Slepian-Wolf Cooperation: A Practical and Efficient Compress-and-Forward Relay Scheme

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Abstract

The *Slepian-Wolf (SW) cooperation* proposed in [1] is probably the first *practical* cooperative scheme that implements the idea of *compress-and-forward*. Through the exploitation of efficient distributed source coding (DSC) technology, the authors of [1] demonstrate the effectiveness of *Slepian-Wolf cooperation* in combating inter-user channel outage in wireless environment. In this paper, we discuss the general framework of *Slepian-Wolf cooperation* using the two most popular DSC technologies: the binning/syndrome approach and the parity approach. We show that the latter is particularly useful in *SW cooperation*, since it is conceptually simpler, provides certain performance advantages, and enables any (system) linear channel code to be readily exploited. Examples using convolutional codes, low-density generator-matrix codes and low-density parity-check codes are demonstrated and practical algorithms for estimating the source-relay correlation and for decoding the compound packets at the destination are discussed.

I. Introduction

User cooperation has become an increasingly hot research topic, due to its substantial gains over non-cooperative communication. Also known as the *relay channel* problem, user cooperation dates back to the late sixties when it was first discussed by van der Meulen [2]. Substantial advances in the theory and the basic coding strategies were made by Cover and El Gamal [3]. It was not until years later that the problem was re-invented, analyzed in detail, and popularized by several research groups (e.g. [4]-[10]).

Consider a cellular type of wireless scenario where multiple users, each equipped with a single antenna, communicate with a common destination. Assume for the time being, the channels experience quasi-static Rayleigh fading, i.e. time-limited channels where time diversity is hard to attain. Due to the lack of diversity, conventional non-cooperative wireless communication sees a high outage rate that decreases only linearly with the increase of signal-to-noise ratio (SNR): $p_{out} \approx 1/(4\text{SNR})$. Through the collaboration of geographically-distributed users, virtual antenna arrays can be formed, promising substantial gains via transmit/receive diversity, beam-forming, spatial multiplexing and power allocation [3]-[10].

The basic cooperative modes include *amplify-and-forward* (AF), where the relay rescales the received analog waveforms and forwards them to the destination, *decode-and-forward* (DF), where the relay demodulates and decodes the received packet and possibly re-encodes it before forwarding it, and *compress-and-forward* (CF), where the relay forwards the quantized/compressed/estimated version of its observations [3][4]. If in CF the relay does not decode the packet, then DF and CF exhibit certain duality and achieve respectively the gains related to multi-antenna transmission and multi-antenna reception [4]. On the other hand, if we view scaling and decoding as special forms of estimation, then CF, in its general sense, subsumes both AF and DF.

This work has been supported by the National Science Foundation under grant CCF-0430634 and by a grant from the Commonwealth of Pennsylvania, Department of Community and Economic Development, through the Pennsylvania Infrastructure Technology Alliance (PITA).

While information-theoretic aspects (e.g. achievable rate regions, capacity bounds) are important, another interesting research direction explores coding solutions that implement these cooperative ideas. A large body of work targets *random* codes. Most notable among them is the work by Kramer *et al* [4], where efficient random coding strategies based on the DF and CF modes are developed and analyzed. In the area of *practical* coding solutions, the existing work has almost exclusively focused on DF (and AF). Examples include *coded cooperation*, *space-time cooperation*, *coded space-time cooperation* and *coded double space-time cooperation* [5]-[10].

This paper discusses a recently developed cooperative strategy, formerly known as *Slepian-Wolf (SW) cooperation*, which exploits the technology of Slepian-Wolf compression, or distributed source coding (DSC), in wireless user cooperation. Originally proposed in [1], *SW cooperation* is probably the first practical coding scheme that falls in the category of compress-and-forward. In [1], the key elements of *SW cooperation* are discussed, examples using low-density parity-check (LDPC) codes (or more precisely, LDPC Slepian-Wolf formulation) are provided, and substantial gains over the existing cooperative schemes (especially in combating the inter-user channel outage) are demonstrated [1].

In this paper, we further the concept of *Slepian-Wolf cooperation*, and discuss a constructive framework that is general enough to allow any systematic linear channel code to be exploited in *Slepian-Wolf cooperation*. We start with the motivation for *Slepian-Wolf cooperation* (Section II). After briefing the background of distributed source coding technology (Section II), we discuss the central idea of *SW cooperation*, and present a general framework for wireless user cooperation (Section III). We demonstrate, through examples of convolutional codes, low-density generator-matrix (LDGM) codes and LDPC codes, the generality of the proposed framework (Section IV). Finally, concluding remarks are provided in Section V).

Throughout the paper, unless otherwise stated, we will assume (1) the relay model comprises three terminals: a source S, a relay R and a destination D, (2) the communication channels between these terminals are spatially-independent block fading Rayleigh channels, where the fading coefficient of each channel remains constant during one round of user cooperation, and (3) all the three terminals operate in a half-duplex mode.

II. Motivation and Background

A. Motivation for Slepian-Wolf Cooperation

The revival of user cooperation in the wireless context is in part spurred by the great success of multiple-input multiple-output (MIMO) technologies. Although user cooperation enables different users to share antennas, the virtual antenna array nevertheless has a fundamental difference from the real antenna array in a MIMO system: while the data to be transmitted are known beforehand to every antenna in the latter case, they need to be conveyed from one antenna to another in the former case. Associated with this is a cost for time, energy and bandwidth, as well as a risk that data may be corrupted or lost during the transmission. We say an *inter-user outage* happens when the relay fails to correctly decode the source packet - even under the protection of a channel code. Clearly, a low inter-user outage is essential to ensure successful user cooperation and subsequently a good cooperative diversity.

However, inter-user outage is not as low as one would like to see. For many practical cases, inter-user outage occurs at a probability of 10^{-2} even with the protection of a convolutional code or an LDPC code[1] [12]. When inter-user outage happens, DF is reduced to a non-

[1]For example, a packet protected by a $(3000, 2000)$ LDPC code may still see 10.4%-1.06% of inter-user outage at inter-user SNRs of 10-22 dB [12].

cooperative mode with a diversity order of only 1. The authors of [12] showed that, at inter-user outage, the asymptotic error probability of AF scales linearly with that of DF by a factor of approximately $1/2$. That is, the availability of a second copy of the packet (foreseeably a quite noisy copy) enables AF to reduce the error rate but only by half. Hence, both strategies perform rather poorly in this worst case scenario, which in term adversely degrades the average performance.

In this paper, we resort to the idea of compress-and-forward, and propose to exploit Slepian-Wolf coding in user cooperation to effectively combat inter-user outage. From the analysis of the achievable rate region [4], one finds that the achievable rate region of DF improves as the relay moves toward the source, and reaches its maximum (i.e. system achieves the capacity) when the relay is near the source. On the other hand, the achievable rate region of CF improves as the relay moves toward the destination, and reaches its maximum when the relay is near the destination. Inter-user outage resembles a case where the relay is farther away from the source (and likely closer up to the destination). CF could therefore expect to outperform DF.

One motivation for *Slepian-Wolf cooperation* is the observation that at inter-user outage, although the relay fails to decode the packet entirely right, it may get most of the bits right most of the time. This holds for most channel codes although to what extent is code dependent. For example, for a $(3000, 2000)$ regular LDPC code, at a low SNR of 7 dB, about 85% of the failed blocks contain less than 5% of errors; when the SNR increases to 13 dB, more than 96% of the failed blocks contain less than 5% of errors[2] Hence, instead of attaining the original source packet X, the relay now gets a copy Y, which is highly correlated with X. Is it possible for the relay to make intelligent use of Y rather than casting it away?

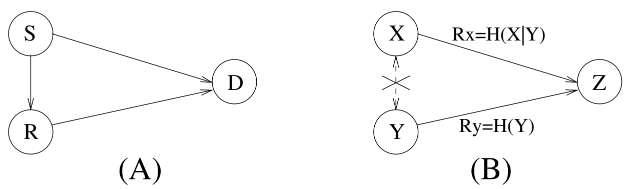


Fig. 1. (A) Relay system. (B) DSC system.

Regarding correlated data at physically-separated places, one technology becomes immediately relevant: Slepian-Wolf coding [11]. Also known as distributed source coding, SW coding concerns the separate compression of two (or more) statistically correlated sources, say X and Y, and the joint decompression of them at a common destination Z; see Figure 1(B). When we compare the system model of SW coding and that of user cooperation (in the case of inter-user outage) in Figure 1, we find that the two seemingly dispatched problems exhibit several interesting similarities: (1) they both have two transmit terminals and one final destination terminal: S, R and D in user cooperation, and X, Y and Z in DSC; (2) a high correlation between data recovered at R and data at S is exactly like the inherent source correlation between X and Y; (3) the (compressed) data from X and Y in DSC are transmitted in orthogonal channels, and so are data from S and R in user cooperation. Further, encoding of DSC is performed *separately* at each source, obviating the need for inter-user coordination in cooperative communication. Hence, there is a good reason to believe that the excellent ideas in SW coding can find new and exciting use in user cooperation. Before proceeding to the framework and the details of *Slepian-Wolf cooperation*, let us first look at

[2]We observe, however, the residual errors appear to be more busty with turbo codes.

the basics of Slepian-Wolf coding.

B. Slepian-Wolf Coding

The theoretical underpinnings of Slepian-Wolf coding were established back in the seventies. In a typical two-source Slepian-Wolf system as shown in Fig. 1(B), assume that X and Y are memoryless binary symmetric sources that are correlated at the same time instant with $\Pr(X \neq Y) = p < 0.5$. This correlation is typically described using a binary symmetric channel (BSC) correlation with a crossover probability p, since one source can be viewed as a noisy version of the other after it is passed through a BSC(p). The achievable rate pairs in this case are specified by the famous Slepian-Wolf theorem [11]: $R_x \geq H(X|Y) = H(p)$, $R_y \geq H(Y|X) = H(p)$ and $R_x + R_y \geq H(X, Y) = 1 + H(p)$. The corner points of the Slepian-Wolf boundary are commonly referred to as asymmetric compression, where one source, say Y, can be coded using a conventional entropy coding technique and transmitted at rate $H(Y)$, the other source X can be compressed to $H(X|Y)$ using "Slepian-Wolf coding", and the destination can recover both X and Y through the joint decoding of Y and $H(X|Y)$.

A key concept in Slepian-Wolf coding is *code binning*, which has been used in the proof of the Slepian-Wolf theory, i.e. the achievability of compressing X to $R_x = H(X|Y)$ [11]. In a nutshell, coding binning refers to the idea of grouping sequences of source X in bins, each indexed with a bin-index. Compression is performed by mapping the X sequence to its bin-index, and decompression is performed by identifying the target bin using the bin-index and subsequently identifying the target X sequence in the bin using the correlated Y sequence.

In practice, bins are constructed using the coset structure of a linear channel code. The idea is to view X^n, a sequence of X with length n, as a virtual codeword of some (n, k) linear channel code. It is then natural to use cosets of this linear channel as bins, to use syndromes S^{n-k} as bin-indexes, and to compress X^n to its corresponding syndrome. If the channel code is capacity approaching on BSC(p), then the rate of the channel code $k/n \rightarrow 1 - H(p)$ (capacity of BSC), and a compression rate of $R_x = (n - k)/n \rightarrow H(p) = H(X|Y)$ is thus achieved.

While the technique of binning can losslessly turn *any* linear channel code to a Slepian-Wolf code, it is not the only means of performing SW compression. Below we describe a different approach which is applicable to a general *systematic* linear channel code and which is particularly attractive to the proposed *Slepian-Wolf cooperation*.

The approach, which we refer to as the *parity approach*, exploits the exact encoding and decoding process of an (n, k) systematic linear channel code for the purpose of SW compression. For each source sequence X^k, it computes its length n codeword, leaves out the systematic part X^k, and transmits only the parity part P^{n-k} to the destination. The decoder recovers X^k by performing channel decoding on Y^k and P^{n-k}, where Y^k is treated as BSC(p) corrupted version of the systematic part. Unlike the binning approach which yields a compression ratio of $n : (n-k)$ for an (n, k) code, the parity approach yields a compression ratio of $k : (n - k)$.

III. A General Framework for Slepian-Wolf Cooperation

A. The System Model of Slepian-Wolf Cooperation

The proposed *Slepian-Wolf cooperation* exploits the asymmetric SW coding technology to combat inter-user outage. In the system model shown in Figure 2(A), the source and the relay will transmit alternatively in three consecutive time slots. In the first time slot, the source sends data X, possibly protected by an error correcting code, to the destination and the relay simultaneously. The relay, upon obtaining a slightly distorted version Y, will invoke *Slepian-Wolf cooperation* by transmitting Y (or $H(Y)$) in the second time slot. Notified by a flag bit, the source will then transmit an additional packet containing $H(X|Y)$ to complete the Slepian-Wolf code. The destination thus attains two spatially diversified copies of X, one from the Slepian-Wolf decoding of $H(Y)$ and $H(X|Y)$, and the other from the initial transmission of X by the source. This basic approach can be improved by letting the source

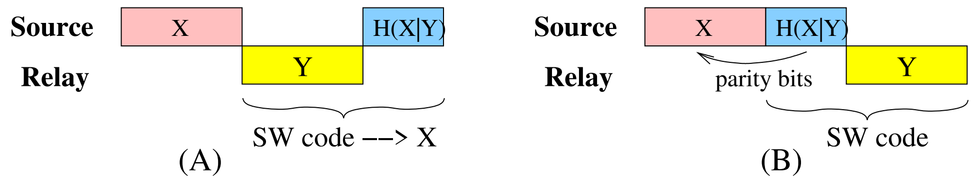


Fig. 2. The system model of *Slepian-Wolf cooperation*. (A) The basic model. (B) The advanced model.

transmit X and $H(X|Y)$ altogether in the first time slot; see Figure 2(B). Since $H(X|Y)$ is needed only when *Slepian-Wolf cooperation* is invoked, will pre-fetching this be wasteful? The answer is no. As discussed in the previous section, $H(X|Y)$ is computed in two ways in practice: (1) in the parity approach, $H(X|Y)$ is in fact the parity bits for X, and can therefore be used to protect X if nothing else; (2) in the binning approach, $H(X|Y)$ is represented using syndromes. From the coding theory, one realizes that syndromes are essentially a special type of parity bits (will be discussed in further detail). Hence, $H(X|Y)$ can first of all be interpreted as parity bits for protecting X, and, in the case of *Slepian-Wolf cooperation*, be combined with Y (or $H(Y)$) to form a Slepian-Wolf code.

To summarize, the proposed cooperative framework operates as follows:

1) In the first time slot, the source computes $H(X|Y)$ using the *parity approach*, and broadcast data X, denoted as $packet_X$, together with $H(X|Y)$, denoted as $packet_{X|Y}$.
2) The relay treats the $packet_{X|Y}$ as parity bits, and performs channel decoding on $packet_X + packet_{X|Y}$ to estimate X. Depending on the estimation result, it chooses one of following three options in the second time slot:
   - If X is decoded successfully, the relay resorts to a DF-based cooperative strategy such as *coded cooperation*.
   - If the decoded data Y contains a small percentage of errors (e.g. below a predefined threshold p_{th}), the relay invokes *Slepian-Wolf cooperation* by forwarding Y or $H(Y)$ (denoted as $packet_Y$), the slightly distorted version of X, to the destination.
   - If the decoded data contains lots of errors, the relay reverts to the *non-cooperative* mode and stays idle.

   In the former two cases, an indicating bit will be piggybacked on the relay packet, so that the destination knows which one happens.
3) The destination will perform a joint decoding on all the packets it received to make a best estimation on X.

B. Practical Issues

We discuss several issues concerning the practicality and the efficiency of the proposed framework.

First, in the initial transmission of X by the source, X may either be raw data or channel-coded data. However, in light of the fact that $packet_{X|Y}$ will provide protection for X, there is no practical benefit for using additional channel coding. Put another way, the (extra) protection power needed for X can be obtained by choosing a proper (Slepian-Wolf) code to compute $packet_{X|Y}$.

Second, in computing $H(X|Y)$ or $packet_{X|Y}$, it is highly recommended that the parity approach, rather than the binning approach, be used. We note that there is a subtle relationship between the binning approach and the parity approach, which will become clear when we discuss the example of LDPC codes. Nevertheless, it is fair to say that the parity approach is not only easier to implement, but tends to perform better in *Slepian-Wolf cooperation*. We realize that the parity approach is restricted to systematic codes, but for any linear channel code, there exists an equivalent systematic code that has the same codeword space and therefore renders the same block error rate. Hence, confirming to systematic codes does not cause any essential compromise to the system performance.

Third, in order to choose between, say, *coded cooperation*, *Slepian-Wolf cooperation* and *no cooperation*, the relay needs to know the decoding quality. Practical systems are typically equipped with a cyclic redundancy check (CRC); it is therefore easy to tell between successful decoding and unsuccessful decoding. In the latter case, an efficient estimation method is needed for the relay to determine the percentage of residual errors. While other methods are possible, [1] showed that the mean of the log-likelihood ratios (LLR) at the decoder output, denoted as $\mu(LLR)$, can be used a figure of merit to describe the relative decoding quality. The relation between $\mu(LLR)$ and the percentage of errors in the block, p_e, is plotted in Figure 3 for an LDPC code and a convolutional code. Each blue dot represents a simulation test, where the mean of the decoder LLRs is computed by averaging over all the bits in the block, and the solid red line represents the "true" mean, which is computed by further averaging over hundreds of thousands of blocks. It is evident that $\mu(LLR)$ is closely related to p_e. In addition to its simplicity and the rather accurate estimation (see [1] for experimental results), another particularly attractive feature about $\mu(LLR)$ is that its relation with p_e is independent to the channel SNRs. Hence, a single lookup table suffices to implement the estimation rule [1],

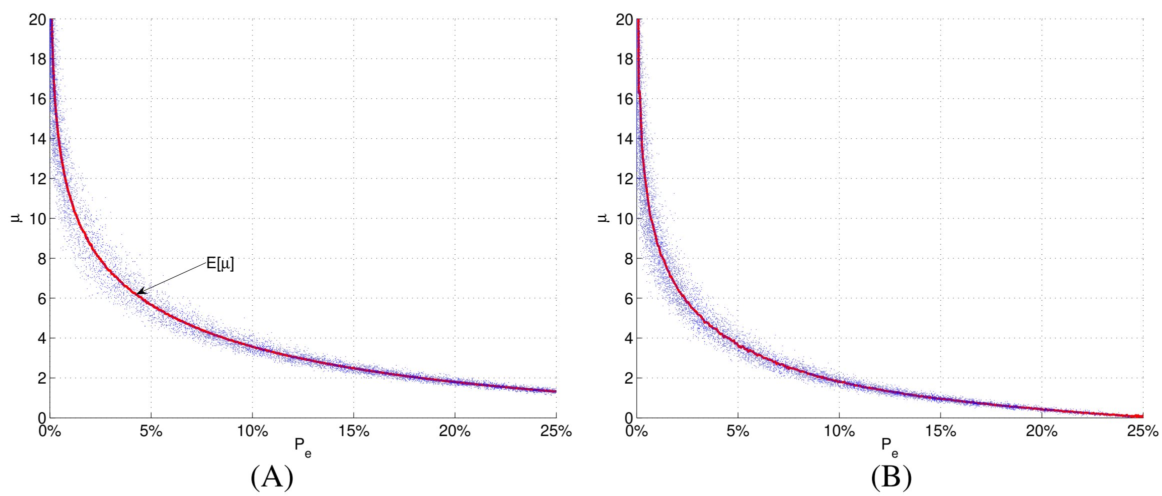


Fig. 3. The relation between the mean of the decoder LLRs, $\mu(LLR)$, and the percentage of errors in the block, p_e. (A) A $(3000, 2000)$ regular LDPC code with column weight 3; message-passing decoding. (B) A rate rate 1/2 systematic convolutional code with generator matrix $(1, 25/31)_{oct}$ and information block size 1000 bits; BCJR decoding.

The last issue concerns the decoding process at the destination in *Slepian-Wolf Cooperation*. By choosing the appropriate Slepian-Wolf code and the thresholds p_{th}, the destination is guaranteed to retrieve a diversity order of two for X. This can be implemented, for example, by Slepian-Wolf decoding of $packet_{X|Y} + packet_Y$ followed by decoding of $packet_X$. A more efficient way, however is to decode the three packets altogether as one codeword. This is possible, since $packet_X$ and $packet_Y$ both contain noisy copies of the source X (note that $packet_Y$ is essentially X passed through a cascade of a BSC(p) and a Rayleigh fading channel), and $packet_{X|Y}$ contains noisy parity bits for X. More detailed discussion is provided in Section IV-C.

IV. EXAMPLES

To demonstrate the generality and the efficiency of the proposed framework, we discuss in the below a few examples using convolutional codes and LDPC codes.

A. Convolutional Codes

Consider a rate 1/2 convolutional code with codeword size $n = 2k$. According to the parity approach, Slepian-Wolf coding can be performed by taking source X in blocks of length k and encoding them using the (n, k) convolutional code, where the parity bits P^{n-k} fulfill the role of $H(X^n|Y^n)$. In the context of user cooperation, the source will, in the first time slot, transmit $packet_{X|Y} = P^{n-k} = P^k$, the parity bits, together with $packet_X = X^k$, the systematic bits, which together forms a complete convolutional codeword (for simplicity, we assume X^k already contains CRC bits.) The relay will decode this convolutional code using, for example, the BCJR algorithm or the soft-output Viterbi algorithm (SOVA): $Y^n = \tilde{X}^n$.

- Upon successful decoding (i.e. $\tilde{X}^n = X^n$), the relay can scramble the source bits X^k, re-encode them using the same convolutional code, and forwards the new set of parity bits Q^{n-k}, to the destination. This *coded cooperation* strategy has essentially delivered a rate $k/(2n - k) = 1/3$ distributed turbo code to the destination, such that X^k can be efficiently recovered using an iterative turbo decoder.
- When the CRC check fails, the relay will estimate the percentage of the residual errors in the block by examining the average decoder LLRs (or using other estimation methods). If the error rate exceeds a threshold p_{th}, which is pre-defined and known to both the relay and the destination, then the relay will discard Y^n since it is badly corrupted and not worthy of further processing. This reduces to a *non-cooperative* mode. Otherwise, the relay decides that Y^n is highly correlated with X^n and invokes *Slepian-Wolf cooperation* by forwarding $packet_Y = Y^n$. The destination has now received two (noisy) copies of the systematic bits, $packet_X = X^k$ and $packet_Y = Y^n = \tilde{X}^n$, and one (noisy) copy of the parity bits, $packet_{X|Y} = P^{n-k}$, it can then perform convolutional decoding to recover X^n.

B. LDPC codes

In [1], examples of LDPC codes are discussed, where $H(X|Y)$ or $packet_{X|Y}$ is computed using the *binning approach*. We will quickly walk through the examples in [1], and show how the same "distributed codeword" can be generated using the *parity approach* in a conceptually simpler way.

SW Cooperation using the binning approach: Let us brief the conventional LDPC Slepian-Wolf formulation using the binning approach. Let $\mathbf{H}$ be the parity check matrix of an (n, k) LDPC code, $X^n \in \{0, 1\}^n$ and $Y^n \in \{0, 1\}^n$ be two memoryless binary symmetric sources with BSC(p_{th}) correlation, and S^{n-k} be the syndrome sequences of X^n, computed by matrix multiplication: $S^{n-k} = \mathbf{H}X^n$. Asymmetric SW coding is performed by compressing X^n to S^{n-k} at a rate of $(n-k)/n$ bit/symbol, and transmitting Y^n at full rate. Since the combination of X^n and S^{n-k} forms a valid codeword of an *extended* LDPC code with parity check matrix $[\mathbf{H}, \mathbf{I}]$, joint decompression is performed by feeding Y^n, the BSC(p_{th}) corrupted version of

X^n, and S^{n-k}, to the message-passing decoder of $[\mathbf{H}, \mathbf{I}]$ (assume the physical transmission channel is noiseless). The process is illustrated in Figure 4(A).

In the context of *Slepian-Wolf cooperation*, the source will broadcast, in the first time slot, $packet_X = X^n$, and $packet_{X|Y} = S^{n-k} = \mathbf{H}X^n$. The relay will estimate X^n by performing message-passing decoding on $[\mathbf{H}, \mathbf{I}]$. If the decoded data, $Y^n = \tilde{X}^n$, contains but less than p_{th} of errors, then the relay forwards $packet_Y = Y^n$. The destination will perform message-passing on $[\mathbf{H}, \mathbf{I}]$ using $packet_X = X^n$, $packet_Y = \tilde{X}^n$ and $packet_{X|Y} = S^{n-k}$.

Since the large number of weight-1 columns in $[\mathbf{H}, \mathbf{I}]$ makes message-passing rather inefficient, [1] proposed an improved LDPC Slepian-Wolf formulation. The idea is to restore the efficiency of the message-passing algorithm by making the parity check matrix possess all the desired features such as random, sparse, having a large girth and few small cycles. Instead of attempting to convert $[\mathbf{H}, \mathbf{I}]$ to a better matrix, which is technically challenging, [1] proposed to start with a new, good matrix $\mathbf{H}^*$ pertaining to a $(2n-k, n-k)$ LDPC code, to diagonalize $\mathbf{H}^*$ to $[\mathbf{P}, \mathbf{I}]$ (using Gaussian Elimination), and to use $\mathbf{P}$ (which may be dense) for computing the "syndrome" S^{n-k} and use $\mathbf{H}^*$ for message-passing decoding. This improved LDPC SW formulation is illustrated in Figure 4(B) and promises a better performance especially in *SW cooperation* [1].

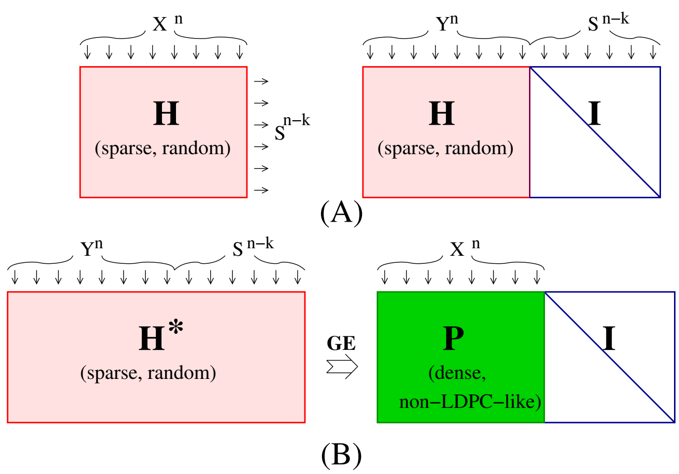


Fig. 4. LDPC Slepian-Wolf coding using the binning approach. (A) The conventional LDPC SW formulation. (B) The improved LDPC SW formulation.

SW Cooperation using the parity approach: Viewed from the perspective of the parity approach, the conventional LDPC binning approach shown in Figure 4(A) is like Slepian-Wolf compression using a $(2n-k, n-k)$ low-density generator-matrix (LDGM) code with parity check matrix $[\mathbf{H}, \mathbf{I}]$. LDGM codes are defined, in the coding literature, as a special class of linear-time encodable LDPC codes whose parity check matrix comprises a random sparse matrix on the left and an identity matrix on the right. Likewise, the improved LDPC binning approach shown in Figure 4(B) is like SW compression using a (typical) $(2n-k, n-k)$ LDPC code with parity check matrix $\mathbf{H}^*$. It is well recognized that random LDPC codes generally outperform LDGM codes, which agrees with the observation that the improved LDPC binning formulation exhibits gains over the conventional LDPC binning formulation.

C. Simulation Results

To demonstrate the efficiency of the proposed framework, we simulate the system performance on block Rayleigh fading channels. We assume that the source-destination channel and the relay-destination channel have the same average quality. Since the block size is typically limited to a few thousand bits in practical systems, we consider a $(3000, 2000)$ random LDPC code (denoted as $LDPC1$) and use the parity approach for Slepian-Wolf coding.

Using the strategy discussed previously, the source will transmit the complete LDPC codeword, with 2000 systematic bits in $packet_X$ and 1000 parity bits in $packet_{X|Y}$. The relay decodes the LDPC code, and uses CRC and $\mu(LLR)$ to compute p_e, the estimated percentage of errors in the block. In the simulation, we used a threshold $p_{th} = 5\%$. Hence, depending on whether $p_e = 0$, $0 < p_e \leq 5\%$ or $p_e > 5\%$, the system will witch respectively to *coded cooperation*, *Slepian-Wolf cooperation*, and *no cooperatoin*.

In the case of *coded cooperation*, the relay re-encodes the 2000 correctly decoded bits using a (different) $(4000, 2000)$ LDPC code (denoted as $LDPC2$), and transmits the new set of 2000 parity bits. The destination then combines $LDPC1$ and $LDPC2$ to form a $(5000, 2000)$ "layer" LDPC code and performs message-passing decoding.

In the case of *Slepian-Wolf cooperation*, the relay simply forwards the 2000 decoded bits (which contain less than 5% errors) in $packet_Y$, and the destination performs message-passing on $LDPC1$. It is crutial that the destination correctly computes the channel LLRs for each received packet, since channel mismatch could lead to disastrous error propagation! Let s be the data bit, r be the received signal, h be the fading coefficient of the Rayleigh channel, and σ^2 be variance of the additive white Gaussian noise (AWGN). Futher, let subscripts sd and rd denote respectively the source-destination channel and the relay-destination channel. $packet_X$ and $packet_{X|Y}$ have gone through a source-relay channel which is a Rayleigh fading channel; the channel LLRs of the data therein (systematic bits and parity bits) can be computed using

$$L_{packet_{X,X|Y}}(s) = \frac{2h_{sd}}{\sigma_{sd}^2} r. \tag{1}$$

$packet_Y$ contains systematic bits, which have traversed the source-relay-destination channel, or, a cascade of a BSC(p_e) and a Rayleigh fading channel. The channel LLRs should therefore be computed using

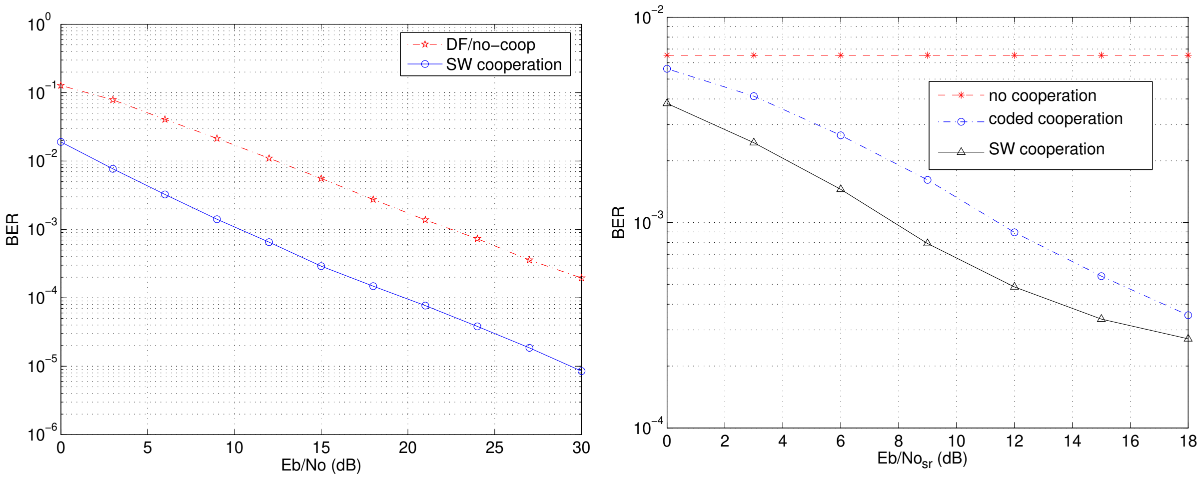
$$L_{packet_Y}(s) = \ln \frac{p_e + (1 - p_e)\mathrm{exp}\left(\frac{2\mathrm{h_{rd}r}}{\sigma_{\mathrm{rd}}^2}\right)}{(1 - p_e) + p_e\mathrm{exp}\left(\frac{2\mathrm{h_{rd}r}}{\sigma_{\mathrm{rd}}^2}\right)}. \tag{2}$$

The simulation results are presented in Figure 5. Figure 5(A) plots the bit error rate (BER) vs the normalized SNR for the case of $0 < p_e \leq 5\%$, i.e the "favorable" inter-user outage case where *Slepian-Wolf cooperation* is fully functional while *code cooperation* degenerates to *no cooperation*. We observe some 13 dB gain provided by *SW cooperation* over the conventional scheme in this favorable case! Note that this gain is evaluated after deducting the additional energy spent in transmitting $packet_Y$ in *SW cooperation*.

To provide a more accurate evaluation of the overall system gain enabled by *Slepian-Wolf cooperation*, we blend in the two other cases of $p_e = 0$ (i.e successful relay decoding) and $p_e > 5\%$ (i.e. severe errors at the relay) and plot in Figure 5(B) the average performance. The two user channels are fixed to a normalized SNR of 14 dB, and the inter-user channel changes from 0 to 18 dB. What is denoted as *coded cooperation* is essentially a mixture of *coded cooperation* ($p_e = 0$) and *no cooperation* ($P_e > 0$), and what is denoted as *Slepian-Wolf cooperation* is essentially a mixture of *coded cooperation* ($p_e = 0$), *SW cooperation* ($0 < p_e \leq 5\%$) and *no cooperation* ($P_e > 5\%$). Again the energy consumption has been normalized, and we observe a system gain of close to 4 dB enabled by *Slepian-Wolf cooperation*. Considering the rather small block size that is used and the extremely low complexity of *Slepian-Wolf cooperation* (virtually no additional complexity over the existing schemes), 4 dB of gain is quite impressive.

V. Conclusion

Slepian-Wolf cooperation, a simple and practical cooperative strategy that implements the idea of compress-and-forward is discussed in detail. We demonstrate, through the exploitation of practical Slepian-Wolf coding strategies and especially the parity approach, how a general (systematic) linear channel code may be exploited in *Slepian-Wolf cooperation* to efficiently combat the inter-user outage. The proposed framework assumes the same universal form as the existing decode-and-forward based schemes such as *coded cooperation*. It therefore provides the relay with a convenient freedom to switch between *no cooperation*, compress-and-forward (i.e. *SW cooperation*) and decode-and-forward (e.g. *coded cooperation*), on-the-fly, depending on the quality of the instantaneous inter-user channel.



(A) (B)

Fig. 5. (A) Performance of *Slepian-Wolf cooperation* in its favorable situation. The X-axis denotes the SNR of the source-destination and the relay-destination channel. (B) Comparison of *Slepian-Wolf cooperation*, *coded cooperation* and *no cooperation*. $E_b/No_{sd} = E_b/No_{rd} = 14$dB. The X-axis denotes the normalized SNR of the inter-user channel.

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