

Homework #3: Chapters 7 and 8

Problem 1:

a) $\text{Clouds} \Rightarrow \text{Rain}$

Clouds	Rain	$\text{Clouds} \Rightarrow \text{Rain}$
T	T	T
T	F	F
F	T	T
F	F	T

neither (*satisfiable*)

b) $(\text{Clouds} \Rightarrow \text{Rain}) \Leftrightarrow ((\text{Clouds} \wedge \text{Hot}) \Rightarrow \text{Rain})$

Clouds	Rain	Hot	$\text{Clouds} \Rightarrow \text{Rain}$	$\text{Clouds} \wedge \text{Hot}$	$((\text{Clouds} \wedge \text{Hot}) \Rightarrow \text{Rain})$	entire sentence
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

neither (*satisfiable*)

c) $\neg(\text{Rain} \Rightarrow \text{Clouds}) \Rightarrow (\text{Clouds} \Rightarrow \text{Rain})$

Clouds	Rain	$\text{Rain} \Rightarrow \text{Clouds}$	$\neg(\text{Rain} \Rightarrow \text{Clouds})$	$\text{Clouds} \Rightarrow \text{Rain}$	$\neg(\text{Rain} \Rightarrow \text{Clouds}) \Rightarrow (\text{Clouds} \Rightarrow \text{Rain})$
T	T	T	F	T	T
T	F	T	F	F	T
F	T	F	T	T	T
F	F	T	F	T	T

valid

Alternate proof:

$\neg(\text{Rain} \Rightarrow \text{Clouds}) \Rightarrow (\text{Clouds} \Rightarrow \text{Rain})$

$\neg(\neg\text{Rain} \vee \text{Clouds}) \Rightarrow (\text{Clouds} \Rightarrow \text{Rain})$

$\neg(\neg\text{Rain} \vee \text{Clouds}) \Rightarrow (\neg\text{Clouds} \vee \text{Rain})$

$\neg\neg(\neg\text{Rain} \vee \text{Clouds}) \vee (\neg\text{Clouds} \vee \text{Rain})$

$(\neg\text{Rain} \vee \text{Clouds}) \vee (\neg\text{Clouds} \vee \text{Rain})$

$(\text{Rain} \vee \neg\text{Rain} \vee \text{Clouds} \vee \neg\text{Clouds})$

$\text{True} \vee \text{True}$

True

d) $\neg(\text{Clouds} \vee \text{Rain} \vee \text{Hot}) \wedge \text{Rain} \wedge \neg\text{Hot}$

Clouds	Rain	Hot	$\text{Clouds} \vee \text{Rain} \vee \text{Hot}$	$\neg(\text{Clouds} \vee \text{Rain} \vee \text{Hot})$	$\neg\text{Hot}$	$\neg(\text{Clouds} \vee \text{Rain} \vee \text{Hot}) \wedge \text{Rain} \wedge \neg\text{Hot}$
T	T	T	T	F	F	F
T	T	F	T	F	T	F
T	F	T	T	F	F	F
T	F	F	T	F	T	F
F	T	T	T	F	F	F
F	T	F	T	F	T	F
F	F	T	T	F	F	F
F	F	F	F	T	T	F

unsatisfiable

e) $(\text{Clouds} \wedge \text{Rain}) \vee (\text{Rain} \Rightarrow \neg\text{Clouds})$

Clouds	Rain	$\text{Clouds} \wedge \text{Rain}$	$\neg\text{Clouds}$	$\text{Rain} \Rightarrow \neg\text{Clouds}$	$(\text{Clouds} \wedge \text{Rain}) \vee (\text{Rain} \Rightarrow \neg\text{Clouds})$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	F	T	T	T
F	F	F	T	T	T

valid

Alternate proof:

$$(\text{Clouds} \wedge \text{Rain}) \vee (\text{Rain} \Rightarrow \neg\text{Clouds})$$

$$(\text{Clouds} \wedge \text{Rain}) \vee (\neg\text{Rain} \vee \neg\text{Clouds})$$

$$\neg\text{Rain} \vee (\text{Clouds} \wedge \text{Rain}) \vee \neg\text{Clouds}$$

$$((\neg\text{Rain} \vee \text{Clouds}) \wedge (\neg\text{Rain} \vee \text{Rain})) \vee \neg\text{Clouds}$$

$$((\neg\text{Rain} \vee \text{Clouds}) \wedge \text{True}) \vee \neg\text{Clouds}$$

$$(\neg\text{Rain} \vee \text{Clouds}) \vee \neg\text{Clouds}$$

$$\neg\text{Rain} \vee (\text{Clouds} \vee \neg\text{Clouds})$$

$$\neg\text{Rain} \vee \text{True}$$

$$\text{True}$$

Problem 2:

a)

P	Q	R	α_1		α_2		α_3	$\alpha_1 \wedge \alpha_2 \wedge \alpha_3$			
P	Q	R	$Q \vee R$	$P \vee R$	$P \vee R \Rightarrow Q$	$\neg Q$	$P \Rightarrow \neg Q$	$K.B.$	$P \Rightarrow R$	$\neg Q \vee P$	
T	T	T	T	T	T	F	F	F	T	T	
T	T	F	T	T	T	F	F	F	F	T	
T	F	T	T	T	F	T	T	F	T	T	
T	F	F	F	T	F	T	T	F	F	T	
F	T	T	T	T	T	F	T	T	T	F	
F	T	F	T	F	T	F	T	T	T	F	
F	F	T	T	T	F	T	T	F	T	T	
F	F	F	F	F	T	T	T	F	T	T	

From above table, we can conclude:

b) KB entails Q , i.e., $KB \models Q$

It is true in both models where the KB is true

c) KB entails $P \Rightarrow R$, i.e., $KB \models P \Rightarrow R$

It is true in both models where the KB is true

d) KB does not entail $\neg Q \vee P$

It is false in both models where the KB is true (and it only needs to be false in at least one)

Problem 3:

$$\forall d, p \text{ Daughter}(d, p) \Leftrightarrow \text{Child}(d, p) \wedge \text{Female}(d)$$

$$\forall s, p \text{ Son}(s, p) \Leftrightarrow \text{Child}(s, p) \wedge \text{Male}(s)$$

$$\forall w, h \text{ Wife}(w, h) \Leftrightarrow \text{Spouse}(w, h) \wedge \text{Female}(w)$$

$$\forall x, z \text{ Grandchild}(x, z) \Leftrightarrow \exists y \text{ Child}(x, y) \wedge \text{Child}(y, z)$$

$$\forall a, d \text{ Great-grandparent}(a, d) \Leftrightarrow \exists b \text{ Parent}(a, b) \wedge \text{Grandparent}(b, d)$$

or

$$\forall a, d \text{ Great-grandparent}(a, d) \Leftrightarrow \exists b, c \text{ Parent}(a, b) \wedge \text{Parent}(b, c) \wedge \text{Parent}(c, d)$$

$$\forall b, x \text{ Brother}(b, x) \Leftrightarrow \text{Sibling}(b, x) \wedge \text{Male}(b)$$

$$\forall s, x \text{ Sister}(s, x) \Leftrightarrow \text{Sibling}(s, x) \wedge \text{Female}(s)$$

(Aunt is defined as the sister of one's father or mother, or the wife of one's uncle.)

$$\forall a, c \text{ Aunt}(a, c) \Leftrightarrow (\exists p \text{ Sibling}(a, p) \wedge \text{Female}(a) \wedge \text{Parent}(p, c)) \vee$$

$$(\exists u \text{ Spouse}(a, u) \wedge \text{Uncle}(u, c))$$

or

$$\forall a, c \text{ Aunt}(a, c) \Leftrightarrow (\exists p \text{ Sister}(a, p) \wedge \text{Parent}(p, c)) \vee (\exists u \text{ Spouse}(a, u) \wedge \text{Uncle}(u, c))$$

(Uncle is defined as a brother of one's father or mother, or an aunt's husband.)

$$\forall u, c \text{ Uncle}(u, c) \Leftrightarrow (\exists p \text{ Sibling}(u, p) \wedge \text{Male}(u) \wedge \text{Parent}(p, c)) \vee$$

$$(\exists a \text{ Spouse}(u, a) \wedge \text{Aunt}(a, c))$$

or

$$\forall u, c \text{ Uncle}(u, c) \Leftrightarrow (\exists p \text{ Brother}(u, p) \wedge \text{Parent}(p, c)) \vee (\exists a \text{ Spouse}(u, a) \wedge \text{Aunt}(a, c))$$

(First cousin is defined as a child of one's aunt or uncle.)

$$\forall c, y \text{ FirstCousin}(c, y) \Leftrightarrow \exists p, x \text{ Child}(c, x) \wedge \text{Sibling}(x, p) \wedge \text{Parent}(p, y)$$

or

$$\forall c, y \text{ FirstCousin}(c, y) \Leftrightarrow \exists x \text{ Child}(c, x) \wedge (\text{Aunt}(x, y) \vee \text{Uncle}(x, y))$$

Note this definition is probably more lax than it should be. If someone who is an aunt or uncle by marriage to one of your blood-relations has a child by a prior marriage, then this definition considers that child to be your first-cousin (some dictionaries include a definition such as "Persons who have a grandparent in common are called first cousins" to rule this situation out). Nevertheless, I'll accept this axiom as is.

Problem 4:

a) $\exists x \text{ knows}(x, \text{Tim}) \wedge \text{loves}(x, \text{Tim})$

b) $\forall x \text{ knows}(x, \text{Sue}) \Rightarrow \text{avoids}(x, \text{Sue})$

c) $\exists x \forall y \text{ loves}(y, x)$

d) $\neg \exists x \forall y \text{ knows}(x, y)$ or
 $\forall x \neg \forall y \text{ knows}(x, y)$ or
 $\forall x \exists y \neg \text{knows}(x, y)$

e) $\exists x \forall y \text{ loves}(x, y) \Rightarrow x = y$ or
 $\exists x \forall y x \neq y \Rightarrow \neg \text{loves}(x, y)$

Problem 5:

$\forall s \text{ NearbyMines}(s,2) \Rightarrow (\exists r,t r \neq t \wedge \text{Adjacent}(s,r) \wedge \text{Mine}(r) \wedge \text{Adjacent}(s,t) \wedge \text{Mine}(t) \wedge$
 $(\forall u u=r \vee u=t \vee \neg \text{Mine}(u) \vee \neg \text{Adjacent}(s,u)))$

or

$\forall s \text{ NearbyMines}(s,2) \Rightarrow (\exists r,t r \neq t \wedge \text{Adjacent}(s,r) \wedge \text{Mine}(r) \wedge \text{Adjacent}(s,t) \wedge \text{Mine}(t) \wedge$
 $(\forall u \text{Adjacent}(s,u) \wedge \text{Mine}(u) \Rightarrow u=r \vee u=t))$