Homework #2: OWL and DL

The following exercises are due at the beginning of class on Tuesday, October 5. This will count for 10% of your overall grade. Note, there is a problem on the back of the page as well.

1. Consider the following OWL document using the RDF syntax. Draw the equivalent graph. For convenience, you may use QNames for your node and edge labels and you may use relative ids for nodes where no namespace is specified.

```
<rdf:RDF xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
        xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#"
        xmlns:owl="http://www.w3.org/2002/07/owl#"
        xmlns:p="http://www.examples.org/person#"
        xml:base="http://www.examples.org/person">
  <owl:Ontology rdf:about="" />
  <owl:Class rdf:ID="Person" />
  <owl:Class rdf:ID="Parent">
     <owl:equivalentClass>
        <owl:Restriction>
           <owl:onProperty rdf:resource="#hasChild" />
           <owl:someValuesFrom rdf:resource="#Person" />
        </owl:Restriction>
     </owl:equivalentClass>
  </owl:Class>
  <owl:Class rdf:ID="Man">
     <rdf:subClassOf>
        <owl:Restriction>
           <owl:onProperty rdf:resource="#hasParent" />
           <owl:allValuesFrom rdf:resource="#Person" />
        </owl:Restriction>
     </rdf:subClassOf>
  </owl:Class>
  <owl:ObjectProperty rdf:ID="hasParent" />
  <owl:ObjectProperty rdf:ID="hasChild">
     <owl:inverseOf rdf:resource="#hasParent" />
  </owl:ObjectProperty>
</rdf:RDF>
```

- 2. One way to specify that two classes are disjoint in OWL is to use the owl:disjointWith property. However, this property is actually redundant. In the OWL RDF-based syntax, write two different axioms (i.e., they use a different combination of constructors) stating that the classes Man and Woman are disjoint. Do not use owl:disjointWith.
- 3. Translate the following DL axioms into OWL DL. Assume that *A*,*B*,*C*, and *D* denote atomic concepts, *P* and *R* denote atomic roles, and *a* an individual.
 - a) $A \equiv B \cap (C \cup D)$
 - b) $A \subseteq \exists P.(B \sqcap C)$
 - c) $\neg A \sqsubseteq \exists P^-.\{a\}$
 - d) $\forall P.(\neg B) \equiv \leq 2 R$

4. (Required for CSE 428 students, extra credit for CSE 397 students) Consider the following simple description logic, where \cdot^{I} is the interpretation function.

Class Constructor Syntax	Semantics
$C \sqcap D$	$(\mathbf{C} \cap \mathbf{D})^I = \mathbf{C}^I \cap \mathbf{D}^I$
$C \sqcup D$	$(\mathbf{C} \sqcup \mathbf{D})^I = \mathbf{C}^I \cup \mathbf{D}^I$
∀R.C	$(\forall R.C)^I = \{x \mid \forall y. \langle x, y \rangle \in R^I \Rightarrow y \in C^I\}$
∃R.C	$(\exists R.C)^I = \{x \mid \exists y. \langle x, y \rangle \in R^I \text{ and } y \in C^I\}$
Axiom Syntax	Semantic Condition
$C \sqsubseteq D$	$C^I \subseteq D^I$

For each of the axioms below, determine which of the three interpretations (I_1 , I_2 , and I_3) in the subsequent table satisfy it (note, zero, one or more interpretations may satisfy any given axiom). Assume A,B,C and D are atomic concepts and P is a role.

- a) $B \sqsubseteq D$
- b) $A \subseteq B \cap \forall P.C$
- c) $D \sqsubseteq B \sqcup \exists P.C$

	Candidate Interpretations ($\Delta^I = \{a,b,c,d\}$)		
Atomic Classes and Roles	I_1	I_2	I_3
A^I	{}	{a}	{b,c}
B^{I}	{a,b}	{a}	{b,c,d}
C^{I}	{b}	{b,d}	{a,b}
\mathbf{D}^{I}	{}	{a,b}	$\{a,b,c,d\}$
P^I	{}	{ <a,b>,<a,c>,<b,d>}</b,d></a,c></a,b>	{ <b,a>,<b,b>,<d,a>}</d,a></b,b></b,a>