Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Tuesday, March 22.

1. [20 pts. total] Consider a knowledge base $KB$ that contains the following propositional logic sentences:

   \[ P \lor R \Rightarrow Q \]
   \[ P \Rightarrow \neg Q \]
   \[ Q \lor R \]

   a) Construct a truth table that shows the truth value of each sentence in $KB$ and indicate the models in which the $KB$ is true.
   
   b) Does $KB$ entail $Q$? Use the definition of entailment to justify your answer.
   
   c) Does $KB$ entail $\neg Q \land R$? Extend the truth table and use the definition of entailment to justify your answer.
   
   d) Does $KB$ entail $P \Rightarrow R$? Use the definition of entailment to justify your answer.

2. [10 pts.] Prove each of the following assertions regarding propositional logic. Hint: “if and only if” proofs require you to prove the claim in both directions.

   a) $\alpha \models \beta$ if and only if the sentence ($\alpha \Rightarrow \beta$) is true in all models.
   
   b) $\alpha \models \beta$ if and only if the sentence ($\alpha \land \neg \beta$) is false in all models.

3. [40 pts.] Building on the kinship domain (Sect. 8.3.2, pp. 301-303), use first-order logic to write axioms defining the binary (i.e., having arity 2) predicates Daughter, Son, Wife, GrandChild, GreatGrandParent, Brother, Sister, Aunt, Uncle, and FirstCousin. Here, a predicate of form $Predicate(x,y)$ should be read in English as “$x$ is the $Predicate$ of $y$.” Only use these predicates and the predicates defined on p. 301-303 of the book in your definitions. Try to ensure that your definitions are as complete as possible without leading to false inferences. You may want to refer to a dictionary to ensure that you understand the full meaning of terms like aunt, uncle and first cousin.

4. [20 pts.] Represent the following sentences in first order logic, assuming that the domain consists only of people. The only predicates you may use are $loves(x,y)$, $knows(x,y)$, and $avoids(x,y)$, where a predicate of form $Predicate(x,y)$ means that “$x$ $Predicate$ $y$.” Choose meaningful constants where appropriate.

   a) Somebody knows and loves Tim.
   
   b) Everybody who knows Sue avoids Sue.
   
   c) There is somebody that everybody loves.
   
   d) Nobody knows everybody.
   
   e) There are some people who love nobody but themselves.

5. [10 pts.] Consider the minesweeper agent example we discussed in class on March 3. Recall that we use NearbyMines($s,n$) to represent the relation between a square $s$ and the number of mines adjacent to it ($n$). We also use Mine($s$) to indicate that square $s$ has a mine, and Adjacent($s,t$) to represent that squares $s$ and $t$ are adjacent to each other. Write an axiom that precisely describes the implications of NearbyMines($s,2$) for any square $s$. You may assume that Adjacent($s,t$) is correctly defined.