Homework #6: Chapters 14-15

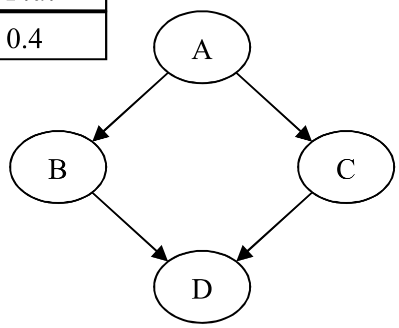
The following exercises are due at the beginning of class on Tuesday, April 15. Note, this homework is continued on the reverse side of the paper.

1. *[20 pts.]* Do exercise 14.11 (a-d) from the book (p. 561).

2. *[25 pts.]* Consider the Bayesian network below, where A, B, C, and D are all Boolean random variables.

P(a)
0.4

A	P(b|A)
T	0.6
F	0.3



A	P(c|A)
T	0.5
F	0.2

B	C	P(d|B,C)
T	T	0.4
T	F	0.2
F	T	0.5
F	F	0.8

Compute the following probabilities and probability distributions, using a <true,false> ordering for all Boolean variable probability distributions. You must give computed numeric answers and show all of your work.

a) $P(a \wedge \neg b \wedge c \wedge d)$
b) $\mathbf{P}(A \mid b \wedge c \wedge \neg d)$
c) $\mathbf{P}(B \mid \neg c \wedge d)$

3. *[20 pts.]* We have a bag of three biased coins *a*, *b*, and *c* with probabilities of coming up heads of 20%, 60% and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1, X_2, and X_3.

    a) Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

    b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails one. Note, you do not need to consider time here.

4. *[25 pts.]* Consider a simple game where a sub is moving in a 3x3 grid. Being on the surface, you cannot see the sub, but have a sonar that produces a ping at a specific location on each time step. At each time step, the sub moves to an adjacent square. Each legal (i.e., in bounds of the grid) square to the north, east, south, or west is equally likely; and each legal diagonal square is half as likely as a move in one of the four compass points. Note, the sub only has 8 legal moves when it is in the center, and could have as few as three legal moves when it is in a corner. The sub must move to a new square at each time step. The ping location for the sonar will be one square north, east, south, or west of the sub's true location with a 10% probability for each such legal adjacent square. Any other squares where the sub is not located will be the ping location with a 5% probability. Otherwise, the ping location will be the true location of the sub. Give a Bayesian network structure (similar to the one in Figure 15.2, p. 569) for this Markov process, including values for all relevant conditional probability tables. Use two discrete variables Sub_t and $Sonar_t$ to represent each state. The domain of both of these variables are the 9 possible squares; identified as (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2) and (3,3).

5. *[10 pts.]* Show that any second-order Markov process can be rewritten as a first-order Markov process with an augmented set of state variables. Can this always be done *parsimoniously*, i.e., without increasing the number of parameters needed to specify the transition model?