Homework #6: Chapters 13 - 16

The following exercises are due at the beginning of class on Wednesday, April 15.

1. *[25 points]* A full joint distribution for the Boolean random variables *A*, *B*, and *C* is specified below. Use a <true,false> ordering for any Boolean variable probability distributions. Assume that the true value of a random variable is the corresponding lower case letter (e.g., *P*(*b*) means *P*(*B*=*true*))

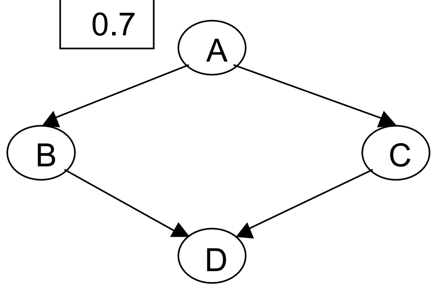
	b		$\neg b$	
	c	$\neg c$	c	$\neg c$
a	0.10	0.01	0.05	0.20
$\neg a$	0.20	0.04	0.15	0.25

Use the distribution to compute the following probabilities and probability distribution. Show your work.

a) $P(a)$
b) $\mathbf{P}(C)$
c) $P(\neg a \wedge b)$
d) $P(\neg c \vee a)$
e) $P(a \mid \neg b \wedge c)$

2. *[40 points]* Consider the Bayesian network below, where *A*,*B*,*C* and *D* are all Boolean random variables.

P(A)
0.7



A	P(B|A)
T	0.3
F	0.8

A	P(C|A)
T	0.2
F	0.3

B	C	P(D|B,C)
T	T	0.1
T	F	0.4
F	T	0.9
F	F	0.6

Compute the probabilities and probability distributions given on the next page. Always use a <true,false> ordering for Boolean variable probability distributions. You must give computed numeric answers and show all of your work.

a) $P(a \wedge \neg b \wedge c \wedge d)$
b) $\mathbf{P}(A \mid b \wedge \neg c \wedge \neg d)$
c) $\mathbf{P}(B \mid c \wedge \neg d)$

3. *[15 points]* Imagine that you have been tasked to build a specialized search engine that can infer the context of a user's query based on the series of keywords he or she enters. Give a Bayesian network structure (similar to the one in Figure 15.2, p. 540) for this Markov process, including values for all relevant conditional probability tables. Use two discrete variables $Context_t$ and $Keyword_t$ to represent each state. The values of $Context_t$ are the three possible contexts: Cars, Football, and Animals. At each time step, a user will only change their context from the previous time step 20% of the time. If they change context, it is equally likely to be changed to either of the other two contexts. At each time step the user enters a single keyword. To keep the problem simple, we limit the domain of $Keyword_t$ to {"auto", "cat", "defense", "dog", "doors", "ecosystem", "ford", "herbivore", "jaguar", "quarterback", "sedan", "team", "touchdown", "other"}. The value "other" represents any word not specified in the domain. Note "jaguar" is considered relevant to all three contexts, while "sedan," "ford", "auto", and "doors" are primarily relevant to Cars, "touchdown," "team," "quarterback", and "defense" are primarily relevant to Football, and "dog," "herbivore," "cat," and "ecosystem" are primarily relevant to Animals. Assume that each word that is relevant to a context has a 15% chance of being the word selected for that context, that the eight words that are not particularly relevant still have a 2% chance each of being selected, and that otherwise the word is classified as "other." Finally, assume that in the absence of all other information, that the context Cars is seven times more likely than Animals, while Football is twice as likely as Animals. In your diagram, clearly indicate your transition model and your sensor model.

4. *[20 points]* A robot soccer player has the option to dribble, pass, or shoot on the goal. If the robot dribbles, there is a 0.2 probability that the other team will steal the ball; otherwise the robot keeps control of the ball. If the robot passes, there is a 0.2 probability that the other team will intercept the pass and a 0.1 probability that the ball will go out of bounds (giving control to the other team); otherwise the pass succeeds. If the robot shoots on the goal, there is a 0.2 probability that it will score, a 0.6 probability that it will be caught by the opponent's goalie, and a 0.2 probability that the shot will go out of bounds (note, in each of the last two situations the other team gains control of the ball). The utility of the robot maintaining control of the ball is 10, the utility of giving it to a teammate is 20, the utility of scoring a goal is 100, and the utility of losing the ball to the other team is – 30.

   What is the expected utility of each action? To maximize the chance for partial credit, be sure to show your work. If the agent follows the principle of maximum expected utility and only considers single actions (as opposed to action sequences), which action will it choose?