Homework #6: Chapters 13-16

1. *[25 points, 5pts. each]*

   a) $P(\neg a)$ = 0.04 + 0.05 + 0.15 + 0.20 = **0.44**

   b) $\mathbf{P}(C)$ = $\langle P(\text{C=true}), P(\text{C=false})\rangle = \langle P(c), 1 - P(c)\rangle$
   = < 0.01 + 0.04 + 0.10 + 0.15, 1 - P(c) > = **< 0.30, 0.70 >**

   c) $P(a \wedge \neg b)$ = 0.10 + 0.25 = **0.35**

   d) $P(c \vee \neg a)$ = 0.01 + 0.04 + 0.10 + 0.15 + 0.05 + 0.20 = **0.55**

   e) $P(a \mid \neg b \wedge c)$ = $P(a \wedge \neg b \wedge c) / P(\neg b \wedge c)$ = 0.10 / (0.10 + 0.15) = **0.4**

2. *[30 points]*

   a) *[5 pts]* $P(w \wedge \neg r \wedge \neg c \wedge h)$ = $P(w|\neg r)P(\neg r|\neg c\wedge h)P(\neg c)P(h)$
   = 0.02 * (1-0.02) * (1-0.35) * 0.25 = **0.003185**

   b) *[10 pts]* $P(h \wedge r \wedge \neg w)$ = $P(h \wedge r \wedge \neg w \wedge \neg c) + P(h \wedge r \wedge \neg w \wedge c)$
   = $P(h)\ P(\neg w|r)P(\neg c)\ P(r|\neg c\wedge h) + P(h)\ P(\neg w|r)P(c)\ P(r|c\wedge h)$
   = $P(h)\ P(\neg w|r)[P(\neg c)\ P(r|\neg c\wedge h) + P(c)\ P(r|c\wedge h)]$
   = (0.25)(1 – 0.7)[(1-0.35)(0.02) + (0.35)(0.6)]
   = (0.25)(0.3)[0.013 + 0.21]
   = 0.016725
   ≈ 0.017

   c) *[15 pts]* $\mathbf{P}(C \mid w) = \alpha\, \mathbf{P}(C,w) = \alpha \sum_{\hat{h}\in H}\sum_{\hat{r}\in R} \mathbf{P}(C,w,\hat{h},\hat{r})$

   $= \alpha \sum_{\hat{h}\in H}\sum_{\hat{r}\in R} \mathbf{P}(C)P(\hat{h})P(\hat{r}\mid C,\hat{h})P(w\mid \hat{r}) = \alpha\, \mathbf{P}(C)\sum_{\hat{h}\in H} P(\hat{h})\sum_{\hat{r}\in R} P(\hat{r}\mid C,\hat{h})P(w\mid \hat{r})$

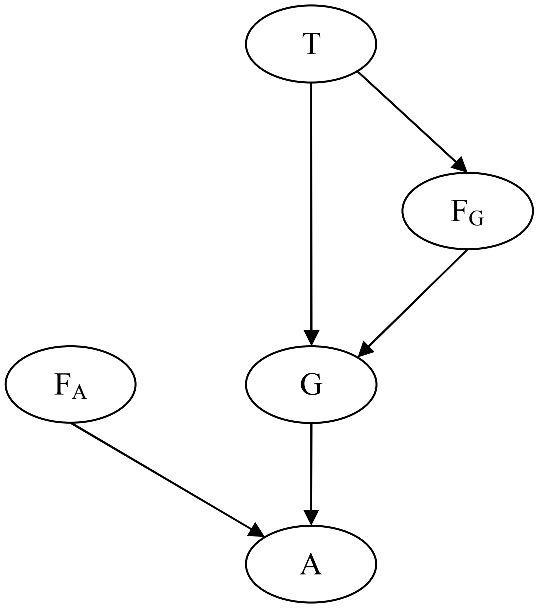
   First, we'll compute for C=true:
   = αP(c)[P(¬h)[P(¬r|c,¬h)P(w|¬r) + P(r|c,¬h)P(w|r)] +
   P(h)[P(¬r|c,h)P(w|¬r) + P(r|c,h)P(w|r)]]
   = α(0.35)[(0.75)[(0.9)(0.02) + (0.1)(0.7)] + (0.25)[(0.4)(0.02) + (0.6)(0.7)]]
   = α(0.35)[(0.75)(0.088) + (0.25)(0.428)]
   = α(0.35)(0.173)
   = α(0.06055)

   Now we compute for C=false:
   = αP(¬c)[P(¬h)[P(¬r|¬c,¬h)P(w|¬r) + P(r|¬c,¬h)P(w|r)] +
   P(h)[P(¬r|¬c,h)P(w|¬r) + P(r|¬c,h)P(w|r)]]
   = α(0.65)[(0.75)[(0.99)(0.02) + (0.01)(0.7)] + (0.25)[(0.98)(0.02) + (0.02)(0.7)]]
   = α(0.65)[(0.75)(0.0268) + (0.25)(0.0336)]
   = α(0.65)(0.0285)
   = α(0.018525)

   P(C|w) = α<0.06055,0.018525>
   = <0.765729, 0.234271>
   ≈ <0.77, 0.23>

3. *[20 pts.]*

a)



b) No, the network is not a polytree. There are two paths from *T* to *G:* one direct path and one via F_G.

c)

F_G	T	G: normal	G: high
F	normal	x	1-x
F	high	1-x	x
T	normal	y	1-y
T	high	1-y	y

d)

F_A	G	A: true	A: false
F	normal	0	1
F	high	1	0
T	normal	0	1
T	high	0	1

4. *[25 pts.]*
Note that the utilities do not exactly correspond to the four atomic events. Two outcomes have a direct correspondence, but the third covers two events. This is summarized in the following table:

Description	**Events**	**Utility**
Agent dies	Dies ∧ Gold, Dies ∧ ¬Gold	-100
Gets the gold and lives	¬Dies ∧ Gold	50
Lives, but does not get gold	¬Dies ∧ ¬Gold	10

We can now compute the expected utility for each action. As a shorthand, we will use *d* for Dies=true, and *g* for Gold=true:

$$EU(A|E) = P(d \wedge g|Do(A),E)(-100) + P(d \wedge \neg g|Do(A),E)(-100) + P(\neg d \wedge g|Do(A),E)(50) + \neg\, P(\neg d \wedge \neg g|Do(A),E)(10)$$
$$= (0.6)(-100) + (0.2)(-100) + (0.05)(50) + (0.15)(10)$$
$$= -60 - 20 + 2.5 + 1.5$$
$$= -76$$

$$EU(B|E) = P(d \wedge g|Do(B),E)(-100) + P(d \wedge \neg g|Do(B),E)(-100) + P(\neg d \wedge g|Do(B),E)(50) + \neg\, P(\neg d \wedge \neg g|Do(B),E)(10)$$
$$= (0.05)(-100) + (0.05)(-100) + (0.1)(50) + (0.8)(10)$$
$$= -5 - 5 + 5 + 8$$
$$= 3$$

$$EU(C|E) = P(d \wedge g|Do(C),E)(-100) + P(d \wedge \neg g|Do(C),E)(-100) + P(\neg d \wedge g|Do(C),E)(50) + \neg\, P(\neg d \wedge \neg g|Do(C),E)(10)$$
$$= (0.25)(-100) + (0.1)(-100) + (0.6)(50) + (0.05)(10)$$
$$= -25 - 10 + 30 + 0.5$$
$$= -4.5$$

Since the action with the greatest expected utility is B (3 versus -76 and -4.5), then the agent should choose **Action B.**