Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Friday, March 2.

1. [25 pts.] Consider a knowledge base \( KB \) that contains the following propositional logic sentences:
   
   \[
   Q \Rightarrow P \\
   P \Rightarrow \neg Q \\
   Q \lor R
   \]

   a) Construct a truth table that shows the truth value of each sentence in \( KB \) and indicate the models in which the \( KB \) is true.
   
   b) Does \( KB \) entail \( R \)? Use the definition of entailment to justify your answer.
   
   c) Does \( KB \) entail \( R \Rightarrow P \)? Extend the truth table and use the definition of entailment to justify your answer.
   
   d) Does \( KB \) entail \( \neg Q \land R \)? Extend the truth table and use the definition of entailment to justify your answer.

2. [10 pts. total] In propositional logic, does an empty knowledge base (i.e., a knowledge base with no sentences in it) entail anything? Explain your answer.

3. [35 pts.] Building on the kinship domain (p. 254), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates \( \text{Daughter} \), \( \text{Son} \), \( \text{Wife} \), \( \text{GrandChild} \), \( \text{GreatGrandParent} \), \( \text{Brother} \), \( \text{Sister} \), \( \text{Aunt} \), \( \text{Uncle} \), and \( \text{FirstCousin} \). Here, a predicate of form \( \text{Predicate}(x,y) \) should be read in English as “\( x \) is the \( \text{Predicate} \) of \( y \).” Only use these predicates and the predicates defined on p. 254-255 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.

4. [20 pts. total] Represent the following sentences in first order logic, assuming that the domain consists only of people. The only predicates you may use are \( \text{loves}(x,y) \), \( \text{knows}(x,y) \), and \( \text{avoids}(x,y) \), where a predicate of form \( \text{Predicate}(x,y) \) means that “\( x \) \( \text{Predicate} \) \( y \).” Choose meaningful constants where appropriate.
   
   a) Somebody knows and loves Tim.
   
   b) Everybody who knows Sue avoids Sue.
   
   c) There is somebody that everybody loves.
   
   d) Nobody knows everybody.
   
   e) There are some people who love nobody but themselves.

5. [10 pts.] Write down a first-order logic sentence such that every world in which it is true contains exactly one object in its domain.