Homework #3: Chapters 7 and 8

The following exercises are due at 3pm on Friday, February 28.

1. [20 pts.] Consider a knowledge base $KB$ that contains the following propositional logic sentences:
   
   $Q \Rightarrow P$
   
   $Q \lor R$
   
   $P \Rightarrow \neg Q$

   a) Construct a truth table that shows the truth value of each sentence in $KB$ and indicate the models in which the $KB$ is true.
   
   b) Does $KB$ entail $\neg Q$? Use the definition of entailment to justify your answer.
   
   c) Does $KB$ entail $P \lor Q$? Extend the truth table and use the definition of entailment to justify your answer.
   
   d) Does $KB$ entail $P \Rightarrow R$? Extend the truth table and use the definition of entailment to justify your answer.

2. [20 pts.] Suppose the wumpus world agent (see Sect. 7.2, p. 236-240) has visited locations [1,1], [2,1], and [1,2] only. Having perceived nothing in [1,1], a stench and a breeze in [2,1], and a stench in [1,2], the agent is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit and at most one can contain a wumpus. Following the example of Figure 7.5 in the book (p. 241), construct the set of possible models (You should find 32 of them). Mark the worlds in which the KB is true and those in which each of the following sentences is true:

   - $\alpha_1 = \text{"There is a wumpus in [2,2]"}$
   - $\alpha_2 = \text{"There is a pit in [3,1]"}$

   a) Does KB entail $\alpha_1$?
   
   b) Does KB entail $\alpha_2$?

3. [40 pts.] Building on the kinship domain (Sect. 8.3.2, pp. 301-303), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates $Daughter$, $Son$, $Wife$, $GrandChild$, $GreatGrandParent$, $Brother$, $Sister$, $Aunt$, $Uncle$, and $FirstCousin$. Here, a predicate of form $Predicate(x,y)$ should be read in English as “x is the $Predicate$ of y.” Only use these predicates and the predicates defined on p. 301-303 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.

4. [10 pts.] Consider the minesweeper agent example we discussed in class on Feb. 20. Recall that we use NearbyMines($s$,n) to represent the relation between a square $s$ and the number of mines adjacent to it (n). We also use Mine($s$) to indicate that square $s$ has a mine, and Adjacent($s,t$) to represent that squares $s$ and $t$ are adjacent to each other. Write an axiom that precisely describes the implications of NearbyMines($s$,2) for any square $s$. You may assume that Adjacent($s,t$) is correctly defined.

5. [10 pts.] Write down a first-order logic sentence such that every world in which it is true contains exactly one object in its domain.