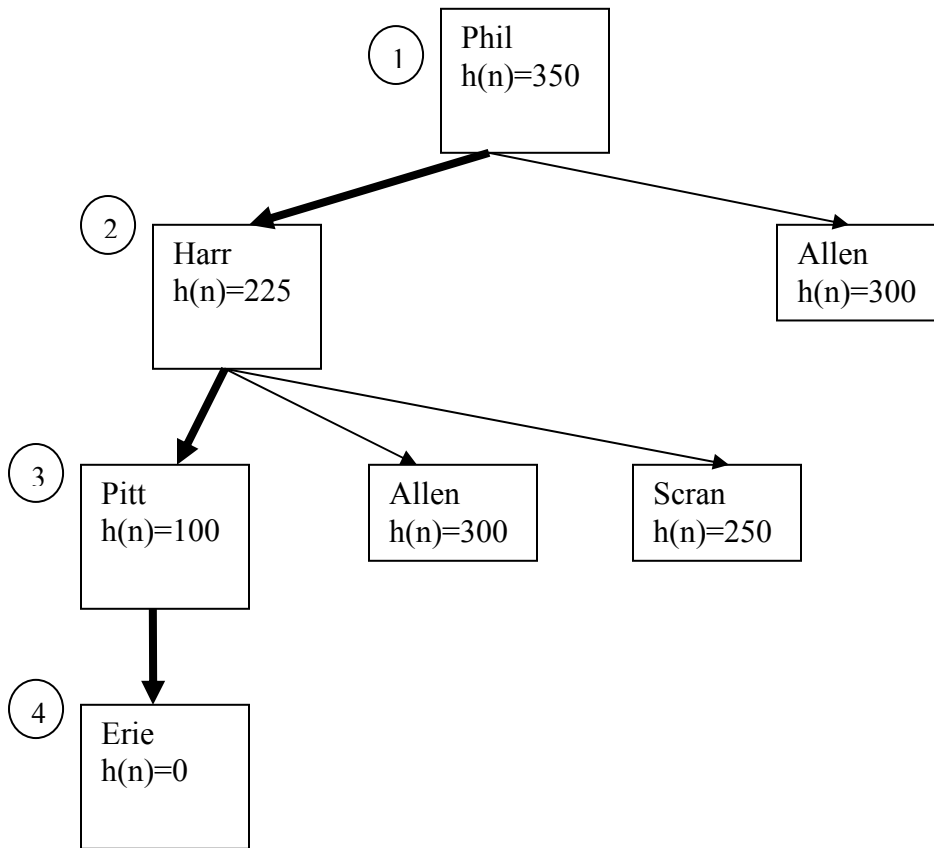


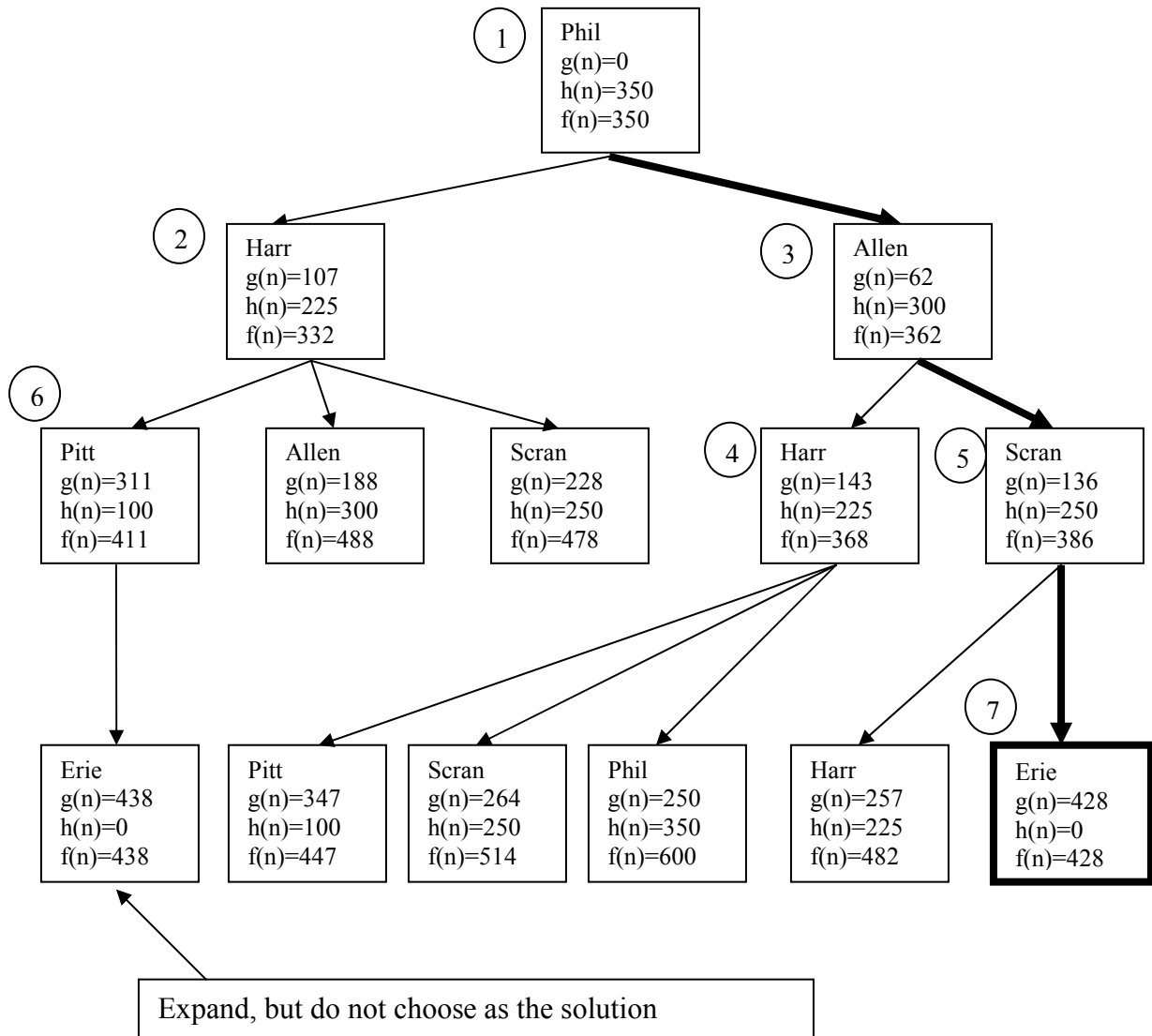
# Homework #2: Chapters 4 and 6

1.



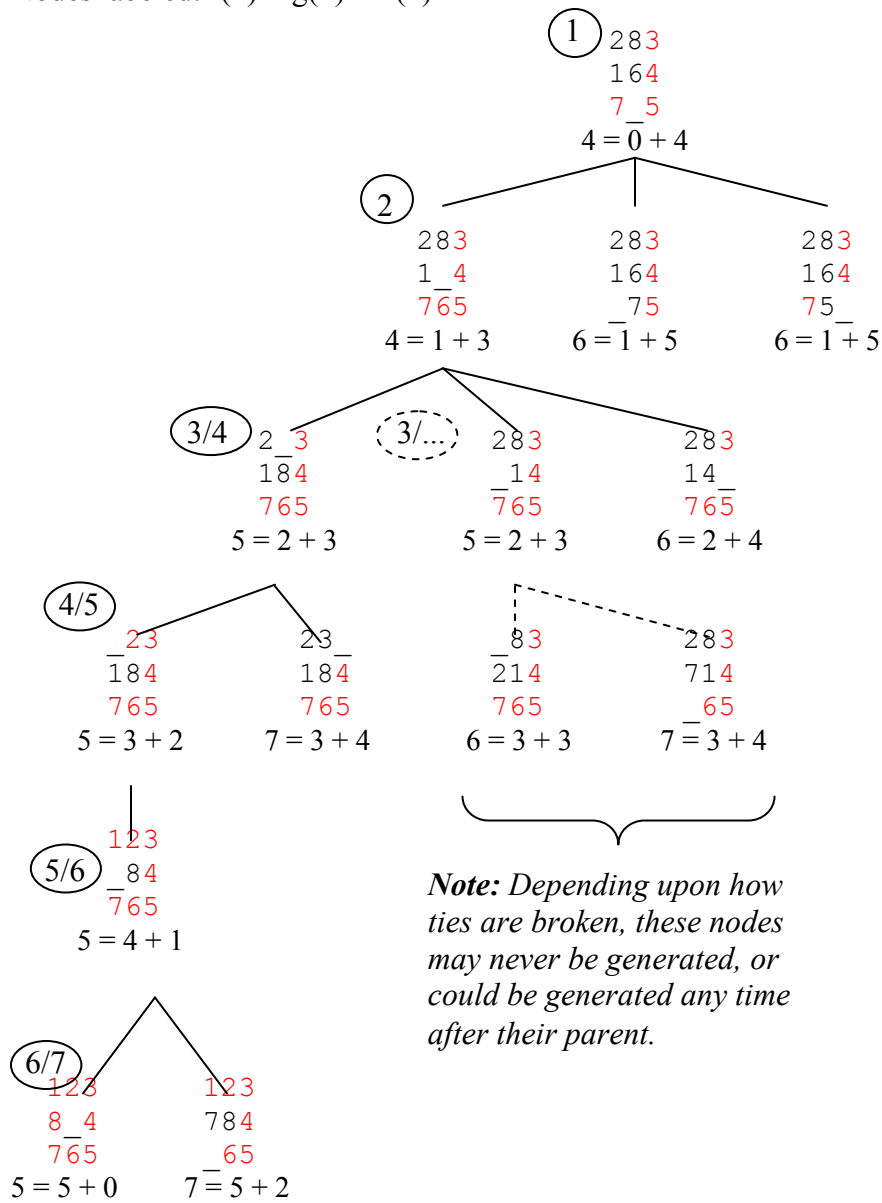
The path is: Phil → Harr → Pitt → Erie  
(note this path has a cost of 438)

2.



3. The tiles in red are in their correct positions.

Nodes labeled:  $f(n) = g(n) + h(n)$



4. i) Using  $h_3$ , every misplaced tile must move at least once. Thus,  $h_3$  can be no less than  $h_1$ , the total number of misplaced tiles. Therefore,  $h_3$  is at least as accurate as  $h_1$ .

Alternative explanation:  $h_1$  can be derived from a relaxation of the relaxed problem used to derive  $h_3$ . “The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.” (p. 107). In this case,  $h_1$  is the cost of an optimal solution to the relaxed problem, which is an admissible heuristic, which is by definition not an overestimate. Therefore  $h_1$  must be less than or equal to  $h_3$  and  $h_3$  is therefore at least as accurate for the actual 8-puzzle problem.

ii) Usually  $h_1$  and  $h_3$  are equally accurate, while  $h_2$  is more accurate than both. However, it is possible to have a situation where the blank is already in the correct spot, and two adjacent tiles are swapped. As such, under  $h_3$ , it takes two moves to get the first misplaced tile in its correct position. The other tile is moved to the blank, then the first misplaced tile is moved to the spot just vacated. Thus,  $h_3$  must be one more than the number of misplaced tiles. Since each tile is only one square away from its correct position, then  $h_3$  is also greater (more accurate) than  $h_2$  (the sum of Manhattan distances).

Here is a specific example:

**state**

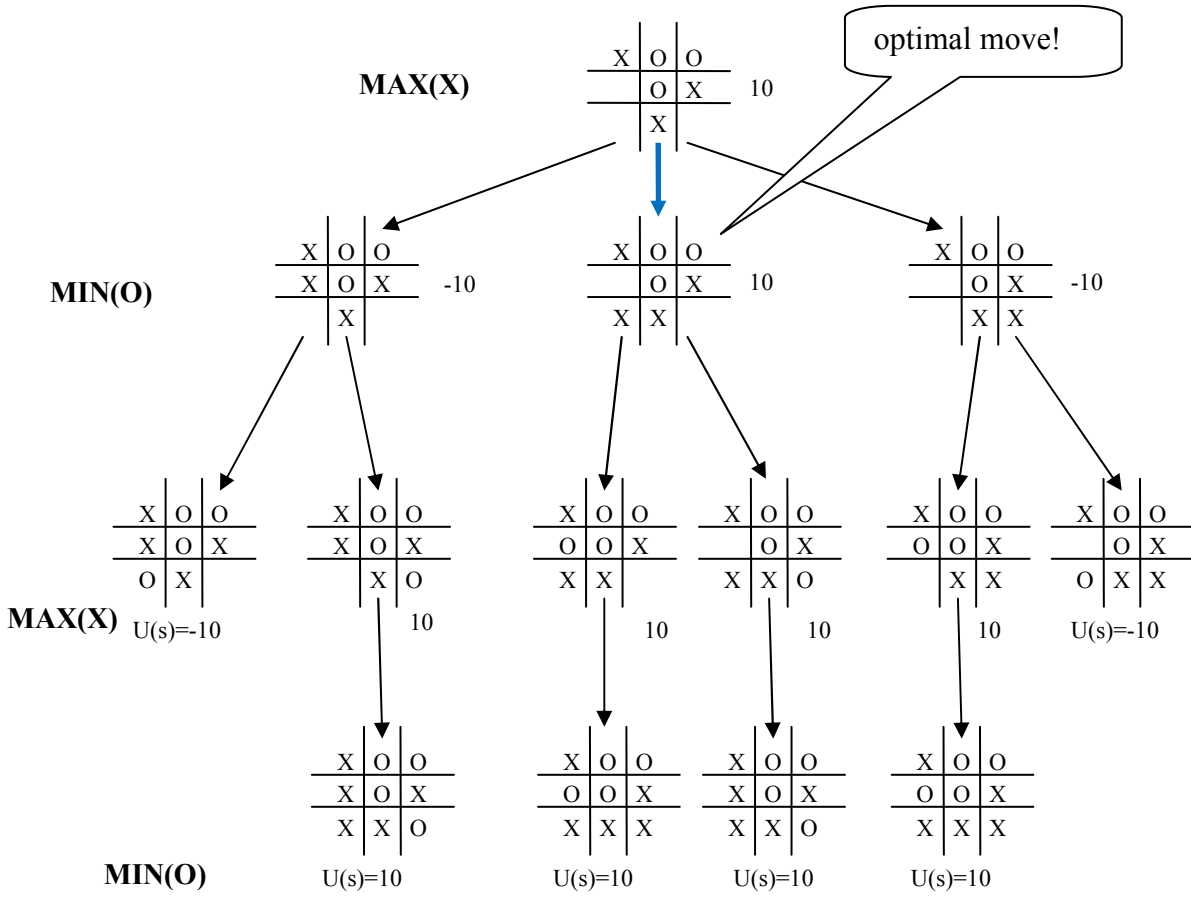
|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 8 | 7 |   |

**goal state**

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 |   |

Here,  $h_1=2$ ,  $h_2=2$ ,  $h_3=3$

5. a)



5b)

MAX

Board 2

|   |  |   |
|---|--|---|
| X |  |   |
| O |  | O |
| X |  | O |
| 0 |  |   |

X's Optimal move according to the 2 ply application of the algorithm

A

|    |  |   |
|----|--|---|
| X  |  | X |
| O  |  |   |
| X  |  | O |
| -3 |  |   |

X's actual best move

B

|    |  |   |
|----|--|---|
| X  |  | X |
| O  |  |   |
| X  |  | O |
| -3 |  |   |

C

|   |  |   |
|---|--|---|
| X |  |   |
| O |  | X |
| X |  | O |
| 0 |  |   |

D

|    |  |   |
|----|--|---|
| X  |  | X |
| O  |  |   |
| X  |  | O |
| -2 |  |   |

E

|    |  |   |
|----|--|---|
| X  |  |   |
| O  |  | X |
| X  |  | O |
| -6 |  |   |

MIN

|  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
|--|---|---|---|---|--|---|---|--|---|----------------|--|--|--|---|--|---|---|--|---|---|--|---|-----------------|--|--|--|---|--|---|---|--|---|---|--|---|------------------|--|--|---|---|--|--|---|--|---|---|--|---|--------------------|--|--|
| <p>A's children</p> <table border="1"> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td>O</td><td> </td><td></td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">1-(3+1) = -3</td></tr> </table>    | X |   | O | O |  |   | X |  | O | 1-(3+1) = -3   |  |  | <table border="1"> <tr><td>X</td><td> </td><td>X</td></tr> <tr><td>O</td><td> </td><td></td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">+3-(3+1) = -1</td></tr> </table>   | X |  | X | O |  |   | X |  | O | +3-(3+1) = -1   |  |  | <p>A's children</p> <table border="1"> <tr><td>X</td><td> </td><td>X</td></tr> <tr><td>O</td><td> </td><td>O</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3+1(2)-3(2) = -1</td></tr> </table> | X |  | X | O |  | O | X |  | O | 3+1(2)-3(2) = -1 |  |  | <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td></td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3+1-1(2) = 2</td></tr> </table>        | X |  |  | O |  |   | X |  | O | 3+1-1(2) = 2       |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1-(3+1) = -3   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| +3-(3+1) = -1  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3+1(2)-3(2) = -1   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3+1-1(2) = 2   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
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| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3-3 = 0  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3(2)+1-3 = 4   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1-1 = 0  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1(2)-3 = -1  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| <p>C's children</p> <table border="1"> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3-3 = 0</td></tr> </table>        | O |   | X | O |  | X | X |  | O | 3-3 = 0        |  |  | <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>O</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3(2)+1-3 = 4</td></tr> </table>    | X |  |   | O |  | O | X |  | O | 3(2)+1-3 = 4    |  |  | <p>D's children</p> <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">1-3 = -2</td></tr> </table>          | X |  |   | O |  | X | X |  | O | 1-3 = -2         |  |  | <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">1(2)-1 = 1</td></tr> </table>         | X |  |  | O |  | X | X |  | O | 1(2)-1 = 1         |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3-3 = 0  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3(2)+1-3 = 4   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1-3 = -2   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1(2)-1 = 1   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| <p>C's children</p> <table border="1"> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>O</td><td> </td><td>X</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3+1-(3+2) = -1</td></tr> </table> | O |   | X | O |  | X | X |  | O | 3+1-(3+2) = -1 |  |  | <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>O</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3-(1(2)+3) = -2</td></tr> </table> | X |  |   | O |  | O | X |  | O | 3-(1(2)+3) = -2 |  |  | <p>E's children</p> <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>O</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">1-(3(2)+1) = -6</td></tr> </table>   | X |  |   | O |  | O | X |  | O | 1-(3(2)+1) = -6  |  |  | <table border="1"> <tr><td>X</td><td> </td><td></td></tr> <tr><td>O</td><td> </td><td>O</td></tr> <tr><td>X</td><td> </td><td>O</td></tr> <tr><td colspan="3" style="text-align: center;">3+1(2)-3(2)-1 = -2</td></tr> </table> | X |  |  | O |  | O | X |  | O | 3+1(2)-3(2)-1 = -2 |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | X |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3+1-(3+2) = -1   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3-(1(2)+3) = -2  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 1-(3(2)+1) = -6  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| O  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| X  |   | O |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |
| 3+1(2)-3(2)-1 = -2   |   |   |   |   |  |   |   |  |   |                |  |  |  |   |  |   |   |  |   |   |  |   |                 |  |  |  |   |  |   |   |  |   |   |  |   |                  |  |  |   |   |  |  |   |  |   |   |  |   |                    |  |  |