** independent of B given M . Assume that the following conditional probabilities hold:

$$P(p|b,m) = 0.9$$

$$P(m|b)=0.9$$

$$P(p|b,\neg m) = 0.5$$

$$P(m|\neg b)=0.7$$

$$P(p|\neg b,m) = 0.8$$

$$P(p|\neg b,\neg m) = 0.3$$

- a) Draw a Bayesian network for the problem.
- b) Assume the student's utility is a function of both buying the book and of passing the course. A common way to represent utilities that depend on multiple attributes in cases such as this is to use an **additive utility function**, e.g. $U(s)=U_{\text{Buy}}(s) + U_{\text{Pass}}(s)$. For this student, $U_{\text{Buy}}(s) = -100$ he buys the book and 0 if not, and $U_{\text{Pass}}(s) = 2000$ if he passes the course and 0 if he does not. Compute the expected utility of buying the book and of not buying it.
- c) What is the rational thing for the student do?