A Class of Linear-Complexity, Soft-Decodable, High-Rate, "Good" Codes: Construction, Properties and Performance

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Abstract — **A new class of high-rate, low-complexity and soft-decodable codes, named *product accumulate* codes, is proposed. The code structure, properties, thresholds and performance are investigated and it is shown that these codes are capable of near-capacity performance.**

I. Introduction

Single-parity check turbo product codes (TPC/SPC) have many appealing properties like intrinsic high-rate, simplicity and soft-decodability. However, a 2-dimensional (2-D) TPC/SPC code has a fixed minimum distance of 4 regardless of block size, and, hence, is not "good"[1]. In this work, we propose the interleaved concatenation of a 2-D TPC/SPC code with a rate-1 recursive inner code, $1/(1+D)$, and name it *product accumulate* (PA) code. The new class of codes are "good"in the maximum likelihood (ML) sense and under iterative decoding. Thresholds calculated using density evolution are within a few tenths a dB away from the Shannon limit, and simulation results confirm this.

II. Structure, Properties and Performance

The encoder and decoder model of PA codes is shown in Fig. 1. It is worth mentioning that several codewords of the turbo product code based on single parity check codes (TPC/SPC) are combined and jointly interleaved before passing to the accumulator. Further, we have replaced the block interleaver between the two levels of SPC codes (in the outer TPC/SPC code) with a random interleaver to improve the distance spectrum. Thus the outer code becomes a parallel concatenation of 2 SPC branches.

Some properties of the proposed codes are:

Linear time encodable and linear time decodable – Encoding of PA codes is linear. Decoding of PA codes is via an iterative approach that exchanges soft information between the outer and inner sub-decoders [2], both of which have linear complexity. In particular, the inner code can be decoded using an efficient message-passing algorithm, which is equivalent to the BCJR algorithm but with much lesser complexity [2].

"Good" in the ML sense – Using the techniques developed by Benedetto *et al* [3], we can show that the interleaving gain of PA codes is proportional to P, the number of SPC codewords combined in each branch [2].

"Good" in the iterative sense – We show that PA codes are also "good" using iterative decoding by computing the thresholds (SNR) above which error rates goes to zero. The thresholds are computed using density evolution with Gaussian approximation [4] [2]. Fig. 2 plots the thresholds along with the simulation results. The thresholds are within a few tenths a dB away from the Shannon limit and the simulations are close to the thresholds for $R \geq 1/2$, both of which are very impressive.

[1]A "good" code is defined as a code for which there exists a threshold above which an arbitrary low bit error rate (BER) can be achieved as block size $N \to \infty$ [1].

III. Conclusion

A class of high-rate provably good codes is proposed. In addition to high performance and low complexity, *Product accumulate* codes are simple to construct and have a very regular structure, which make them very appealing for implementation.

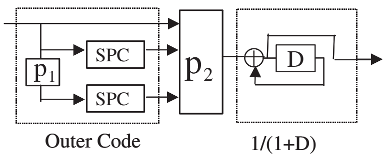

Figure 1: System model of PA codes

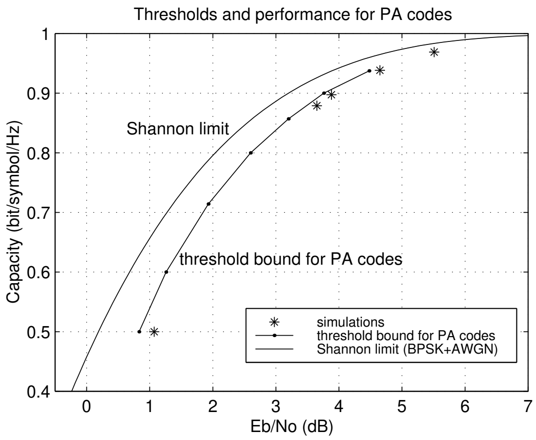

Figure 2: Thresholds and performance of PA Codes

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