Homework #2: Chapters 6, 7, 8

The following exercises are due at the beginning of class on Wednesday, February 25.

1. **[30 points total]** This problem exercises the basic concepts of game playing, using tic-tac-toe as an example. Define \( X_n \) as the number of rows, columns, or diagonals with exactly \( n \) X’s and no O’s. Similarly, define \( O_n \) as the number of rows, columns, or diagonals with exactly \( n \) O’s and no X’s. Use the evaluation function 
   \[\text{Eval}(s) = 10X_3(s) + 3X_2(s) + X_1(s) - (10O_3(s) + 3O_2(s) + O_1(s)).\]
   
   a) How many possible games of tic-tac-toe are there? You may approximate your answer by specifying reasonable upper and lower bounds. Explain your answer.
   
   b) Show the whole game tree starting from board shown on the right down to a depth of 2. Assume that X will make the next move and that X is the MAX player.
   
   c) Mark on your tree the evaluations of all the positions at depth 2.
   
   d) Use the minimax algorithm to choose the best move for X to make next.
   
   e) Circle the nodes that would *not* be evaluated if alpha-beta pruning were used. Assume that the nodes are generated in the optimal order for alpha-beta pruning.

2. **[10 points]** Suppose the wumpus world agent has visited locations \([1,1]\), \([2,1]\), and \([1,2]\) only. Having perceived nothing in \([1,1]\), a stench and a breeze in \([2,1]\), and a breeze in \([1,2]\), the agent is now concerned with the contents of \([1,3]\), \([2,2]\), and \([3,1]\). Each of these can contain a pit and at most one can contain a wumpus. Following the example of Figure 7.5 in the book (p. 202), construct the set of possible worlds (You should find 32 of them). Mark the worlds in which the KB is true and those in which each of the following sentences is true:
   
   \[\alpha_1 = \text{“There is a wumpus in } [3,1]\text{”}\]
   
   Does KB entail \( \alpha_1 \)?
   
   \[\alpha_2 = \text{“There is a pit in } [2,2]\text{”}\]
   
   Does KB entail \( \alpha_2 \)?

3. **[15 points total]** Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 in the book (p. 210).
   
   a) Clouds \( \rightarrow \) Rain
   
   b) \((\text{Clouds } \rightarrow \text{Rain}) \leftrightarrow ((\text{Clouds } \land \text{Hot}) \rightarrow \text{Rain})\)
   
   c) \(\neg (\text{Rain } \rightarrow \text{Clouds}) \rightarrow (\text{Clouds } \rightarrow \text{Rain})\)
   
   d) \(\neg (\text{Clouds } \lor \text{Rain } \lor \text{Hot}) \land \text{Rain } \land \neg \text{Hot}\)
   
   e) \((\text{Clouds } \land \text{Rain}) \lor (\text{Rain } \rightarrow \neg \text{Clouds})\)

4. **[15 points total]** Represent the following sentences in first order logic using quantifiers, the predicates \(\text{loves}(x,y), \text{knows}(x,y), \text{and avoids}(x,y)\), the binary = predicate, and any necessary constants. Here, a predicate of form \(\text{Predicate}(x,y)\) means that “\(x\) \(\text{Predicate}\) \(y\)”
   
   a) Somebody knows and loves Tim.
   
   b) Everybody who knows Sue avoids Sue.
   
   c) There is somebody that everybody loves.
   
   d) Nobody knows everybody.
   
   e) There are some people who love nobody but themselves.

5. **[10 points]** The wumpus world agent has developed a better sense of smell. The agent now perceives a *Stench* when the wumpus is in a horizontally or vertically adjacent square and when the wumpus is in a square diagonal to the agent. Write out the axioms required for reasoning about the wumpus’s location, using a constant symbol \(\text{Wumpus}\) and a binary predicate \(\text{In}(\text{Wumpus}, \text{Location})\). Remember that there is only one wumpus.

6. **[20 points]** Building on the kinship domain (p. 254), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates \(\text{Daughter}, \text{Son}, \text{Wife}, \text{GrandChild}, \text{GreatGrandParent}, \text{Brother}, \text{Sister}, \text{Aunt}, \text{Uncle}, \text{and FirstCousin}\). Here, a predicate of form \(\text{Predicate}(x,y)\) should be
read in English as “x is the *Predicate* of y.” Only use these predicates and the predicates defined on p. 254-255 of the book in your definitions.