Homework #4: Chapters 9 and 12

The following exercises are due at the beginning of class on Tuesday, March 27

1. [15 points, 3 points each] For each pair of atomic sentences, give the most general unifier (in normal form) if it exists. Assume that x, y, and z are variables, while other symbols are either predicates, constants, or functions as required by their use in the sentences. Do not standardize apart the sentences.
   a) \( P(B,A,B), P(x,y,z) \)
   b) \( P(x,x), Q(A,A) \)
   c) \( \text{Older}(\text{Father}(y), y), \text{Older}(\text{Father}(x), \text{John}) \)
   d) \( Q(y,G(A,B)), Q(G(x,z),y) \)
   e) \( P(f(x), y, g(A)), P(f(y), A, z) \)

2. [40 points total] Consider the first-order logic sentences defined below.
   \[
   \forall x,y \quad P(x,y) \land Q(y,x) \implies R(x,y) \\
   \forall x,y \quad S(x,\text{Bob}) \land S(y,x) \implies P(x,y) \\
   \forall x,y \quad S(x,y) \implies Q(y,x) \\
   \forall x,y \quad T(x,y,x) \implies Q(x,y) \\
   T(\text{Alice}, \text{Dawn}, \text{Alice}) \\
   T(\text{Eve}, \text{Carl}, \text{Eve}) \\
   T(\text{Alice}, \text{Bob}, \text{Dawn}) \\
   T(\text{Carl}, \text{Carl}, \text{Alice}) \\
   S(\text{Bob}, \text{Alice}) \\
   S(\text{Carl}, \text{Bob}) \\
   S(\text{Dawn}, \text{Carl}) \\
   S(\text{Carl}, \text{Dawn}) \\
   S(\text{Alice}, \text{Dawn}) \\
   S(\text{Eve}, \text{Carl})
   \]

   Use backward chaining to find \textbf{ALL} answers for the following queries. When matching rules, proceed from top to bottom, and evaluate subgoals from left to right. You must show your search tree using the same form I did in class: Each node should contain a list of subgoals remaining to be proven, and each child is a subsequent recursive call. Also label each arc with the rule that was matched and give the substitutions that permit the match. Note, the form of the proof tree shown in Fig. 9.7 of the book (p. 338) is unacceptable, because it does not show when backtracking occurs.
   a) [15 points] \( \exists x \ Q(\text{Alice}, x) \)
   b) [25 points] \( \exists x,y \ R(x,y) \)

3. [15 points] Use the forward-chaining algorithm to find all conclusions that can be made from the following knowledge base. Clearly delineate what was derived at each iteration of the algorithm and for each new conclusion indicate which sentences were used to derive it.
   \[
   \text{HasPrereq}(\text{cse017}, \text{cse002}) \\
   \text{HasPrereq}(\text{cse109}, \text{cse017}) \\
   \text{HasPrereq}(\text{cse216}, \text{cse109}) \\
   \text{HasPrereq}(\text{cse303}, \text{cse109}) \\
   \forall x,y \quad \text{HasPrereq}(x,y) \implies \text{MustTakeBefore}(y,x) \\
   \forall x,y,z \quad \text{MustTakeBefore}(x,y) \land \text{MustTakeBefore}(y,z) \implies \text{MustTakeBefore}(x,z)
   \]
4. [15 points] Construct a semantic network representation for the following sentence: “Mary gave the green flowered vase to her cousin.” Break the sentence down such that each object is represented by a separate bubble and each property of an object is represented by a different labeled link. See Figures 12.5 and 12.6 (p. 455) in the book for examples.

5. [15 points] Consider the following description logic expression. Write a first-order logic sentence that says there is at least one object that satisfies the conditions defined by the expression:

\[
And(Man, \text{AtLeast}(3, \text{Son}), \text{AtMost}(2, \text{Daughter}),
\quad All(\text{Son}, And(\text{Unemployed}, \text{Married}, All(\text{Spouse}, \text{Doctor}))),
\quad All(\text{Daughter}, And(\text{Professor}, Fills(\text{Department}, \text{Physics})))).
\]