

## Homework #2: OWL and DL

The following exercises are due at the beginning of class on Tuesday, September 30. This will count for 10% of your overall grade. Although this homework will be mostly graded for completeness, selected problems will be graded for correctness. Note, there is a problem on the back of the page as well.

1. Consider the following OWL document using the RDF syntax. Draw the equivalent graph. For convenience, you may use QNames for your node and edge labels and you may use relative ids for nodes where no namespace is specified.

```
<rdf:RDF xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
  xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#"
  xmlns:owl="http://www.w3.org/2002/07/owl#">
  <owl:Ontology rdf:about="" />
  <owl:Class rdf:ID="Person" />
  <owl:Class rdf:ID="Parent">
    <owl:equivalentClass>
      <owl:Restriction>
        <owl:onProperty rdf:resource="#hasChild">
          <owl:someValuesFrom rdf:resource="#Person">
        </owl:Restriction>
      </owl:equivalentClass>
    </owl:Class>
  <owl:Class rdf:ID="Man">
    <rdf:subClassOf>
      <owl:Restriction>
        <owl:onProperty rdf:resource="#hasChild">
          <owl:allValuesFrom rdf:resource="#Person">
        </owl:Restriction>
      </ rdf:subClassOf >
    </owl:Class>
  <owl:ObjectProperty rdf:ID="hasParent">
  <owl:ObjectProperty rdf:ID="hasChild">
    <owl:inverseOf rdf:resource="#hasParent">
  </owl:ObjectProperty>
</rdf:RDF>
```

2. One way to specify that two classes are disjoint in OWL is to use the `owl:disjointWith` property. However, this property is actually redundant. In the OWL RDF-based syntax, write two different axioms (i.e., they use a different combination of constructors) stating that the classes `Man` and `Woman` are disjoint. Do not use `owl:disjointWith`.
3. Translate the following DL axioms into OWL DL. Assume that  $A, B, C$ , and  $D$  denote atomic concepts,  $P$  and  $R$  denote atomic roles, and  $a$  an individual.
  - a)  $A \equiv B \sqcap (C \sqcup D)$
  - b)  $A \sqsubseteq \exists P \neg C$
  - c)  $\neg A \sqsubseteq B \sqcap \exists P . \{a\}$
  - d)  $\forall P . (B \sqcup C) \equiv \leq 2 R$

4. Consider the following simple description logic, where  $\cdot^I$  is the interpretation function.

Class Constructor Syntax	Semantics
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\forall R.C$	$(\forall R.C)^I = \{x \mid \forall y. \langle x,y \rangle \in R^I \Rightarrow y \in C^I\}$
$\exists R.C$	$(\exists R.C)^I = \{x \mid \exists y. \langle x,y \rangle \in R^I \text{ and } y \in C^I\}$
Axiom Syntax	Semantic Condition
$C \sqsubseteq D$	$C^I \subseteq D^I$

For each of the axioms below, determine which of the three interpretations ( $I_1$ ,  $I_2$ , and  $I_3$ ) in the subsequent table satisfy it (note, zero, one or more interpretations may satisfy any given axiom). Assume A,B,C and D are atomic concepts and P is a role.

- $B \sqsubseteq D$
- $A \sqsubseteq B \sqcap \forall P.C$
- $D \sqsubseteq B \sqcup \exists P.C$

Atomic Classes and Roles	Candidate Interpretations ( $\Delta^I = \{a,b,c,d\}$ )		
	$I_1$	$I_2$	$I_3$
$A^I$	$\{\}$	$\{a\}$	$\{b,c\}$
$B^I$	$\{a,b\}$	$\{a\}$	$\{b,c,d\}$
$C^I$	$\{b\}$	$\{b,d\}$	$\{a,b\}$
$D^I$	$\{\}$	$\{a,b\}$	$\{a,b,c,d\}$
$P^I$	$\{\}$	$\{\langle a,b \rangle, \langle a,c \rangle, \langle b,d \rangle\}$	$\{\langle b,a \rangle, \langle b,b \rangle, \langle d,a \rangle\}$