An Information Theoretic Analysis for Adaptive-Network-Coded-Cooperation (ANCC) in Wireless Relay Networks

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***Abstract*— We conduct an information theoretic analysis for the adaptive-network-coded-cooperation (ANCC) protocol over large wireless relay networks. The ANCC protocol adaptively encodes the data from different terminals using a single network code by matching the instantaneous network topology with the code graph of a low-density-parity-check (LDPC) code. The ergodic capacity and the outage probability of this protocol are analyzed for both finite and infinite network sizes, and closed-form expressions are derived for the infinite case. Comparison with the existing protocols including repetition and space-time-coded-cooperation confirm that ANCC is both low-complexity and high-performance.**

I. Introduction

User cooperation is an effective technique to combat (slow) channel fading in single-antenna multi-user wireless systems, where the transmit terminals are allowed to share antennas to form a virtual antenna array. Early researches focus on the protocols for the 3-terminal basic scenario, such as repetition, coded-cooperation and space-time-coded-cooperation (STCC) [1]. Motivated by the potential to achieve a larger diversity gain, user cooperation in the context of large wireless networks is gaining increasing interests [2][3]. However, existing schemes do not scale well, and are therefore inefficient, expensive or wasteful to operate in a large network. For example, the repetition protocol discussed in [2] becomes intolerably bandwidth inefficient as the network size increases. Space-time-coded-cooperation requires inter-user synchronization at the symbol level [2], which is technically challenging especially among a large number of distributed terminals.

With the recent advances in cross-layer designs, network coding [7][8], an extension and generalization of routing, begins to find it way to user cooperation, an area predominated by physical layer technologies. A number of papers emerged recently, proposing network-coding-assisted cooperation protocols [3]-[6]. Most of them, however, use *predefined, fixed* network codes [3]-[5], with the ideal assumption that the transmission link between any two cooperating users is free from outage. Since practical networks consisting of wireless fading channels are subject to random link failure and topology change, fixed network codes will therefore break. For example, when a relay fails to retrieve a packet needed for its designated coding operation, the entire network code will have to abort. A natural remedy to this critical issue is *adaptation*. Having a set of network codes, each mapped to a possible network topology, is a possibility, but works only for small networks. A more practical and efficient solution is to allow network codes to be generated distributedly, adaptively and on-the-fly, as is proposed by the *adaptive-network-coded-cooperation* (ANCC) protocol in [6]. The key idea here is to couple *networks-on-graphs*, i.e. instantaneous network topologies described in graphs, with the well-known class of *codes-on-graphs*, i.e. low-density-parity-check (LDPC) codes and LDPC-like codes [6].

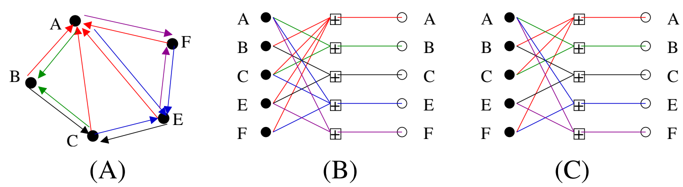

Fig. 1. (A) A network graph describing network topology. (B) The corresponding bipartite code graph. (C) A thinned code graph.

To explain how ANCC works, let us take a simple but generalizable example where 5 users A, B, C, E, F communicate to a common destination D. For convenience, we use "packet" and "symbol" interchangeably. In the first phase, each user broadcasts its wireless data. Assume that for the time being, the inter-user connectivity is shown in Figure 1(A), where a directed edge indicates a successful transmission (destination D not shown). Without much manipulation, we can transform this 5-node network graph to a bipartite code graph that specifies a $(10, 5)$ LDPC-like code, as illustrated in Figure 1(B). In the code graph, the 5 black circles, 5 boxes and 5 white circles represent the source symbols originated from the 5 users, the network coding operations to be performed by these users, and the relay symbols to be forwarded by these users, respectively. Hence, in the second phase, each user can compute and forward the check-sum of its correctly-decoded symbols, in a de-centralized and adaptive manner. A small bit-map field will be included in each relay packet, so that the destination knows how the checks are formed and can correspondingly replicate the code graph and perform message-passing decoding.

To make the message-passing algorithm effective, instead of performing the check sum on *all* the decoded symbols, each node can (randomly) pick only *a few* symbols, thus "thinning" the code graph and eliminating the chances for short cycles. For example, in Figure 1(A), user A and user E may ignore the source symbol from user E and user C when performing their respective coding operation. The new code graph, which now happens to be free of length-4 cycles, is shown in Figure 1(C).

The efficiency and practicality of ANCC is initially demonstrated in [6] by simulations. This paper performs information-theoretic analysis in terms of both capacity and outage. For comparison purpose, we also analyze repetition-cooperation

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and STCC. We test the impact of the number of users, and evaluate the scalability of these protocols with network size. In the boundary case where the network size goes to infinity, closed-form capacity and outage expressions are derived for all the three protocols. Our results provide a strong theoretic support for the excellent performance of ANCC.

The remainder of the paper is organized as follows. Section II presents the system model. The capacity and outage for ANCC, STCC and repetition are analyzed in Section III, IV and V, respectively. Numerical evaluation is discussed in Section VI. Finally, Section VII concludes the paper.

II. System Model

We consider m terminals transmitting data to a common destination. The transmission can be divided into two phases.

In the first phase (broadcasting phase), the terminals broadcast their data in a round Robin fashion. In the second phase (relaying phase), the terminals help forward each other's data using either repetition-cooperation, STCC or ANCC.

We assume that all the transmission channels follow block fading model with channel fading coefficient α and additive complex Gaussian noise Z with zero mean and variance N_0. We model α as zero-mean, independent, circularly symmetric complex Gaussian random variables with unit variance, so the magnitudes $|\alpha|$ is Rayleigh distributed, and the channel power $u = |\alpha|^2$ is exponentially distributed with probability density function (pdf)

$$p_u(y) = e^{-y}. \qquad (y > 0) \tag{1}$$

The sum of n independent channel powers satisfies $\chi-square$ distribution with pdf

$$p_\chi(y) = \frac{y^{n-1}}{(n-1)!}e^{-y}. \qquad (y > 0) \tag{2}$$

The transmit SNR γ is defined as $\gamma = P/N_0$, where P is the average power constraint of each terminal.

We assume that the fading coefficient α is known to the receivers but not the transmitters. This implies that instantaneous power adaptation among different users or over different time slots is not possible. Let decode-set, $D(i)$, denote the set of terminals that can successfully detect terminal i's data, and subscript (i, d) denote the channel from terminal i to the destination.

III. Adaptive-network-coded-cooperation (ANCC)

In the ANCC protocol, in the second phase, each terminal (randomly) selects a small number of others' data that have been correctly decoded, adds them together in the binary domain, and transmits them to the destination. At the destination, the data from all the terminals in both transmission phases form a systematic codeword, where the data transmitted in the broadcasting phase are taken as the systematic bits, and the data transmitted in the relaying phase are considered as the parity bits. Thus, all the terminals' data are encoded into a single codeword of some network code in a distributed manner. Unlike STCC, in ANCC, the synchronization among different terminals is only needed at the packet level but not the symbol level, which makes ANCC practical in implementation.

A. Mutual Information

In the ANCC protocol, each packet occupies 2 time slots; hence, to normalize, both the mutual information and transmit SNR need to be scaled down by a factor of 2. On the other hand, each terminal transmits a different part of the network codeword, which amounts to utilizing independent channels. The total mutual information obtained in the second phase equals the sum of m Shannon formulas with m terminals' instantaneous SNR. Furthermore, since the network code provides equal protection (on average) to all the terminals, the mutual information carried across in the second phase will contributed evenly to each terminal. Consequently, we arrive at the following result for the mutual information of ANCC per terminal [1]

$$I_{ancc} = \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \frac{1}{2m}\sum_{r=1}^{m}\log\Big(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\Big). \tag{3}$$

B. Ergodic Capacity

Following the definition in [9], the ergodic capacity of a cooperation protocol is given by the expectation carried out with mutual information of random channels. Since the mutual information of ANCC is not related to a certain decode-set (see (3)), we can write the ergodic capacity of ANCC directly as the expectation of the mutual information on random channels as follows:

$$C_{ancc} = E[I_{ancc}]. \tag{4}$$

From (3), and since the expectation of a sum of random variables is equal to the sum of the expectations of these random variables, (4) becomes

$$\begin{aligned} &C_{ancc} \\ &= \frac{1}{2}\int_0^\infty \log\Big(1+\frac{\gamma}{2}y\Big)\, e^{-y}dy + \frac{1}{2m}\sum_{r=1}^{m}\int_0^\infty \log\Big(1+\frac{\gamma}{2}y\Big)\, e^{-y}dy, \\ &= \int_0^\infty \log\Big(1+\frac{\gamma}{2}y\Big)\, e^{-y}dy = \exp\Big(\frac{2}{\gamma}\Big)\, \mathrm{Ei}\Big(\frac{2}{\gamma}\Big) / \ln(2), \end{aligned} \tag{5}$$

where Ei(.) is the *exponential-integral function* defined as

$$\mathrm{Ei}(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}dt. \qquad (x > 0) \tag{6}$$

It is worth noting that the capacity (per terminal) of ANCC is not a function of the network size m. This is because the underlying assumption here (as well as in other papers in this area) is that all the terminals transmit over their designated time slots just as much information as promised by their instantaneous single-channel capacities.

[1] Note that the network code in use is not necessarily optimal, but each packet is assumed to be optimally channel coded. If the network code is also optimal (which will hold for large m), then $I_{ancc} = \frac{1}{m}\sum_{r=1}^{m}\log\big(1+\gamma|\alpha_{r,d}|^2\big)$.

C. Outage Probability

Outage probability, $\Gamma(R)$, is defined as the probability that a system fails to instantaneously support a predefined information rate of R. Because the mutual information of ANCC is unrelated to the decode-set, we can write the outage probability from the definition directly as

$$\Gamma_{ancc}(R) = Pr[I_{stcc} < R],$$
$$= Pr\left[\frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{1}{2m}\sum_{r=1}^{m}\log\left(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\right)<R\right]. \tag{7}$$

To evaluate (7), we define

$$f_1 \triangleq \frac{1}{2}\log\left(1+\frac{\gamma}{2}u_r\right), \tag{8}$$

where u_r is an exponential random variable with unit variance. From the Jacobi law, the pdf of f_1 can be written as

$$p_{f_1}(y) = p_u\left(f_1^{-1}(y)\right)\frac{\partial f_1^{-1}(y)}{\partial y} = \frac{4\ln(2)}{\gamma}2^{2y}e^{-2(4^y-1)/\gamma}. \tag{9}$$

In addition, we define $f_2 \triangleq \frac{1}{2m}\log\left(1+\frac{\gamma}{2}u_r\right)$, (10)

whose pdf can be written as

$$p_{f_2}(y) = p_u\left(f_2^{-1}(y)\right)\frac{\partial f_2^{-1}(y)}{\partial y},$$
$$= \frac{4m\ln(2)}{\gamma}2^{2my}e^{-2(4^{my}-1)/\gamma}. \tag{11}$$

Since the pdf of a sum of independent random variables is equal to the convolution of the individual pdf, we have ('*' denotes convolution)

$$\Gamma_{ancc}(R) = \int_0^R p_{f1}(y) * \overbrace{p_{f2}(y) * \ldots * p_{f2}(y)}^{m}\, dy. \tag{12}$$

This expression is hard to simplify, so we resort to numerical integral techniques to evaluate the outage for different m. The results are presented in Section VI.

Next, we consider the outage probability when m tends to infinity. From the law of large numbers, we have

$$\lim_{m\to\infty}\frac{1}{m}\sum_{r=1}^{m}\log\left(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\right) = E\left[\log\left(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\right)\right],$$
$$= \int_0^\infty \log\left(1+\frac{\gamma}{2}y\right)\exp(-y)\,dy = \frac{\exp\left(\frac{2}{\gamma}\right)\mathrm{Ei}\left(\frac{2}{\gamma}\right)}{\ln(2)}. \tag{13}$$

Therefore, we can write

$$\lim_{m\to\infty}\Gamma_{ancc}(R)$$
$$= Pr\left[\frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{\exp\left(\frac{2}{\gamma}\right)\mathrm{Ei}\left(\frac{2}{\gamma}\right)}{2\ln(2)}<R\right],$$
$$= 1-\exp\left[-\frac{2}{\gamma}\left(2^{2R-\exp(2/\gamma)\mathrm{Ei}(2/\gamma)/\ln(2)}-1\right)\right]. \tag{14}$$

IV. Space-time-coded-cooperation (STCC)

The STCC protocol works like a genie-aided multi-input multi-output (MIMO) system, where in the relaying phase, all the terminals that have received a clean copy of terminal i's data utilize a suitable (optimal) space-time code to transmit this information simultaneously.

A. Mutual Information

Similar to ANCC, each packet in STCC takes up 2 time slots, so the mutual information needs to be divided by 2. Furthermore, assuming the space-time codes in use are optimal, the mutual information obtained in the relaying phase needs to be evaluated using the Shannon capacity formula with the energy gathered from all the terminals in terminal i'th decode-set $D(i)$ [2]. To normalize the average transmit power, each terminal in $D(i)$ is allowed to use only $1/|D(i)|$ of the energy to transmit the space-time code as compared to ANCC, where $|\cdot|$ denotes the cardinality of the set. The mutual information for each terminal in the STCC protocol is:

$$I_{stcc} = \frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{1}{2}\log\left(1+\frac{\gamma}{2}\sum_{r\in D(i)}\frac{|\alpha_{r,d}|^2}{|D(i)|}\right). \tag{15}$$

B. Ergodic Capacity

Unlike the case of ANCC, the mutual information of STCC is related to the decode-set, so the ergodic capacity is given by the expectation over the mutual information with random channels as follows:

$$C_{stcc} = E\left[I_{stcc}\right]$$
$$= \sum_{n=0}^{m-1} E\Big[I_{stcc}\big||D(i)|=n\Big]Pr\left[|D(i)|=n\big|R\right], \tag{16}$$

where $I_{stcc}\big||D(i)|=n$ is the mutual information conditioned on the decode-set size n, and $Pr\left[|D(i)|=n\big|R\right]$ is the decode-set size probability given by

$$Pr\left[|D(i)|=n\big|R\right]$$
$$= \binom{m-1}{n}Pr[r\in D(i)]^n Pr[r\notin D(i)]^{m-1-n},$$
$$= \binom{m-1}{n}e^{-\frac{2n}{\gamma}(2^{2R}-1)}\left[1-e^{-\frac{2}{\gamma}(2^{2R}-1)}\right]^{m-n-1}. \tag{17}$$

Since the channel powers in the first term and the second term of (15) follow the exponential and $\chi-square$ distributions respectively, the expectation of conditional mutual information can be derived as

$$E\Big[I_{stcc}\big||D(i)|=n\Big] = \frac{1}{2}\int_0^\infty \log\left(1+\frac{\gamma}{2}y\right)\exp(-y)dy$$
$$+\frac{1}{2}\int_0^\infty \log\left(1+\frac{\gamma}{2n}y\right)\frac{y^{n-1}}{(n-1)!}e^{-y}dy. \tag{18}$$

Equating the transmission rate to the ergodic capacity, we obtain the ergodic capacity of STCC C_{stcc} as the solution to the following equation with parameter R:

$$\sum_{n=0}^{m-1} E\left[I_{stcc}\big||D(i)|=n\right] Pr\left[|D(i)|=n\big|R\right]=R, \quad (19)$$

which can be computed numerically.

Specifically, when m tends to infinity, use the law of large numbers, we have

$$\sum_{r\in D(i)} \frac{|\alpha_{r,d}|^2}{|D(i)|}=E\left[|\alpha_{r,d}|^2\right]=1. \quad (20)$$

Thus, we can write

$$\begin{aligned}&\lim_{m\to\infty} C_{stcc}=\lim_{m\to\infty} E\left[I_{stcc}\right],\\ &=\int_0^{\infty}\frac{1}{2}\log(1+\frac{\gamma}{2}y)e^{-y}dy+\frac{1}{2}\log(1+\frac{\gamma}{2}),\\ &=\frac{1}{2}\exp\left(\frac{2}{\gamma}\right)\mathrm{Ei}\left(\frac{2}{\gamma}\right)/\ln(2)+\frac{1}{2}\log\left(1+\frac{\gamma}{2}\right). \quad (21)\end{aligned}$$

C. Outage Probability

From the total probability law, the outage probability of STCC $\Gamma_{stcc}(R)$ can be computed using

$$\Gamma_{stcc}(R)=\sum_{n=0}^{m-1} Pr\left[|D(i)|=n\big|R\right] Pr\left[I_{stcc}<R\big||D(i)|=n\right], \quad (22)$$

where $Pr\left[|D(i)|=n|R\right]$ is given in (17). To calculate the conditional outage probability $Pr\left[I_{stcc}<R\big||D(i)|=n\right]$ in (22), we take a similar approach as in the case of ANCC and define

$$f_3 \triangleq \frac{1}{2}\log\left(1+\frac{\gamma}{2n}\sum_{r=1}^{n}u_r\right), \quad (23)$$

whose pdf is given by

$$\begin{aligned}p_{f_3}(y)=&p_\chi\left(f_3^{-1}(y)\right)\frac{\partial f_3^{-1}(y)}{\partial y},\\ =&\frac{\ln(2)2^{n+1}n^n 4^y(4^y-1)^{n-1}e^{-2n(4^y-1)/\gamma}}{(n-1)!\gamma^n}. \quad (24)\end{aligned}$$

Since the pdf of a sum of independent random variables equals the convolution of their independent pdfs, we get

$$\begin{aligned}&Pr\left[I_{stcc}<R\big||D(i)|=n\right]\\ =&\int_0^R p_{f1}(y)*p_{f3}(y)\,dy,\\ =&\frac{8\ln(2)^2(2n)^n}{(n-1)!\gamma^{n+1}}\int_0^R y16^y(4^y-1)^{n-1}e^{-2(n+1)(4^y-1)/r}\,dy. \quad (25)\end{aligned}$$

When the number of terminals m tends to infinity, gathering (15), (20) and (22), we obtain

$$\begin{aligned}&\lim_{m\to\infty}\Gamma_{ancc}(R),\\ =&Pr\left[\frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{1}{2}\log\left(1+\frac{\gamma}{2}\right)<R\right],\\ =&1-\exp\left[-\frac{2}{\gamma}\left(2^{2R-\log(1+\gamma/2)}-1\right)\right]. \quad (26)\end{aligned}$$

V. Repetition-cooperation

In repetition-cooperation, each user's data is expected to be repeated by all the other terminals, and m time slots are therefore assigned to each packet [2]. Due to the possible inter-user outage, only the terminals in the decode-set $D(i)$ are able to relay for i, so there are idle and wasted time slots. The mutual information in this case can be written as

$$I_{rep}=\frac{1}{m}\log\left(1+\frac{\gamma}{m}|\alpha_{i,d}|^2+\frac{\gamma}{m}\sum_{r\in D(i)}|\alpha_{r,d}|^2\right). \quad (27)$$

A. Ergodic Capacity

Similar to the analysis of STCC, we equate the ergodic capacity to the transmission rate and get the capacity C_{rep} as the solution to the following equation:

$$\sum_{n=0}^{m-1} E\left[I_{rep}\big||D(i)|=n\right] Pr\left[|D(i)|=n\big|C_{rep}\right]=C_{rep}, \quad (28)$$

where the decode-set size probability is

$$\begin{aligned}&Pr\left[|D(i)|=n\big|R\right]\\ =&\binom{m-1}{n}e^{-\frac{mn}{\gamma}(2^{mR}-1)}\left[1-e^{-\frac{m}{\gamma}(2^{mR}-1)}\right]^{m-n-1}. \quad (29)\end{aligned}$$

Since the fading coefficients for different user channels are i.i.d. distributed, the sum of different channel powers satisfies the χ-square distribution. The expectation of the conditional mutual information can be written as

$$E\left[I_{rep}\big||D(i)|=n\right]=\frac{1}{m}\int_0^{\infty}\log\left(1+\frac{\gamma}{m}y\right)\frac{y^n}{n!}\exp(-y)dy. \quad (30)$$

Specifically, when m tends to infinity, the mutual information of repetition-cooperation tends to zero. Thus, we can write

$$\lim_{m\to\infty} C_{rep}=0. \quad (31)$$

B. Outage Probability

For repetition-cooperation, since $D(i)$ is a random set, from the total probability law we get

$$\Gamma_{rep}(R)=\sum_{n=0}^{m-1} Pr\left[|D(i)|=n\right] Pr\left[I_{rep}<R\big||D(i)|=n\right], \quad (32)$$

where $Pr\left[|D(i)|=n\right]$ is the decode-set size probability and $Pr\left[I_{rep}<R\big||D(i)|=n\right]$ is the conditional outage probability.

From (27), the conditional outage probability satisfies

$$Pr\left[I_{rep}<R\big||D(i)|=n\right]=\int_0^{\frac{m}{\gamma}\left(2^{mR}-1\right)}\frac{y^n\exp(-y)}{n!}dy, \quad (33)$$

and in the limit of infinite m,

$$\lim_{m\to\infty} Pr[I_{rep}<R]=Pr\left[0<R\right]=1. \quad (34)$$

VI. Numerical Results

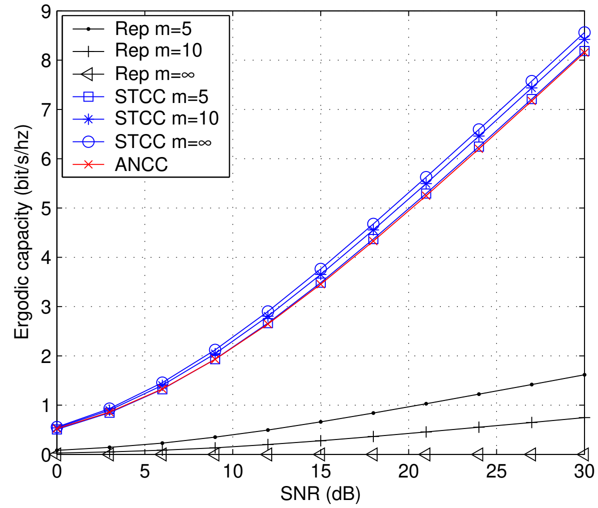
Following the analytical expressions in the previous section, we provide in this section a numerical evaluation of and comparison between different cooperation protocols.

The ergodic capacities are illustrated in Figure 2. From this figure, we observe that repetition-cooperation performs the worst. Its ergodic capacity decreases with the increase of number of terminals m, and eventually drops to zero as m tends to infinity. For STCC, when the number of terminals is small, e.g., $m = 5$, it performs very close to ANCC. Since its capacity increases with m whereas C_{ancc} is not a function of m, STCC slightly outperforms ANCC in large networks (at the cost of significantly increased complexity). We also observe that the capacity of STCC for a moderate network size, say $m = 10$, already gets quite close to the asymptotic limit achieved when $m = \infty$. This suggests that the closed-form expression in (21) can serve as a good prediction for the ergodic capacity of STCC when m is large. For ANCC, we see that it provides a constant capacity regardless of the number of terminals m. Its performance is consistently close to STCC even if STCC has infinite number of terminals.

The outage probabilities are demonstrated in Figure 3 with transmission rate threshold $R = 1/2$. It is not surprising that repetition-cooperation exhibits the worst outage performance. The slope of the outage curves improves with m, indicating an increased diversity gain, but the curves are shifted further and further to the right, indicating a loss in coding gain which outweighs the diversity gain. Both STCC and ANCC achieve much better outage performance than repetition or direct transmission (no cooperation), and their outage probabilities drop quickly with the increase of SNR, showing a good diversity order. It is interesting to note that ANCC slightly outperforms STCC at small m, say $m = 5$, and STCC slightly outperforms ANCC at large m, say $m = 100$. From the numerical results, it appears that $m = 20$ is a threshold, where ANCC and STCC exhibit about the same outage probabilities. That the outage of STCC improves faster with m than ANCC is in part attributed to the fact that the mutual information per terminal of STCC increases with m whereas the mutual information per terminal of ANCC is constant with respect to m.

VII. Conclusions

The simplicity, practicality and efficiency of adaptive-network-coded-cooperation were demonstrated by simulations in [6]. This paper provides a theoretical support for its excellent performance by analyzing its ergodic capacity and outage probability. Closed-form expressions are derived for the limiting case where the network size increases without bound. For any finite network size where analysis is not tractable, we performed numerical evaluations. We have also compared ANCC with repetition and space-time-coded-cooperation. We demonstrate that ANCC is superb to repetition and on par with STCC in both capacity and outage. Considering that ANCC obviates the need for stringent inter-user synchronization (which is extremely costly in large wireless networks), it is therefore a very attractive protocol in practice.



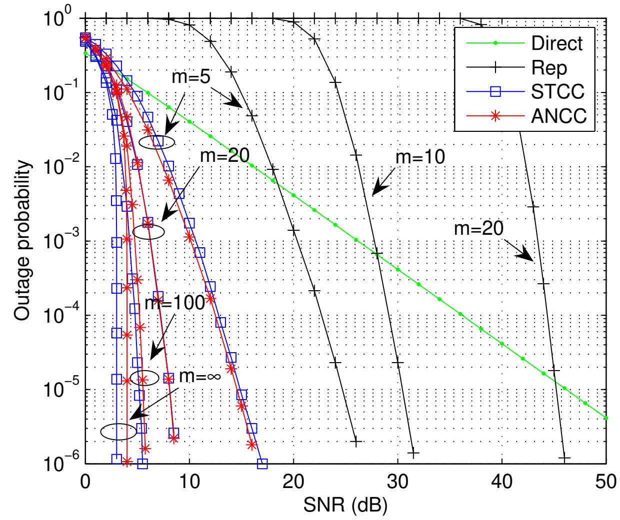


Fig. 3. Outage probabilities with rate threshold $R = 1/2$.

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