Homework #3: Chapters 7 and 8

The following exercises are due at the beginning of class on Tuesday, March 7.

1. [25 pts. total] Consider a knowledge base \( KB \) that contains the following propositional logic sentences:

\[
P ∨ ¬Q \\
¬R ⇒ Q \\
(Q ∨ R) ⇒ P
\]

a. Construct a truth table that shows the truth value of each sentence in \( KB \) and indicate the models in which the \( KB \) is true.

b. Does \( KB \) entail \( P \)? Use the definition of entailment to justify your answer.

c. Does \( KB \) entail \( R \)? Use the definition of entailment to justify your answer.

d. Does \( KB \) entail \( R ⇒ P \)? Extend the truth table and use the definition of entailment to justify your answer.

2. [10 pts.] In propositional logic, does an empty knowledge base (i.e. a knowledge base containing only the sentence \text{true}) entail anything? Explain your answer.


4. [30 pts.] For each of the following sentences, decide if the accompanying first-order logic sentence is a good translation. If it is not good, explain why and correct it.

a. No two people have the same social security number

\[
∀x,y \ Person(x) ∧ Person(y) ⇒ ¬∃n \ HasSSN(x,n) ∧ HasSSN(y,n)
\]

b. John’s social security number is the same as Mary’s

\[
∃n \ HasSSN(John,n) ∧ HasSSN(Mary,n)
\]

c. Everyone’s social security number has nine digits.

\[
∀x,n \ Person(x) ⇒ (HasSSN(x,n) ∧ Digits(n,9))
\]

d. Rewrite your answers to parts a–c using the function symbol \text{SSN} rather than the predicate symbol \text{HasSSN} (if the original sentence was good, rewrite the original).