

Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Tuesday, March 22.

1. [20 pts. total] Consider a knowledge base KB that contains the following propositional logic sentences:

$$\begin{aligned} P \vee R &\Rightarrow Q \\ P &\Rightarrow \neg Q \\ Q &\vee R \end{aligned}$$

- Construct a truth table that shows the truth value of each sentence in KB and indicate the models in which the KB is true.
 - Does KB entail Q ? Use the definition of entailment to justify your answer.
 - Does KB entail $\neg Q \wedge R$? Extend the truth table and use the definition of entailment to justify your answer.
 - Does KB entail $P \Rightarrow R$? Use the definition of entailment to justify your answer.
2. [10 pts.] Prove each of the following assertions regarding propositional logic. Hint: “if and only if” proofs require you to prove the claim in both directions.
- $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is true in all models.
 - $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is false in all models.
3. [40 pts.] Building on the kinship domain (Sect. 8.3.2, pp. 301-303), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates *Daughter*, *Son*, *Wife*, *GrandChild*, *GreatGrandParent*, *Brother*, *Sister*, *Aunt*, *Uncle*, and *FirstCousin*. Here, a predicate of form $Predicate(x,y)$ should be read in English as “ x is the $Predicate$ of y .” Only use these predicates and the predicates defined on p. 301-303 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.
4. [20 pts.] Represent the following sentences in first order logic, assuming that the domain consists only of people. The only predicates you may use are $loves(x,y)$, $knows(x,y)$, and $avoids(x,y)$, where a predicate of form $Predicate(x,y)$ means that “ x $Predicate$ y .” Choose meaningful constants where appropriate.
- Somebody knows and loves Tim.
 - Everybody who knows Sue avoids Sue.
 - There is somebody that everybody loves.
 - Nobody knows everybody.
 - There are some people who love nobody but themselves.
5. [10 pts.] Consider the minesweeper agent example we discussed in class on March 3. Recall that we use $NearbyMines(s,n)$ to represent the relation between a square s and the number of mines adjacent to it (n). We also use $Mine(s)$ to indicate that square s has a mine, and $Adjacent(s,t)$ to represent that squares s and t are adjacent to each other. Write an axiom that precisely describes the implications of $NearbyMines(s,2)$ for any square s . You may assume that $Adjacent(s,t)$ is correctly defined.