Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Friday, February 20.

1. [10 pts.] Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 in the book (p. 210).
   a) Clouds ⇒ Rain
   b) (Clouds ⇒ Rain) ⇔ ((Clouds ∧ Hot) ⇒ Rain)
   c) ¬(Rain ⇒ Clouds) ⇒ (Clouds ⇒ Rain)
   d) (Clouds ∨ Rain ∨ Hot) ∧ Rain ∧ ¬Hot
   e) (Clouds ∧ Rain) ∨ (Rain ⇒ ¬Clouds)

2. [25 pts.] Consider a knowledge base KB that contains the following propositional logic sentences:
   
   \[ Q ∨ R \\
   P ∨ R ⇒ Q \\
   P ⇒ ¬Q \]

   a) Construct a truth table that shows the truth value of each sentence in KB and indicate the models in which the KB is true.
   b) Does KB entail Q? Use the definition of entailment to justify your answer.
   c) Does KB entail P ⇒ R? Extend the truth table and use the definition of entailment to justify your answer.
   d) Does KB entail ¬Q ∨ P? Extend the truth table and use the definition of entailment to justify your answer.

3. [35 pts.] Building on the kinship domain (p. 254), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates Daughter, Son, Wife, GrandChild, GreatGrandParent, Brother, Sister, Aunt, Uncle, and FirstCousin. Here, a predicate of form Predicate(x,y) should be read in English as “x is the Predicate of y.” Only use these predicates and the predicates defined on p. 254-255 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.

4. [20 pts.] Represent the following sentences in first order logic, assuming that the domain consists only of people. The only predicates you may use are loves(x,y), knows(x,y), and avoids(x,y), where a predicate of form Predicate(x,y) means that “x Predicate y.” Choose meaningful constants where appropriate.
   a) Somebody knows and loves Tim.
   b) Everybody who knows Sue avoids Sue.
   c) There is somebody that everybody loves.
   d) Nobody knows everybody.
   e) There are some people who love nobody but themselves.
5. [10 pts.] Consider the minesweeper agent example we discussed in class. Recall that we use NearbyMines\((s,n)\) to represent the relation between a square \(s\) and the number of mines adjacent to it \((n)\). We also use Mine\((s)\) to indicate that square \(s\) has a mine, and Adjacent\((s,t)\) to represent that squares \(s\) and \(t\) are adjacent to each other. Write an axiom that precisely describes the implications of NearbyMines\((s,2)\) for any square \(s\). You may assume that Adjacent\((s,t)\) is correctly defined.