Is Amplify-and-Forward Practically Better than Decode-and-Forward or Vice Versa?

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Abstract— **This paper evaluates the relative merit of amplify-and-forward (AF) and decode-and-forward (DE), the two basic modes of cooperative communications, in practical systems. Specifically, the paper considers the case where the two user channels are slow fading with similar channel qualities and the inter-user channel is some 10 dB better. Through the evaluation of the excess information rate, the analysis of the worst-case error rate, and simulations using practical turbo codes, it is consistently shown that the two modes are practically on par with each other. Furthermore, the study points to inter-user outage as the detrimental factor, and location, rather than the specific cooperative strategy, as the key element in cooperative communications.**

I. Introduction

Spatial diversity techniques that can effectively mitigate the performance deterioration caused by fading, without imposing delay or bandwidth expansion, are clearly desirable from efficiency and system usability points of view. Spatial diversity is obtained when signals transmitted from antennas separated far enough to experience independent fading channels. Since it is not always practical to deploy multiple antennas at a mobile due to size or other constraints, user cooperation has been proposed [1]-[3], where multiple users share antennas which form a virtual antenna array. Aiming at increasing the channel capacity [1] and/or decreasing the outage probability [2], several interesting two-user cooperative protocols have been proposed, among which are the two basic modes: amplify-and-forward (AF) and decode-and-forward (DF).

The performance of the two modes has been the interest of research for a while. First, a simple diversity-vs-multiplexing analysis conducted in [1] indicates that the AF mode is superb than the DF mode with the same multiplexing gain but a better diversity order. However bit error rate (BER) simulation results reported later on suggest the opposite [4]. More recently, egordic capacity evaluations of the "compound" cooperation channel [5] that AF and DF can each outperform the other depending on the underlying channel condition. Specifically, it points to the inter-user channel as the determining factor that cuts the capacity region in two: when the inter-user channel is statistically worse than the two user channels, AF offers a higher capacity; otherwise, DF does. The above results have certainly sheded useful light on the relative merit of the two modes, but how well does the theory (e.g. capacity result) translate to practical gains?

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The purpose of this paper is to understand which mode is *practically* better, or how much of the gains promised by the theory can be realized in practice and how. Since user cooperation is most useful when channels are varying very slowly (i.e. hard to obtain time diversity in a single user channel), from now on we will consider all channels as block fading. To begin, note that in practice, the inter-user channel tends to be a much nicer channel than either of the user channels (since it is natural for a mobile user to partner with one that is close by). Thus, using the results in [5] arrive at a crude engineering rule that DF should be the default mode performance-wise. However, despite the *statistically* high quality of the inter-user channel, there will be a small probability where the *instantaneous* inter-user channel is so noisy that it prevents the package from getting to the relay node reliably. In such an outage case, the relay node is helpless in the DF mode, but can at least amplify and forward the (noisy) package in the AF mode. This suggests a need to conditionally turn to the AF mode. Hence, the one question that confronts us is: shall we adopt a mixed DF-AF scheme, switching to DF when the relay node correctly demodulates/decodes the packet and AF otherwise?

In the succeeding sections, we will answer this question by quantifying the performance of these strategies in practical scenarios analytically and experimentally. We adopt an *error probability* approach, in addition to mutual information, since it best reflects the practical concern. Rather than focusing on the best case (i.e. when cooperation is successful), we consider all possible scenarios and the worst case in particular. An especially refreshing result we obtained is that, when the inter-user channel is at outage, (the lower bound of) the error rate of the AF and the DF modes differ only by a factor of 2 (approximately) for a fixed information rate. This analytical result is also verified by simulations using turbo codes. Hence, if we agree that the (average) performance of a system is dominated by the worst case, then we come to another interesting result: AF and DF are practically the same. Meaning, it does not make much difference in performance using one mode or the other, and there is certainly no practical benefit in considering a mixed-mode system (since the gain is not worth the trouble).

II. System Model and Preliminaries

A. System Model

We consider two-user symmetric cooperation in uplink wireless transmission. Among the various possible strategies of user cooperation (e.g. [1][4]), we consider the simplest type where, after the source user transmits a package in one time slot, its partner relays the package in the next time slot, and the destination combines both packages and makes a joint decision. Since only the relay will transmit (if at all) at the second time slot (i.e. no concurrent transmission), there is no concern for inter-user synchronization, which makes the system simple and practical.

Let “home channel”, “inter-user channel” and “relay channel” denote the channels between the source and the destination, the source and the relay, and the relay and the destination, respectively. Let h_{SD}, h_{SR} and h_{RD} denote the respective path gain. The general form of a signal received over a specific channel at time t is given by,

$$r(t) = \sqrt{E_s}h(t)c(t) + n(t), \tag{1}$$

where E_s is the signal energy, $h(t)$ the Rayleigh distributed path gain, and $n(t)$ the additive white Gaussian noise (AWGN). We consider block fading channels, where $h(t)$ remains constant for the duration of one round of user cooperation (4 consecutive time slots). Between channels and cooperation rounds, $h(t)$'s are independent. Further, we assume that the home channel and the relay channel have the same average signal-to-noise (SNR) ratio, while the inter-user channel is, for example, 10 dB better.

B. Cooperative Modes

1) Input-Output Relation of the AF Mode: As the name suggests, in the AF mode, the relay node simply amplifies the received signal and forwards it to the base station. Here we assume that the power of the signal retransmitted at the rely node is scaled uniformly with respect to all the bits in the package, such that the average (re-)transmission energy per signal equals E_S. In time slot 1, the signals received at the relay and the destination are

$$\begin{aligned} y_{R,1} &= \sqrt{E_S}h_{SR}x_1 + n_0, & (2)\\ y_{D,1} &= \sqrt{E_S}h_{SD}x_1 + n_{1,D}. & (3)\end{aligned}$$

where n_0 and $n_{i,D}, i = 1, 2$ denote the zero-mean complex AWGN at the inter-user channel and home channel with variances equal to $N_1/2$ and $N_0/2$ per dimension, respectively.

During time slot 2, the equivalent signal to be retransmitted by the relay contains a unit average power:

$$x^{AF}_{r,2} = \sqrt{\frac{E_S|h_{SR}|^2}{E_S|h_{SR}|^2 + N_1}}x_1 + \sqrt{\frac{N_1}{E_S|h_{SR}|^2 + N_1}}n_0 \tag{4}$$

The relay signal received at the destination is given by [5]

$$\begin{aligned} y^{AF}_{D,2} &= \sqrt{E_s}h_{RD}x^{AF}_{r,2} + n_{2,D}, \\ &= h_{RD}h_{SR}\sqrt{\frac{E_S^2}{E_S|h_{SR}|^2 + N_1}}x_1 + \tilde{n} \qquad (5)\end{aligned}$$

where $\tilde{n}$ is a zero mean complex Gaussian noise with variance of $(\frac{N_0}{2} + \frac{N_1^2|h_{RD}|^2E_s}{2(E_S|h_{SR}|^2+N_1)})$ per dimension. The destination combines $y_{D,1}$ and $y^{AF}_{D,2}$ using maximal ratio combination rule before decoding.

2) Input-Output Relation of the DF Mode: The signal transmission of the DF mode in the first time slot is the same as that of the AF mode (see 2 and 3).

During time slot 2, the relay first demodulates and decodes the received signal. Upon success, it re-encodes the data (possibly using a different code) and forwards it to the destination. Hence, the destination receives

$$y^{DF}_{D,2} = \sqrt{E_s}h_{RD}x_{r,2} + n_{2,D} \tag{6}$$

In the outage case where the relay fails to decode the data correctly, it cannot help its partner for the current cooperation round. It may select to either stay silent (to save energy) or transmits its own data (to improve the channel utilization) [4]. To ease the analysis, we will assume the former in the next Section. However, our simulation in Section IV will show that to be idle is in fact not a bad choice, since the performance improvement brought by the latter is so minor that it is not worth the additional energy consumed.

III. Performance Comparison between AF and DF

A. Achievable Information Rate

Assume that perfect channel side information, h_{SD}, h_{SR} and h_{RD}, are available at the respective destination. For the AF mode, it is easy to see that the achievable (instantaneous) information rate is upper bounded by the (instantaneous) mutual information of the compound channel[1] [1]:

$$R^{AF} \le I^{AF} = \frac{1}{2}log_2(1 + \|\gamma\|^2) \ \ \text{bit/s/Hz} \tag{7}$$

where

$$\gamma = \Big[\underbrace{\frac{\sqrt{E_s}|h_{SR}|}{\sqrt{N_1}}}_{\text{1st time slot}}, \underbrace{\frac{E_s^{3/2}|h_{RD}h_{SR}|}{\sqrt{E_S|h_{RD}|^2N_1 + (E_S|h_{SR}|^2+N_1)N_0}}}_{\text{2nd time slot}}\Big]$$

For the DF mode, we first note that the quality of the inter-user channel plays a key role. Define $R_0 \triangleq \frac{1}{2}log_2(1 + \frac{E_S}{N_1}|h_{SR}|^2)$. When the inter-user channel is at outage, i.e. $R \ge R_0$ (R is the instantaneous information rate), then the relay channel cannot be utilized, and consequently the achievable information rate is determined by the quality of the home channel $\frac{1}{2}log_2(1 + \frac{E_S}{N_0}|h_{SD}|^2)$. Otherwise, since the relay obtains a correct copy of the data, it can convey at least part of this information to the destination. As such, the information rate is limited by $\min\{\frac{1}{2}\mathrm{log_2}(1 + \frac{E_S}{N_1}|h_{SR}|^2), \frac{1}{2}\mathrm{log_2}(1 + \frac{E_S}{N_0}(|h_{SD}|^2 + |h_{RD}|^2)\}$. Hence, to summarize, the (instantaneous) information rate for the DF mode is upper bounded by

$$R \le \begin{cases} \frac{1}{2}log_2(1+\frac{E_S}{N_0}|h_{SD}|^2), \ \text{if} \ \frac{|h_{SR}|^2}{N_1} \le \frac{|h_{SD}|^2}{N_0}, \\ \frac{1}{2}log_2(1+\frac{E_S}{N_1}|h_{SR}|^2), \ \text{if} \ \frac{|h_{SD}|^2}{N_0} < \frac{|h_{SR}|^2}{N_1} \le \frac{|h_{SD}|^2+|h_{RD}|^2}{N_0}, \\ \frac{1}{2}log_2\big(1+\frac{E_S(|h_{SD}|^2+|h_{RD}|^2)}{N_0}\big), \ \text{if} \ \frac{|h_{SR}|^2}{N_1} < \frac{|h_{SD}|^2+|h_{RD}|^2}{N_0}. \end{cases}$$

By averaging the above mutual information results on the distribution of the channel gains, h_{SD}, h_{RD} and h_{SR}, capacities can be obtained for both AF and DF modes. Table I lists

[1]There is a factor $\frac{1}{2}$ in all the mutual information results, since two consecutive time slots are used for each package.

the *excess* information rate the DF mode has over the AF mode for the case when the inter-user channel is 10 dB better than either of the user channels. It is quite striking that that excess information rate is quite small and remains (near) invariant as the quality of the user channels changes. This seems to suggest that AF and DF are comparable at a wide range of user channel condition. However, since capacity results are most relevant to the cases when inter-block coding using very long and powerful codes or adaptive-rate transmission is employed, how well does this match to practical systems with fixed-rate transmission and short block sizes? For further insight into practical performances, we turn to error rate analysis based on outage events, as well as simulations using practical codes.

TABLE I

EXCESS INFORMATION RATE OF DF OVER AF

SNR(dB)	10	15	20	25	30	35
Excess(bit/s/Hz)	0.118	0.127	0.130	0.130	0.131	0.133

B. Worst-Case Error Rate

Let us first consider the worst case scenario, where the relay-channel is at outage. Assume that a capacity approaching channel code is used.

In the DF mode, since only the source has delivered a copy of the package where the instantaneous SNR at the destination is $E_s|h_{SD}|^2/N_0$, decoding error happens when the instantaneous home channel cannot support the the instantane transmission rate R, i.e. $|h_{SD}|^2/N_0 \leq 2^{2R}-1$. Hence, error rate at outage events is given by (note $R \geq R_0$ $log_2(1+\frac{E_S}{N_1}|h_{SR}|^2)$):

$$P^{DF}(\varepsilon|\text{outage}) \geq \Pr\left(\frac{E_S}{N_0}|h_{SD}|^2 \leq 2^{2R}-1\right),$$
$$= \int_0^{N_0(2^{2R}-1)/E_S} f_{|h_{SD}|^2}(x)dx,$$

where $f_{|h_{SD}|^2}$ is the probability density function (pdf) c central Chi-Square distribution with 2 degrees of freedom

In the AF mode, the effective SNR for the second time slot (the concatenation of the inter-user channel and the relay channel) is given by

$$\frac{E_S^2|h_{RD}|^2|h_{SR}|^2}{N_1E_S|h_{RD}|^2+(E_S|h_{SR}|^2+N_1)N_0} \leq \frac{E_S|h_{SR}|^2}{N_1}. \quad (9)$$

Since the instantaneous SNR of the inter-user channel is low (i.e. at outage), it is safe to assume that the effective SNR is dominated by the inter-user channel (i.e. the relay channel is good with a high probability and the left hand side of (9) approaches the right hand side). Joint decoding error thus happens when the combined SNR of the first time slot and the second time slot cannot support the instantaneous data rate:

$$P^{AF}(\varepsilon|\text{outage}) \geq \mathrm{P}\left(\frac{\mathrm{E_S}}{\mathrm{N_0}}|\mathrm{h_{SD}}|^2+\frac{\mathrm{E_S}}{\mathrm{N_1}}|\mathrm{h_{SR}}|^2 \leq 2^{2\mathrm{R}}-1\right),$$
$$= \int_0^{(2^{2R}-1)/E_S} f_{|h_{SD}|^2/N_0+|h_{SR}|^2/N_1}(x)dx, \quad (10)$$

where the pdf of $(\frac{|h_{SD}|^2}{N_0}+\frac{|h_{SR}|^2}{N_1})$ can be obtained by convolving the the pdf's of two central Chi-Square distributions.

Using numerical evaluations, we can obtain the (frame) error rate of the two modes when the inter-user is at outage. To help illustrate, instead of plotting the actual error rates in y-axis in Fig. 1, we plot the ratio of the error rates: $P^{AF}(\varepsilon|\text{outage})/\mathrm{P}^{\mathrm{DF}}(\varepsilon|\text{outage})$. The x-axis denotes the SNR of the home channel and the inter-user channel is always 10 dB higher. It is remarkable to observe that, for the region of practical interest, i.e. $10-40$ dB, $P^{DF}(\varepsilon|\text{outage})$ scales linearly with $P^{AF}(\varepsilon|\text{outage})$ by a factor of approximately 2. Such a result makes clear the fact that (1) DF does not perform much differently from AF in such worst case scenarios, and (2) there is no practical need to conditionally switch to AF. This makes intuitive sense, since when the relay fails to decode, the packet has already experienced a severe fade on the inter-user user channel, and further distortion induced by the relay channel could easily wipe out its usefulness completely.

Furthermore, if we agree that the (average) performance of a system tends to be dominated by the worst case, then the above result also indicates that the performance of AF and DF could be on par with each, provided that such worst cases are not too rare. For this, we tested systems coded by practical convolutional/turbo codes. For an inter-user channel SNR of 17-19 dB, simulations show that worst cases happen at a probability of 4%-5%, which is certainly nonnegilible.

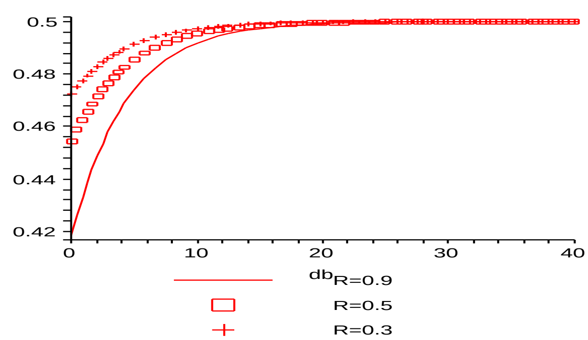

Fig. 1. Ratio of error rates between AF and DF at inter-user outage.

IV. PRACTICAL CODED SYSTEMS

Note that the above analysis in (10) and (8) assumes an optimal channel code. Will the result still hold if practical, imperfect codes are used? This section answers the above question by simulating practical systems.

We use a similar turbo coding strategy as in [6], i.e. a turbo code is used across the two segments of the transmission. This specific DF mode is also known as coded cooperation [4]. At the first time slot, the source encodes and sends the data using a (2000,1000) recursive systematic convolutional (RSC) code. For the DF mode, upon successful decoding, the relay scrambles the data before re-encoding them using the same RSC code. The two segments thus form a rate 1/4 turbo code with 2 copies of the systematic bits at the destination. For the AF mode, the relay simply repeats the initial RSC codeword, which results in two copies of the same RSC codeword (or a convolutional-repetition code) at the destination.

For a thorough understanding, let us first examine the bit error rate (BER) conditioned on different cooperative situations. Such an exercise is particularly useful for the DF mode, since depending on the inter-user channel condition and the cooperative rule, the relay could either sit idle, transmit its own data or relay for its partner [4]. This brings up 4 possible reception combinations in a symmetric DF cooperation cycle (see Fig. 2): (1) the destination receives only one copy of the package from the home channel, (2) the destination receives two copies, but both are from the home channel, (3) the destination receives two copies that are respectively from the home channel and the relay channel (marked with "desired"), and (4) the destination receives three copies, two of which from the home channel and third from the relay channel. Clearly, except for the third case which represents a successful cooperation, all the others cases correspond to a possible consequence of the inter-user outage. For the AF mode, regardless of the inter-user outage, the destination will receive two package copies, each from a different user channel. Nevertheless, to be in parallel with the DF mode, we have separately evaluated the two cases. Several interesting observations are made. First, when the inter-user channel is at outage, all strategies of the DF mode perform badly due to the lack of diversity; and the AF mode performs equally poorly, although it presumably has a higher diversity order. This result matches well with the analysis in Section III. (The factor of 2 in the BER analysis is not observable here, probably due to the fading on the relay channel and the imperfectness of the code.) Second, when the inter-user is not at outage, we see that DF outperforms AF (solid curves) but not at a remarkable degree. It should also be noted that part of this gain may be due to the strength of turbo codes (in the DF mode) over convolutional-repetition code (in the AF mode). Third, when comparing the two sets of curves: "DF - 1 copy" vs "DF - 2 copies", and "DF - desired" vs "DF - 3 copies", we observe that the performances are marginally different between the peers. This clearly states the importance of the spatial diversity rather than the channel utilization, i.e. transmission without additional diversity gain is useless. It also suggests that the strategy for the relay to send its own data at inter-user outage is unattractive as it consumes more energy.

To form an absolute basis for comparison, Fig. 3 plots the overall BER curves where the BERs are computed by weighted summation of the relevant BER curves in Fig. 3 and the weight coefficients are determined by the outage probability of the inter-user channel obtained in simulations. Three strategies are considered, the AF mode and the DF mode where the relay sits idle (marked with "DF-passive") and transmits its own package (marked with "DF-active"). As expected, the average performances of the two modes are very similar, with the DF mode being slighter better (about 1 dB). We say then that DF and AF are practically the same.

V. Conclusion

We have taken an error probability approach, in addition to mutual information, to evaluate the performance of AF and DF modes in practical situations. We observe that outage events at the inter-user channel are not as rare as can be safely ignored. At inter-user outage, the performance of AF and DF modes are not much different and are both pretty bad. This in turn leads to a comparable average performance between the two modes (with DF being slightly better than AF), which also matches pretty well with the capacity results.

This result is interesting. It suggests that the results obtained by looking at only the successful cooperation case, as in most existing papers, are partial and overly optimistic. More importantly, it points to location, rather than the specific cooperative strategy, as the key to achieve a good cooperative diversity. In other words, to fully realize the diversity order promised by the theory, a mobile node should probably partner with one that is as close as possible (thus making the inter-user channel Gaussian like). Finally, when viewed from the multi-hop routing diversity point of view, this indicates that the first hop is more important than all subsequent hop(s).

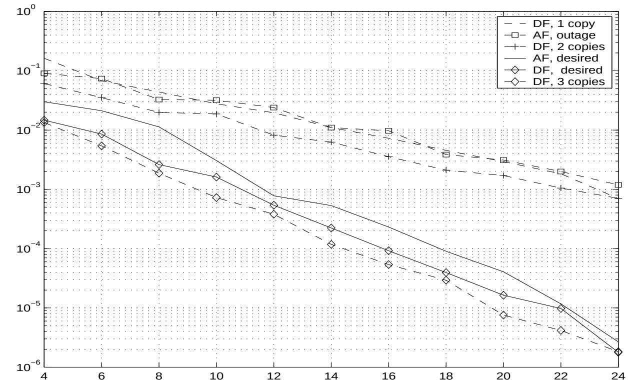

Fig. 2. Conditional performance for different cases of the AF and DF mode.

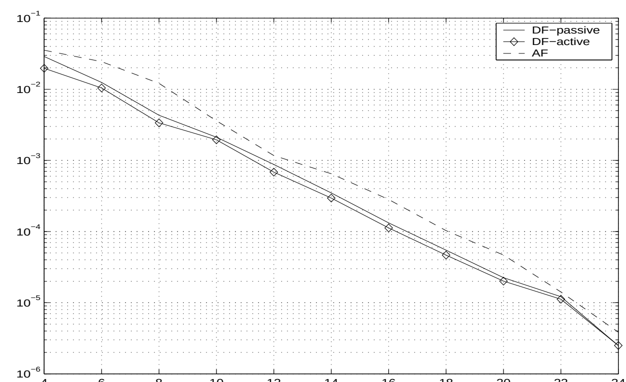

Fig. 3. Average performance of the AF and DF mode.

References

[1] N. Lanenan, "Cooperative Diversity in Wireless Networks: Algorithms and Architectures," *Ph.D. dissertation,* Massachusetts Institute of Technology, Cambridge, MA, Aug. 2002.

[2] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity-part I:system Description," *IEEE Trans. on Commun.*, pp. 1927-1938, Nov 2003.

[3] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity-part II:implementation aspects and performance analysis," *IEEE Trans. on Commun.*, pp. 1939-1948, Nov 2003.

[4] T. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," *IEEE Trans. on Signal Processing*, pp 362-372, Feb 2004

[5] R.U.Nabar, H.Bolcskei and F.W. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Selected Area.*, Aug 2004

[6] Valenti, M.C.; Bin Zhao, "Distributed turbo codes: towards the capacity of the relay channel," *Vehicular Technology Conference, Oct 2003*