Thresholds for Iterative Equalization of Partial Response Channels Using Density Evolution

Jing Li, Krishna R. Narayanan, Costas N. Georghiades
Electrical Engineering Department
Texas A&M University, College Station, TX 77843-2138
e-mail: `{jingli,krishna,georghia}@ee.tamu.edu`

Erozan Kurtas
Seagate Technology, Seagate Research
2403 Sidney St., Pittsburgh, PA 15203
e-mail: `erozan.m.kurtas@seagate.com`

Abstract — **A unified framework is presented to calculate the thresholds of different outer codes, including serial turbo, low density parity check codes, single-parity check turbo product codes, on partial response channels using density evolution with Gaussian approximation.**

Low density parity check (LDPC) codes, single-parity check turbo product codes (TPC/SPC) and convolutional codes (also known as the serial turbo) have demonstrated impressive coding gains over partial response (PR) channels using iterative equalization and decoding. To facilitate the understanding and to optimize the above systems, we investigate their thresholds using Density evolution (DE) with Gaussian approximation (GA) [1]. The systems studied take a unified model where the (precoded) PR channel is viewed as a rate-1 convolutional code to form a serial concatenation with the outer code (Fig. 1). Unlike TPC/SPC and serial turbo cases where precoder and random interleaver are used for interleaving gain, LDPC codes do not need either the precoder nor the interleaver [2].

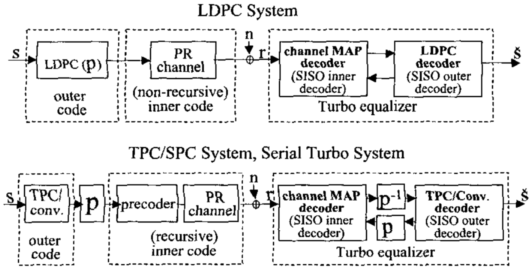

Figure 1: System model

By examining the distribution of messages and calculating the average amount of incorrect messages passed in each step, an SNR threshold value can be determined using density evolution, above which the fraction of incorrect messages vanishes as iteration goes to infinity. By assuming that messages follow a Gaussian distribution, and by enforcing the *consistency condition* [1], it follows that the variance of the message equals twice the mean. Thus, only the mean needs to be examined. This greatly reduces the complexity with minor sacrifice in accuracy [1]. We need to track the messages passed from the inner to the outer code ($m_i^{(q)}$) at the q_{th} iteration and vice versa ($m_o^{(q)}$). For MAP decoding of convolutional codes, Monte Carlo simulations are used to evaluate the mean. For regular LDPC codes of column weight s and row weight t, the mean of messages is examined after l rounds of check-to-bit and bit-to-check updates. For $((K_1+1)(K_2+1), K_1K_2)$ TPC/SPC codes, the mean is examined after 1 row update and 1 column update.

$$\textbf{LDPC:}\ \text{bit-to-check}: \quad m_b^{(q,l)} = m_i^{(q)} + (s-1)\cdot m_c^{(q,l-1)}, \quad (1)$$

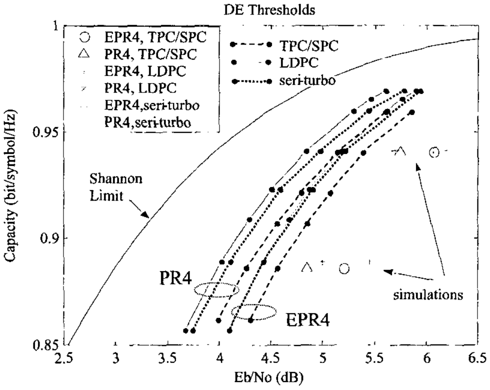

Figure 2: Thresholds (calculated using DE method)

$$\text{check-to-bit}: \quad m_c^{(q,l)} = \psi^{-1}([\psi(m_b^{(q,l)})]^{t-1}), \quad (2)$$

$$\text{LDPC-to-MAP}: \quad m_o^{(q)} = s\cdot m_c^{(q,L)}. \quad (3)$$

$$\textbf{TPC/SPC:}\ \text{row-code}: \quad m_{c_1}^{(q)} = \psi^{-1}(\,[\psi(m_i^{(q)})]^{K_1}), \quad (4)$$

$$\text{column-code}: \quad m_{c_2}^{(q)} = \psi^{-1}(\,[\psi(m_i^{(q)} + m_{c_1}^{(q)})]^{K_2}), (5)$$

$$\text{TPC/SPC-to-MAP}: \quad m_o^{(q)} = m_{c_1}^{(q)} + m_{c_2}^{(q)}. \quad (6)$$

where $m_c(q,0) = 0, \forall q$, and $\psi(x)$ is defined as: $\psi(x) = \frac{1}{\sqrt{4\pi x}}\int_{-\infty}^{\infty}\tanh(\frac{u}{2})e^{-\frac{(u-x)^2}{4x}}du$, for $x > 0$ and 0 otherwise.

We study PR4 and EPR4 channels and high rate codes with data block size 4K bits for use in high-density magnetic recording systems.

Choice of L – Each turbo iteration involves 1 round of channel MAP decoding followed by a maximum of L rounds of bit-check/check-bit updates in LDPC decoder. Large L leads to better thresholds but involves more complexity. By visualizing the plot of L vs corresponding thresholds, we found $L = 4$ or 5 to be a good tradeoff for EPR4 channels and $L = 7$ or 8 for PR4 channels.

Analytical Bounds – Fig. 2 shows the thresholds and the simulation results of LDPC, TPC/SPC and serial turbo over PR4/EPR4 channels. Both the serial turbo and the TPC/SPC systems use the precoder $1/(1 \oplus D^2)$. Due to the finite block size, the simulations are around 0.5 to 1 dB away from the bounds.

REFERENCES

[1] S.-Y.Chung, R. Urbanke and T. J. Richardson, "Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation", submitted to *IEEE Trans. Info. Theory*

[2] J. Li, E. Kurtas, K. R. Narayanan, and C. N. Georghiades, "On the performance of turbo product and LDPC codes over partial-response channels", to appear *Proc. Intl. Conf. Commun.*, Finland, June, 2001

0-7803-7123-2/01/$10.00 ©2001 IEEE