On the Bit-Error Rate of Product Accumulate Codes in Optical Fiber Communications

Jing Li, *Member, IEEE*, Yi Cai, *Member, IEEE*, Krishna R. Narayanan, *Member, IEEE*, Allen Lucero, Alexei Pilipetskii, and Costas N. Georghiades, *Fellow, IEEE*

***Abstract*—Product accumulate (PA) codes were proposed as a class of high-rate low-complexity, capacity-approaching codes on additive white Gaussian noise (AWGN) channels. In this paper, we investigate the performance of the PA codes on intensity modulated optical fiber channels where the amplified spontaneous emission (ASE) noise dominates all other noise sources. We consider binary ON–OFF keying (OOK) modulation and iterative soft-decision message-passing decoding for the PA codes. Three channel models for the ASE noise dominated channel are investigated: asymmetric chi-square, asymmetric Gaussian, and symmetric Gaussian channels (i.e., AWGN). At low signal-to-noise ratios (SNRs), due to the lack of tight bounds, code performance is evaluated using simulations of typical PA coding schemes. For high SNRs that are beyond simulation capabilities, we derive the pairwise error probability of the three channels and explore an average upper bound on the bit-error rate over the ensemble of PA codes. We show that AWGN channels, although fundamentally different from chi-square channels, can serve as a reference to approximate the performance of high-rate PA codes.**

***Index Terms*—Amplified spontaneous emission (ASE) noise, forward-error correction (FEC), iterative decoding, message passing decoding, optical fiber communication, serially concatenated codes, union bounds.**

I. Introduction

BANDWIDTH- and power-efficient forward-error correction (FEC) codes are desirable for optical fiber communications. FEC codes have been applied to or proposed for optical fiber communication systems, including the hard-decision decoding Reed–Solomon (RS) codes [1]–[3], concatenated RS/convolutional codes [4], concatenated RS/RS codes [5]–[7], low-density parity check (LDPC) codes [8], [9], and soft-decision iterative decoding block turbo codes [5]. The trend of improving code performance is actualized by code concatenation, soft-decision decoding, and iterative decoding techniques.

Product accumulate (PA) codes (Fig. 1) were proposed as a class of high-rate low-complexity capacity-approaching codes on additive white Gaussian noise (AWGN) channels [10], [11].

Manuscript received June 11, 2003; revised September 18, 2003.

J. Li was with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128 USA. She is currently with the Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, PA 18015 USA (e-mail: jingli@ece.lehigh.edu).

Y. Cai, A. Lucero, and A. Pilipetskii are with Tyco Telecommunications, Eatontown, NJ 07724 USA (e-mail: ycai@tycotelecom.com; alucero@tycotelecom.com; apilipetskii@tycotelecom.com).

K. R. Narayanan and C. N. Georghiades are with the Department of Electrical Engineering, Texas A&M University, College Station, TX 77843-3128 USA (e-mail: krishna@ee.tamu.edu; georghia@ee.tamu.edu).

Digital Object Identifier 10.1109/JLT.2003.821766

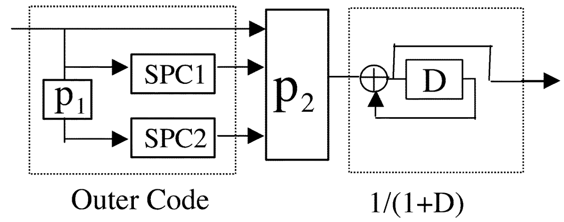

Fig. 1. Code structure of PA codes.

In this paper, we investigate the performance of iterative soft-decision of PA codes based on different fiber optical channel models.

We consider optically amplified fiber communication systems using ON–OFF keying (OOK) modulation where the signal is modulated to be either zero intensity or an optical pulse of duration T_s. Under low-power operations, amplified spontaneous emission (ASE) noise from optical amplifiers is the dominant source of noise in the system (especially undersea long-haul systems). An analytically tractable theoretical model for ASE noise (after photodetector) is the asymmetric chi-square model [12], [13] as defined later in Section II. In this paper, we consider three memoryless channel models:

1) asymmetric channel with uncorrelated chi-square distributed ASE noise;
2) asymmetric channel with Gaussian noise (an approximation to the chi-square model);
3) symmetric channel with Gaussian noise (i.e., AWGN), which is widely employed in coding research.

We would like to mention [8] and [9], where LDPC codes from combinatorial designs are investigated for fiber optical channels. It is interesting to point out that PA codes are essentially a special class of LDPC codes, namely, a class of *differentially coded, structured, high-rate* LDPC codes [11]. Like random LDPC codes, PA codes have also demonstrated performance close to the capacity on a variety of channels [11]. Unlike random LDPC codes, PA codes are linear-time encodable (as well as linear-time decodable), making them suitable for high-speed applications.

At low signal-to-noise ratios (SNRs), due to the lack of tight bounds, performance evaluation is based on simulations of typical PA coding schemes. For high SNRs that are beyond simulation capabilities, we derive the pairwise error probability (PEP) of the aforementioned channels and explore an average upper bound on the bit-error rate (BER) over the ensemble of PA codes. We show that symmetric Gaussian noise channel (i.e., AWGN), although fundamentally different from the chi-square model, can serve as a convenient reference to approximate the performance of high-rate PA codes.

The rest of this paper is organized as follows. Section II presents the three channel models under investigation. Section III discusses the iterative soft decoding of PA codes. Section IV derives and computes the average union bounds of PA codes on the different channel models. Section V presents the analytical and simulation results. Section VI concludes this paper.

II. Channel Models

A. Asymmetric Channels With Chi-Square Noise

Consider $M = B_o/B_e > 1$ as the number of modes per polarization state in the received optical spectrum, B_o as the optical bandwidth, and B_e as the electrical bandwidth of the system at the detector. As discussed in [13], prior to square-law detection, the noise n_i can be mathematically represented as a Fourier series expansion with Fourier coefficients that are assumed to be independent Gaussian random variables with zero mean and variance $N_0/2$. After passing through an optical amplifier, the received signal (the integral of the output of the photodetector) is given by

$$x = \sum_{i=1}^{2M}(s_i + n_i)^2 \tag{1}$$

where s_i and n_i denote the signal and the ASE noise projected to $2M$ orthonormal basis. Signal energy is $\sum_{i=1}^{2M} s_i^2 = 2E_s$ for transmitting "1" and $\sum_{i=1}^{2M} s_i^2 = 0$ for transmitting "0", where E_s is the average energy of the transmitted signals (assuming equal symbol probability).

Completing the square in the integral, the first-order statistics of the optical channel can be modeled as the chi-square distribution with $2M$ degrees of freedom [13], [12]. The closed-form probability density function (pdf) of received signal symbols "1" and "0" after square-law detection ($x > 0$) is given by [13]

$$f_1(x) = \frac{1}{N_0}\left(\frac{x}{2E_s}\right)^{(M-1)/2} e^{-(x+2E_s)/N_0} I_{M-1}\left(\frac{2\sqrt{2xE_s}}{N_0}\right) \tag{2}$$

$$f_0(x) = \frac{1}{N_0}\frac{(\frac{x}{N_0})^{M-1}\exp(\frac{-x}{N_0})}{(M-1)!} \tag{3}$$

where $I_{M-1}(\cdot)$ denotes the $(M-1)_{th}$ modified Bessel function of the first kind. The means and variances of signal "1" and "0" can thus be derived as

$$\mu_1 = MN_0 + 2E_s, \quad \sigma_1^2 = MN_0^2 + 4E_sN_0 \tag{4}$$

$$\mu_0 = MN_0, \qquad \sigma_0^2 = MN_0^2 \tag{5}$$

B. Asymmetric Channels With Gaussian Approximation

Observe that x in (1) is the sum of $2M$ independent random variables, and the application of the central limit theorem (for large M) yields a Gaussian approximation for both symbols. Therefore, it is convenient to approximate the signals as Gaussian distributed with the same mean and variance of the chi-square densities.

Defining Q factor as $Q = (|\mu_1 - \mu_0|)/(\sigma_1 + \sigma_0)$ and normalizing N_0 to 1, the noise parameters can be rewritten as functions of the system parameters B_o, B_e, and Q as

$$\mu_1 = MN_0 + 2E_s = \frac{B_o}{B_e} + 2Q\sqrt{\frac{B_o}{B_e}} + 2Q^2 \tag{6}$$

$$\sigma_1 = \sqrt{MN_0^2 + 4E_sN_0} = \sqrt{\frac{B_o}{B_e}} + 2Q \tag{7}$$

$$\mu_0 = MN_0 = \frac{B_o}{B_e} \tag{8}$$

$$\sigma_0 = \sqrt{MN_0^2} = \sqrt{\frac{B_o}{B_e}}. \tag{9}$$

Thus, for a given Q factor and system parameters B_o, B_e, the Gaussian approximation of ASE noise distribution is given by

$$f_1(x) = \mathcal{N}\left(\frac{B_o}{B_e} + 2Q\sqrt{\frac{B_o}{B_e}} + 2Q^2, \left(\sqrt{\frac{B_o}{B_e}} + 2Q\right)^2\right) \tag{10}$$

$$f_0(x) = \mathcal{N}\left(\frac{B_o}{B_e}, \frac{B_o}{B_e}\right). \tag{11}$$

C. Symmetric Channels With Gaussian Approximation

By assuming $\sigma_1 = \sigma_0$, the asymmetric channel is reduced to the well-known AWGN channel in conventional communications. Since ON–OFF signaling is used in fiber communications rather than antipodal signaling, there is a 3.010-dB difference compared to conventional results using BPSK modulation on AWGN channels.

The pdfs of the received signals for the chi-square channels and the asymmetric and symmetric Gaussian approximations can be found in [17, Fig. 1], which gives a feel of how the original chi-square channel looks like and how well the Gaussian channels approximate the original channel. It can also be seen from the plot that, for the same Q factor, the pdf curves of the different channel models will have different optimal hard-decision thresholds as well as the resulting error probabilities.

III. Iterative Soft Decoding of PA Codes

Product accumulate codes proposed in [10] and [11] are a class of interleaved serial concatenated codes where the inner code is a rate-1 recursive convolutional code $1/(1+D)$ (also known as the accumulator) and the outer code is a parallel concatenation of two single-parity check (SPC) codes (Fig. 1).

The decoding of product accumulate codes is via an iterative procedure employing the turbo principle. Soft information in log-likelihood ratio (LLR) form iterates among different component codes. An efficient sum-product algorithm (also known as the message-passing algorithm) and its reduced-complexity approximation, the min-sum algorithm, are described in [11]. Since the sum-product and min-sum algorithms are a class of generic algorithm that is independent from the channel model, they allow the same decoder to be "self-adaptive" to different channel models provided that proper input log-likelihood ratios (obtained from the channels) are fed into the decoder. This decoupling of the decoder from the transmission channel is convenient and useful in real systems where an "optimal" decoder is easily retained while the modeling of transmission channel is modified.

For simple hard decoding, the optimal threshold γ can be found numerically by letting $f_0(\gamma) = f_1(\gamma)$. However, to maximize the error correction power of the codes, we employ soft decoding. For the aforementioned three channels, the input LLRs of received signal x defined as $L_{ch}(x) = \Pr(0|x)/\Pr(1|x)$ are given by (assuming equally probable occurrence of "1"s and "0"s)

$$\text{Chi-square}: L_{ch}(x) = \left(\frac{\sqrt{2xE_s}}{N_0}\right)^{M-1} \cdot \frac{\exp(\frac{N_0}{2E_s})}{(M-1)! I_{M-1}(\frac{\sqrt{8xE_s}}{N_0})} \tag{12}$$

$$\text{Asym. Gauss}: L_{ch}(x) = \log\frac{\alpha}{\beta} - (\alpha-\beta)((\alpha+\beta)x - 2\alpha\beta^2)x \tag{13}$$

$$\text{Sym. Gauss}: L_{ch}(x) = \frac{2E_s - 2x\sqrt{E_s}}{\sigma^2} \tag{14}$$

where $\beta = \sqrt{B_0/B_e}$ and $\alpha = \beta + 2Q$. Note that the symmetric Gaussian channel uses ON–OFF signaling instead of the conventional antipodal signaling. As alluded earlier, an equivalent and more convenient calculation method is to assume antipodal signaling with $L_{ch}(x) = (2\sqrt{E_s}/\sigma^2)x$ and then shift the performance curve by 3.010 dB.

IV. Analytical Bounds

A. Average Maximum Likelihood Bounds

Union bounds, although loose at low SNRs, have been shown to be useful at high SNRs that are beyond simulation capabilities. They are particularly helpful in determining error floors as well as illustrating the effect of interleaver sizes. Here, we apply the union bounding technique to the ensemble of (N, K) PA codes by averaging over all possible interleavers. Denote $h_{\min}$ as the minimum Hamming distance of the PA code ensemble, $P_2(h)$ as the pairwise error probability for codewords of Hamming distance h apart, and $A_{w,h}$ and A_h as the input output weight enumerator (IOWE) and the output weight enumerator (OWE) averaged over the ensemble of PA codes, respectively. It is well known that the average upper bounds of word error rate (WER) and BER can be computed using (see, for example, [15])

$$P_w \le \sum_{h=h_{\min}}^{N} \bar{A}_h P_2(h) = \sum_{h=h_{\min}}^{N} \sum_{w=1}^{K} \bar{A}_{w,h} P_2(h) \tag{15}$$

$$P_e \le \sum_{h=h_{\min}}^{N} \sum_{w=1}^{K} \frac{w}{K} \bar{A}_{w,h} P_2(h) \tag{16}$$

where K and N are the user data block size and the codeword length, respectively.

B. Average Input Output Weight Enumerator $\bar{A}_{w,h}$

Viewing product accumulate codes as a hybrid concatenation, the average input output weight enumerator $\bar{A}_{w,h}$ can be computed using [15]

$$\bar{A}_{w,h} = \sum_{h_1}\sum_{h_2} A^{\text{SPC1}}_{w,h_2} \frac{A^{\text{SPC2}}_{w,h_1-h_2}}{\binom{K}{w}} \frac{A^{1/(1+D)}_{h_1,h}}{\binom{N}{h_1}} \tag{17}$$

where $A^{\text{SPC1}}_{w,h}$, $A^{\text{SPC2}}_{w,h}$, and $A^{1/(1+D)}_{w,h}$ are IOWEs of the parallel branch SPC1, SPC2, and the inner code $1/(1+D)$, respectively (Fig. 1).

For each parallel branch where q $(t+1, t)$ SPC codewords are combined, the IOWE function is given by (assuming even parity check) [11], [15]

$$A^{\text{SPC}}(w,h) = \left(\sum_{j=0}^{t} \binom{t}{j} w^j h^{2\lceil j/2\rceil}\right)^q \tag{18}$$

where the coefficient of the term $w^u h^v$ denotes the number of codewords with input weight u and output weight v in an SPC branch. It should be noted that the first branch includes the systematic bits while the second contains only the parity bits. In other words, we have $A^{\text{SPC1}}_{u,v} = A^{\text{SPC}}_{u,v}$ and $A^{\text{SPC2}}_{u,v} = A^{\text{SPC}}_{u,v+u}$. The IOWE of the inner $1/(1+D)$ code (or the accumulator) is given by [15]

$$A^{1/(1+D)}_{w,h} = \binom{N-h}{\lfloor \frac{w}{2} \rfloor}\binom{h-1}{\lceil \frac{w}{2}\rceil - 1}. \tag{19}$$

The computation of the average IOWE $\bar{A}_{w,h}$ is generally tedious work with concatenated coding schemes. For PA codes, although we do not have a closed-form expression for (17), each component code is so simple that a numerical approach can be used to approximate the weight distribution quite well [15]. For short block sizes, it is convenient to examine the entire weight distribution, although the performance bound is dominated by the first few terms (i.e., low weight codewords). For fairly large block sizes, due to the computational complexity and the potential numerical issue, we focus on the low weight terms. Hence, the bounds we computed here are truncated union bounds.

C. Pair-Wise Error Probability $P_2(h)$

Pair-wise error probability $P_2(h)$ is a function of the channel characteristics, the modulation scheme, and the decoding strategy. Below we derive the *average* PEP of the aforementioned channels. By average, we assume "1"s and "0"s are transmitted with equal probability and that there is an equal probability that the "1"s and "0"s are in error. Throughout the discussion, unless otherwise stated, we assume OOK modulation (signal energy either zero or $2E_s$) and soft decoding.

1) Symmetric Gaussian: For OOK signaling on symmetric Gaussian channels with noise variance σ^2, the average Euclidean distance of two codewords of Hamming distance h apart is given by $\sqrt{2hE_s}$. It thus follows that the pairwise error probability of soft decoding is

$$P_2(h) = \mathrm{Q}\left(\sqrt{\frac{hE_s}{2\sigma^2}}\right) \tag{20}$$

where $\mathrm{Q}(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-x^2/2}dx$ is the complementary distribution function of a zero-mean unit variance Gaussian random variable.

2) Asymmetric Gaussian: With asymmetric Gaussian channels, the optimal decision threshold γ for a transmitted bit should satisfy $f_0(\gamma) = f_1(\gamma)$ in (11) and (10) [17]. Although a numerical approach is possible, the solution of the optimal threshold has a quite complex form. It is convenient to set the threshold such that probabilities of space error (bit "0" in error) and mark error (bit "1" in error) are the same (i.e., $P(1|0) = P(0|1)$). Under the assumption of equally likely space and mark errors, the convenient choice of the threshold allows a simple derivation of PEP where the typical all-zeros codeword can be approximately as a reference. We note that this approximation may cause overestimation of the error rate [18], but we have traded the accuracy of the resulting PEP with the simplicity of the PEP evaluation.

Since each length-N codeword can be viewed as a point in an N-dimensional code space, to evaluate the distance between two codewords that differ in h bit positions, we can conveniently ignore the $N-h$ irrelevant dimensions and consider only the reduced h-dimensional subspace. Since each noisy bit follows Gaussian distribution and all bits are orthogonal to each other, it then follows that the joint pdfs of these two (noise-corrupted) codewords can be approximated as

$$f(\mathbf{c}_0) = \mathcal{N}(MN_0\sqrt{h},\, MN_0^2) \tag{21}$$
$$f(\mathbf{c}_h) = \mathcal{N}(MN_0\sqrt{h}+2\sqrt{h}E_s,\, MN_0^2+4E_sN_0). \tag{22}$$

The customary threshold γ^* for estimating codewords can be obtained by letting

$$\mathrm{Q}\left(\frac{\gamma^* - MN_0\sqrt{h}}{\sqrt{MN_0^2}}\right) = \mathrm{Q}\left(\frac{MN_0\sqrt{h} + 2\sqrt{h}E_s - \gamma^*}{\sqrt{MN_0^2 + 4E_sN_0}}\right). \tag{23}$$

We then derive the threshold as

$$\gamma^* = MN_0\sqrt{h} + \frac{2\sqrt{h}E_s\sqrt{MN_0^2}}{\sqrt{MN_0^2} + \sqrt{MN_0^2 + 4E_sN_0}} \tag{24}$$

and the corresponding pairwise error probability $P_2(h)$ as

$$P_2(h) \approx \mathrm{Q}\left(\frac{\frac{2\sqrt{h}E_s}{N_0}}{\sqrt{M} + \sqrt{M + \frac{4E_s}{N_0}}}\right). \tag{25}$$

It should be noted that we have simplified the computation of PEP by evaluating only the typical all-zeros codeword and a weight h codeword. Although this is not exact for asymmetric channels, it is a reasonable approximation due to the linear codeword space and the assumption that equal probability of marks and spaces will occur and that the customary decision threshold will be used. Further, since we considered sequence/codeword detection rather than bit detection, the optimal (customary) decision threshold in (24) is thus dependent on h, the distance between the two codewords.

3) Chi-Square: Unlike the Gaussian distribution, which is symmetric and which has characteristic bell-shaped probability density curve, chi-square distribution does not possess such properties to be exploited for the evaluation of a soft-decoding PEP. In this paper, we use a hard-decision PEP as an upper bound for a soft-decision PEP.

For each noise-corrupted bit of energy zero or $2E_s$, the receiver makes a decision by comparing it with a threshold γ. The probabilities that a "1" is decided when a "0" is sent, and a "0" is decided when a "1" is sent, are given by [14], [13], [16]

$$P(1|0) = \int_\gamma^\infty f_0(x)dx = e^{-\gamma/N_0}\sum_{k=0}^{M-1}\frac{1}{k!}\left(\frac{\gamma}{N_0}\right)^k \tag{26}$$
$$P(0|1) = \int_0^\gamma f_1(x)dx = 1 - \mathcal{Q}_M\left(\sqrt{\frac{4E_s}{N_0}}, \sqrt{\frac{2\gamma}{N_0}}\right) \tag{27}$$

where $\mathcal{Q}_M(a,b)$ is the generalized Marcum $\mathcal{Q}$ function of order M defined as

$$\mathcal{Q}_M(a,b) = \int_b^\infty \frac{x^M}{a^{M-1}}\exp\left(-\frac{x^2+a^2}{2}\right)I_{M-1}(ax)dx. \tag{28}$$

There is no simple, closed-form expression for calculating the generalized Marcum $\mathcal{Q}$ function, but highly reliable and efficient numerical methods can be found in [16] and the references therein. The optimum threshold γ can also be solved numerically (in an iterative fashion) by letting $f_0(x) = f_1(x)$ or

$$\left(\frac{\sqrt{2E_s\gamma}}{N_0}\right)^{M-1} = e^{-2E_s/N_0}(M-1)!I_{M-1}\left(\frac{2\sqrt{2E_s\gamma}}{N_0}\right). \tag{29}$$

Using the asymptotic expansion of I_M reveals that the optimal normalized threshold $(\gamma - MN_0)/2E_s$ approaches 1/4 for large $E_s/(N_0M^2)$ [12], [13].

The average probability of a bit in error is given by

$$p = \frac{P(0|1) + P(1|0)}{2}. \tag{30}$$

It then follows that the PEP of two codewords of length N and Hamming distance h apart is (with hard decoding)

$$P_2(h) = p^h(1-p)^{N-h} \approx p^h \tag{31}$$

where the approximation can be made for small p (or large SNRs).

V. Results

In all the simulations provided, we assume perfect channel knowledge on the receiver side. Thus, the performance of the PA decoder is optimized according to different channel models.

Figs. 2–4 plot the simulations of a rate 0.8, block size 16-K PA code on the AWGN, the asymmetric Gaussian, and the chi-square noise channels, respectively. We use $M = 4$ and OOK signaling. BER performance after 5, 10, 15, 20, and 25 iterations is shown. Channel conditions are measured using gross Q^2 (in dB) as defined before. For AWGN channels, the conventional

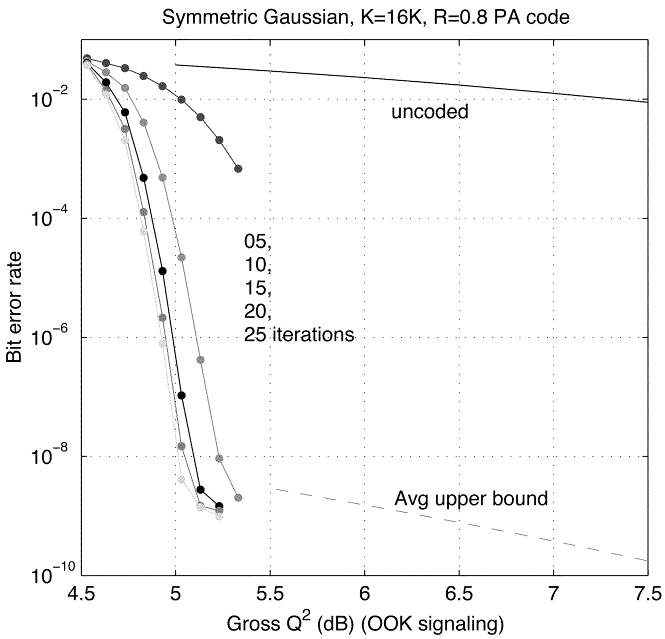

Fig. 2. Performance and bounds of PA codes on AWGN channels: code rate 0.8, data block size 16 K.

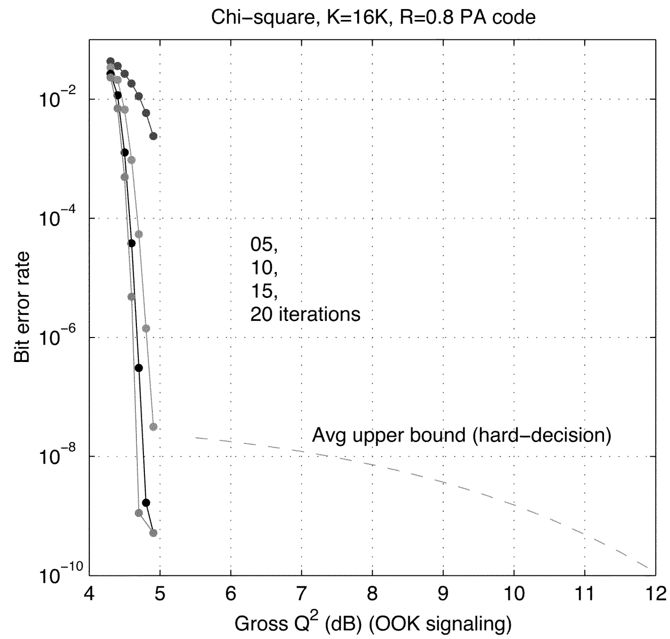

Fig. 4. Performance and bounds of PA codes on asymmetric chi-square channels: code rate 0.8, data block size 16 K.

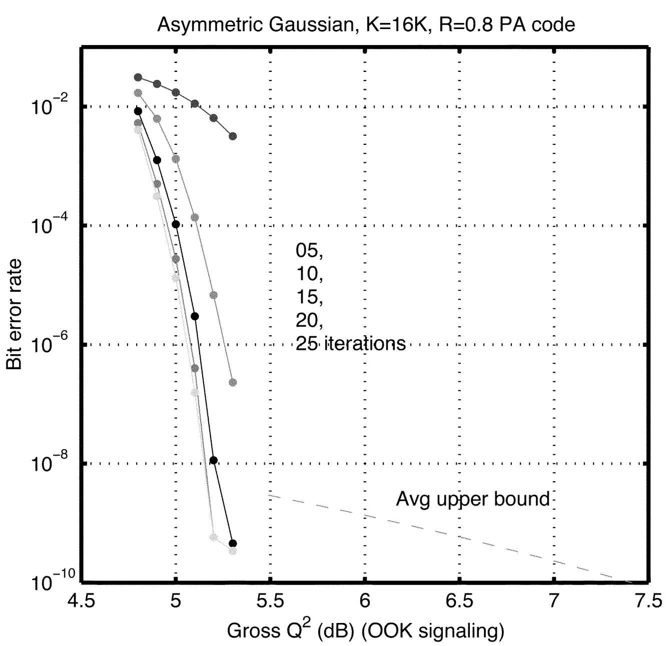

Fig. 3. Performance and bonds of PA codes on asymmetric Gaussian channels: code rate 0.8, data block size 16 K.

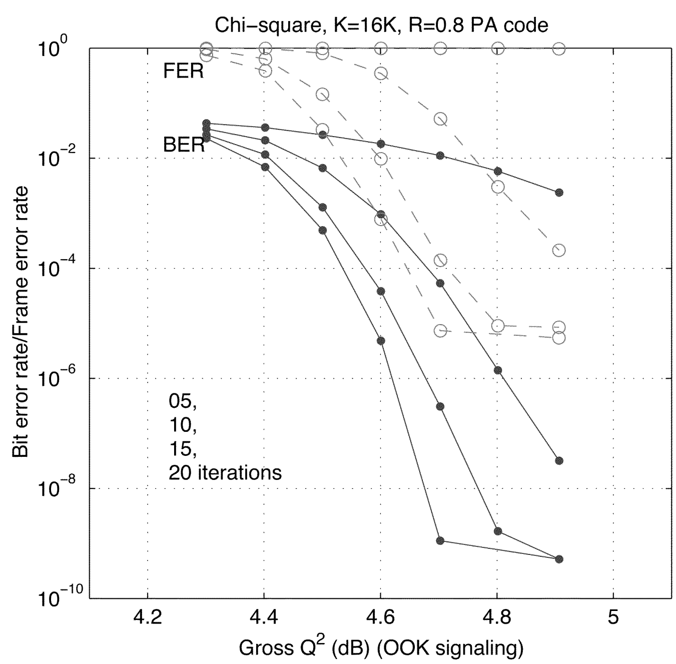

Fig. 5. BER and frame error rate of PA codes on asymmetric chi-square channels: code rate 0.8, data block size 16 K.

E_s/N_0 of BPSK signaling and the gross Q^2 in our simulations are approximately offset by 3 dB. The observations are made over 10^{11} bits for high SNRs. In each simulation point, more than 50 codeword error events are collected, so the results are fairly reliable. As can be seen, PA codes yield impressive performance for all three channels, with error floors as low as BER of 10^{-9} to 10^{-10}. Comparing to the uncoded OOK systems, which require 15 dB to achieve BER of 10^{-8} on AWGN channels, the rate 0.8 PA code can achieve as many as 9-dB gains (after the code rate penalty). It should be noted that for fiber optical systems where the target BER is as low as 10^{-15}, an error floor at 10^{-9} is far from satisfactory. A possible solution is to use code concatenation, that is, wrapping another (high-rate) RS code on top of the PA code to (hopefully) clear up the residue errors.[1] This requires an evaluation of the error bursts after PA decoder. As an example, we plot in Fig. 5 both the bit error rate and the frame error rate (FER or codeword error rate) of the aforementioned rate 0.8, data block size 16-K PA code on Chi-square channels. That the FER curve is significantly higher than the

[1]This is the same strategy that is being seriously tested and evaluated for future high-density digital data recording systems, where the required the BER is no higher than 10^{-15}.

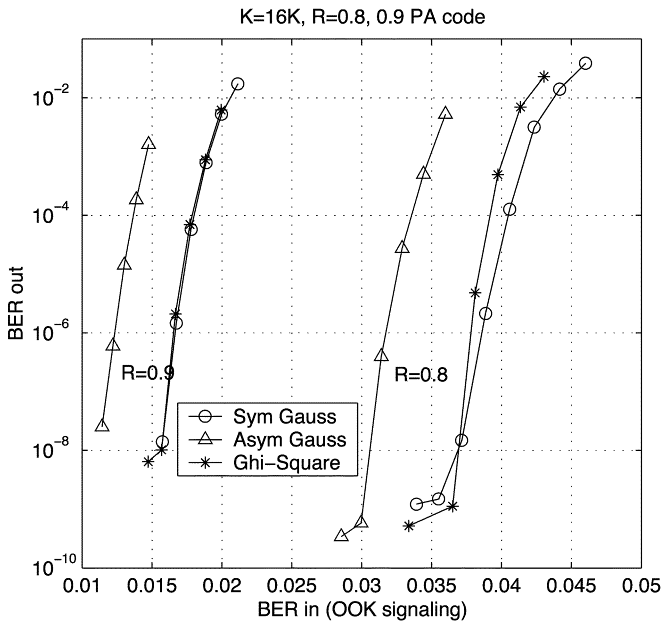

Fig. 6. Performance of high-rate PA codes on different channels: code rate 0.8 and 0.9, data block size 16 K, 20 iterations.

BER curve indicates the error events are actually very short (at high SNRs), although the codeword is quite long. Further examinations of the error pattern we collected in the simulation reveal that the majority of the erroneous frames/codewords contain less than five bit errors (in the data block). What exactly caused these error events is not entirely clear. A tentative explanation is that the specific interleavers we used in this PA code resulted in a few very short loops in the code graph, causing the iterative decoder to trap in these "local minimums" once in a while. On one side, this suggests that there is room for us to improve the PA code by optimizing the interleavers. On the other, it suggests that such a PA code may work harmoniously with an outer RS wrap, opening the possibility of achieving really low BER.

To facilitate the evaluation, analytical bounds are computed and presented along with the simulations. Since the bounds assume ML decoders (rather than the practical iterative decoders), and since they are averaged over all possible interleavers (thus may well be dominated by the worse case interleaver), the accuracy of the bound for a PA code with a specific interleaving scheme is questionable. In fact, the performances are seen to be slightly better than the average bounds. Nevertheless, they shed useful insights into what to expect of PA codes in general for regions beyond simulation capabilities.

Since Q^2 represents different channel conditions for different channels, we plot the performances on different channels in terms of BER-in versus BER-out for a fair comparison. As shown in Fig. 6, the performances of PA codes on the asymmetric Gaussian channels appear worse. It is interesting to observe that the performances on the chi-square and AWGN channels match quite well for rate 0.9 PA codes and show a slight difference for rate 0.8 PA codes. This shows that a conventional AWGN channel can be used as a convenient reference to approximate the performance of PA codes on a chi-square channel, which agrees with the results in [19].

VI. Conclusion

Product accumulate codes have been investigated with three different channel models for optical fiber communications. Extensive simulations down to quite low BERs provided benchmarks of the performance of high-rate PA codes. Theoretical analysis provided insight into the regions that are beyond simulation capabilities. We also showed that the conventional AWGN channel can be used as a convenient reference to determine the code performance on chi-square channels at high code rates.

As a concluding remark, we mention that although PA codes have several advantages like relatively simple decoding complexity (compared to turbo codes), easy construction (compared to random LDPC codes), and flexibility in changing code rates/lengths (compared to LDPC codes), it is nevertheless imprudent and unconvincing to conclude at this point that PA codes are a better candidate than turbo codes or LDPC codes. The simulations and analysis provided in this paper are intended to inform readers of this class of high-rate high-performance codes that might find promising application in optical fiber communications. More in-depth and thorough evaluation (like how exactly PA codes perform when concatenated with an outer RS wrap, how much performance degradation will incur if 2-b/3-b quantization is used in the PA decoder) is needed before conclusive remarks can be made.

References

[1] J. L. Pamart, E. Lefranc, S. Morin, G. Balland, Y. C. Chen, T. M. Kissell, and J. L. Miller, "Forward error correction in a 5 Gbit/s 6400 km EDFA based system," *Electron. Lett.*, vol. 30, no. 4, pp. 342–343, Feb. 1994.

[2] S. Yamamoto, H. Takahira, and M. Tanaka, "5 Gbps optical transmission terminal equipment using forward error correction code and optical amplifier," *Electron. Lett.*, vol. 30, no. 3, Feb. 1994.

[3] H. Kidorf, N. Ramanujam, I. Hayee, M. Nissov, J. Cai, B. Pedersen, A. Puc, and C. Rivers, "Performance improvement in high capacity, ultra-long distance, WDM systems using forward error correction codes," in *OFC/IOOC'00 Tech. Dig.*, Baltimore, MD, Mar. 2000, pp. ThS3-1–ThS3-3.

[4] A. Puc, F. Kerfoot, A. Simons, and D. L. Wilson, "Concatenated FEC experiment over 5000 km long straight line WDM test bed," in *OFC/IOOC'99 Tech. Dig.*, San Diego, CA, Feb. 1999, pp. ThQ6-1–THQ6-3.

[5] O. Ait Sab and V. Lemaire, "Block turbo code performances for long-haul DWDM optical transmission systems," in *OFC/IOOC'00 Tech. Dig.*, Baltimore, MD, Mar. 2000, pp. ThS5-1–ThS5-3.

[6] H. Taga, H. Yamauchi, T. Inoue, and K. Goto, "Performance improvement of highly nonlinear long-distance optical fiber transmission system using novel high gain forward error correcting code," in *OFC/IOOC'01 Tech. Dig.*, Anaheim, CA, Mar. 2001, pp. TuF3-1–TuF3-3.

[7] O. Ait Sab, "FEC techniques in submarine transmission systems," in *OFC/IOOC'01 Tech. Dig.*, Anaheim, CA, Mar. 2001, pp. TuF1-1–TuF1-3.

[8] B. Vasic and I. B. Djordjevic, "Iterative decodable block codes in long-haul optical transmission systems," *J. Opt. Commun.*, vol. 23, no. 5, pp. 182–186, Oct. 2002.

[9] B. Vasic, I. B. Djordjevic, and R. Kostuk, "Low-density parity check codes and iterative decoding for long haul optical communication systems," *J. Lightwave Technol.*, vol. 21, pp. 438–446, Feb. 2003.

[10] J. Li, K. R. Narayanan, and C. N. Georghiades, "A class of linear-complexity, soft-decodable,high-rate, good codes: Construction, properties and performance," in *Proc. Int. Symp. Inform. Theory*, Washington, DC, June 2001, pp. 122–122.

[11] ——, "Product accumulate codes: A class of capacity-approaching, low complexity codes," IEEE Trans. Inform. Theory, 2001, submitted for publication.

[12] D. Marcuse, "Derivation of analytical expressions for the bit-error probability in lightwave systems with optical amplifiers," *J. Lightwave Technol.*, vol. 8, pp. 1816–1823, Dec. 1990.

[13] P. A. Humblet and M. Azizoglu, "On the bit error rate of lightwave systems with optical amplifiers," *J. Lightwave Technol.*, vol. 9, pp. 1576–1582, Nov. 1991.

[14] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1983.

[15] J. Li, K. R. Narayanan, and C. N. Georghiades, "Generalized product accumulate codes: Performance and analysis," in *Proc. IEEE GLOBECOM*, San Antonio, TX, Nov. 2001, pp. 975–979.

[16] D. A. Shnidman, "The calculation of the probability of detection and the generalized Marcum Q-function," *IEEE Trans. Inform. Theory*, vol. 35, pp. 389–400, Mar. 1989.

[17] Y. Cai, J. M. Morris, T. Adali, and C. R. Menyuk, "On turbo code decoder performance in optical-fiber communication systems with dominating ASE noise," *J. Lightwave Technol.*, vol. 21, pp. 727–733, Mar. 2003.

[18] O. K. Tonguz and L. G. Kazovsky, "Theory of direct-detection lightwave receivers using optical amplifiers," *J. Lightwave Technol.*, vol. 9, pp. 174–181, Feb. 1991.

[19] Y. Cai, J. Chen, J. Li, A. Pilipetskii, A. Lucero, and M. Nissov, "On channel models for predicting soft-decision error correction performance in optically amplified systems," in *OFC/IOOC'03 Tech. Dig.*, Atlanta, GA, Mar. 2003, pp. 532–533.

Jing Li (S'98–M'03) received the B.S. degree in computer science (with honors) from Peking (Beijing) University, China, and the M.E. and Ph.D. degrees in electrical engineering from Texas A&M University, College Station.

She was a Research Intern with Seagate Research Laboratory and Tyco Telecommunications Laboratory in 2000 and 2001. She is currently with the Electrical and Computer Engineering Department, Lehigh University, Bethlehem, PA. Her research interests are in the area of telecommunication and information technology, with focus on iterative coding, source/channel coding, and coding and signal processing for wireless, free-space optical, and fiber optical communications as well as data storage systems.

Dr. Li received the Chinese Academy of Science Striving Scholarship, the Ethel Ashworth-Tsutsui Memorial Award for Research, and the J. W. Van Memorial Scholarship for Academic Excellence. She is listed in *Who's Who Among Students in American Universities and Colleges.*

Yi Cai (S'98–M'01) was born on May 12, 1970. He received the B.S. degree in optical engineering from the Beijing Institute of Technology, Beijing, China, in 1992, the M.S. degree in signal and information processing from the Shanghai Institute of Technical Physics, Chinese Academy of Sciences, Shanghai, China, in 1998, and the Ph.D. degree in electrical engineering from the University of Maryland—Baltimore County, Baltimore, in 2001.

He is currently a Senior Member of Technical Staff with Tyco Telecommunications Laboratories, Eatontown, NJ. His research interests include forward-error correction, line coding, and adaptive signal processing, and their applications in optical fiber communications.

Krishna N. Narayanan (S'92–M'98) was born in Madurai, India. He received the Ph.D. degree in electrical engineering from the Georgia Institute of Technology, Atlanta, in 1998.

He is currently an Assistant Professor in the Electrical Engineering Department, Texas A&M University, College Station. His research interests are in communication theory with emphasis on coding and signal processing for wireless/optical communications and digital magnetic recording.

Dr. Narayanan is an Associate Editor for IEEE COMMUNICATIONS LETTERS and an Editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He received the CAREER Award from the National Science Foundation in 2001.

Allen Lucero received the M.S. and Ph.D. degrees in physics from the University of Connecticut, New Haven, in 1989 and 1993, respectively.

He completed a two-year Postdoctoral Fellowship at Bell Laboratories in 1995. He subsequently accepted an MTS position at AT&T Advanced Technologies Systems, where he concentrated on development of low power consumption EDFAs and system design. In 1997, he left to assist in the establishment of the system and components test laboratories for Corning, Inc., Somerset, NJ. He joined Tyco Telecommunications, Eatontown, NJ, in 2000, where his current interests include 40-Gb/s transport over novel dispersion maps and long-haul Raman assisted systems.

Dr. Lucero is a Member of Phi Beta Kappa.

Alexei N. Pilipetskii received the M.S. degree from Moscow State University, Moscow, Russia, in 1985 and the Ph.D. degree from the Academy of Sciences of Russia in 1990, both in physics.

From 1985 to 1994, he was with the Fiber Optics Department, General Physics Institute, Moscow, where worked on the nonlinear effects in fibers. From 1994 to 1997, he was an Assistant Research Professor at the University of Maryland—Baltimore County, where he worked on problems of fiber-optic communication lines. In 1997, he joined Tyco Submarine Systems Ltd. (now Tyco Telecommunications), Eatontown, NJ, where he presently works as a Director of the System Modeling and Signal Processing Research Group. His main area of expertise is signal transmission in long-haul WDM systems.

Dr. Pilipetskii was a Technical Program Committee Member for the 2000–2001 OFC conference and Chair of the Digital Transmission Systems Subcommittee for the 2002 OFC conference.

Costas N. Georghiades (S'82–M'82–SM'90–F'98) was born in Cyprus in 1955. He received the B.E. degree (with distinction) from the American University of Beirut, Lebanon, in 1980 and the M.S. and D.Sc. degrees from Washington University, St. Louis, MO, in 1983 and 1985, respectively, all in electrical engineering.

Since September 1985, he has been with the Electrical Engineering Department, Texas A&M University, College Station, where he is a Professor and holder of the J. W. Runyon, Jr., Endowed Professorship and Director of the Telecommunications and Signal Processing Group. His general interests are in the application of information, communication and estimation theories to the study of communication systems. He was Program Chair for the Communication Theory Symposium within IEEE GLOBECOM 2001

Dr. Georghiades is a Member of Sigma Xi and Eta Kappa Nu. He is a registered Professional Engineer in Texas. He was an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, Publications Editor and Associate Editor for Communications for the IEEE TRANSACTIONS ON INFORMATION THEORY, Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, and Associate Editor for the IEEE COMMUNICATIONS LETTERS. He has been involved in organizing a number of conferences, including as Technical Program Chair for the 1999 IEEE Vehicular Technology Conference and the 2001 Communication Theory Workshop.