ON THE ACHIEVABLE INFORMATION RATE OF ASYMMETRIC OPTICAL FIBER CHANNELS WITH AMPLIFIER SPONTANEOUS EMISSION NOISE

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ABSTRACT

This work is motivated by the need to understand the fundamental limit of channel capacities of optical fiber communication channels where amplified spontaneous emission (ASE) noise is dominant and to understand how much has been achieved by the state-of-the-art forward error correction (FEC) coding techniques. Two most commonly used channel models, the Chi-square channel and its Gaussian approximation, are examined. Both soft receiver with continuous output and hard receiver with 1-bit and 2-bit quantized output are investigated for several values of the system parameter M, which is the number of signal photons at the input of the ideal high gain optical amplifier that produces the noise. We show how bit quantization affects the channel capacity and how the capacity scales with different values of M. Finally, we report the performance of some of the best-known codes like turbo codes on optical fiber communication channels.

INTRODUCTION

Consider an optically amplified fiber communication system using amplitude shift keying (ASK) modulation where the signal is modulated to be either zero intensity or an optical pulse of duration T_s. A practical receiver consists of an optical bandpass filter of bandwidth B_o, a photo-detector and an electrical filter of bandwidth B_e that integrates over the bit period T_s. Under low-power operations, amplified spontaneous emission (ASE) noise from optical amplifiers dominates all other sources of noise. It has been shown that the performance of such an optical system is identical to that of a radio system with square-law detection [1], and that the most accurate theoretical model for ASE noise (after photodetector) is the asymmetric Chi-square model [2] [1] as defined later in Section 2. Whereas much has been understood about the additive white Gaussian noise (AWGN) channels, the binary symmetric channels (BSC) and the erasure channels, relatively little has been reported on Chi-square channels [3]. The motivation of this work is to investigate the ultimate performance limit on optical fiber communications, as well as to report how much has been achieved by some of the best-known forward error correction (FEC) codes like turbo codes [5].

Since the properties of Gaussian distributions are more convenient and better understood than Chi-square densities, it is common to approximate the distribution of ASE noise with asymmetric Gaussian densities. In this work, we consider two memoryless channel models: asymmetric channel with uncorrelated Chi-square distributed ASE noise and asymmetric channel with Gaussian noise (an approximation to the Chi-square model). For comparison purpose, we also include the results of traditional additive white Gaussian noise (AWGN) channels.

Due to the asymmetry of the channel, the Shannon limit (the ultimate capacity) is achievable only when the transmission source follows an optimized distribution where the probabilities of "0"s and "1"s are not necessarily the same. It is thus of practical interest to also compute the "practical capacity" where "0"s and "1"s are transmitted with equal probability.

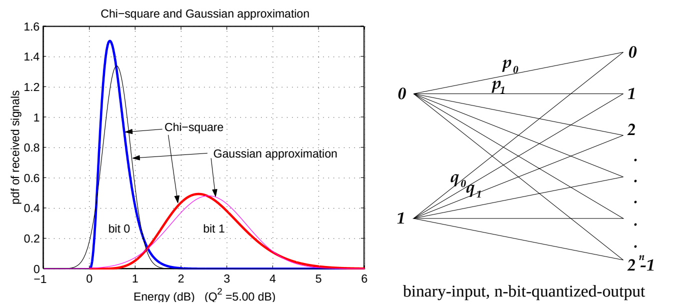

Fig. 1. (A) Soft-output Chi-square channel and the asymmetric Gaussian approximated channel. (B) Binary-input, 2^n-output discrete memoryless channels.

We consider both the soft receiver (continuous output) and hard receiver (quantized output). The continuous output infers the best capacity obtainable (at the cost of complexity). In practice, due to the limitation on the internal data size in hardware/software and in particular the concern for complexity, quantization is unavoidable. For example, using a simple and efficient hard detector with 1-bit quantization, the communication channels are reduced to binary asymmetric channels (BAC), on which Reed-Solomon (RS) codes have been widely used to correct errors. For higher capacity and for better performance with more sophisticated error correction codes like turbo codes and low density parity check (LDPC) codes, more quantization levels are needed. At present stage, due to the concern for complexity and signal processing speed, research focus has been limited to no more than a few bits and 2 bits in particular. Hence, in addition to continuous output and binary output, we also investigate the important case of hard receiver with 2-bit quantization. We show how quantization affects the capacity and how capacity scales with different values of the system parameter $M = \frac{B_o}{B_e}$.

To give a state-of-the-art view, we also report some of the latest results on FEC coding for optical fiber communication channels. We show how much has been achieved and how much is yet to be achieved.

SYSTEM MODEL

A. Soft Receiver (Continuous Output)

<u>*Asymmetric Chi-square Channels*</u> – The first order statistics of the optical fiber communication channels (with dominant ASE noise) after square-law detector are most accurately modeled as the Chi-square distribution with $2M$ degrees of freedom [1] [2], where $M = B_o/B_e > 1$ is the number of modes per polarization state in the received optical spectrum, and B_o and B_e are the optical and electrical bandwidth of the system at the detector, respectively. The closed-form probability density function (pdf) of *mark* (i.e., signal "1") and *space* (i.e., signal "0") at the receiver side are given by ($y \geq 0$) the non-central and central Chi-Chi-square

distributions [1] (see Fig. 1(A))

$$f_0(y) \triangleq p(y|0) = \frac{1}{N_0} \frac{(y/N_0)^{M-1} \exp(-y/N_0)}{(M-1)!}, \quad (1)$$

$$f_1(y) \triangleq p(y|1) = \frac{1}{N_0}\left(\frac{y}{2E_s}\right)^{\frac{M-1}{2}} \exp\left(-\frac{y+2E_s}{N_0}\right) I_{M-1}\left(\frac{2\sqrt{2yE_s}}{N_0}\right), \quad (2)$$

where $I_{M-1}(\cdot)$ denotes the $(M-1)_{th}$ modified Bessel function of the first kind. The means and variances of signal "1" and "0" are given by

$$\mu_0 = MN_0, \qquad \sigma_0^2 = MN_0^2, \quad (3)$$
$$\mu_1 = MN_0 + 2E_s, \qquad \sigma_1^2 = MN_0^2 + 4E_sN_0. \quad (4)$$

The (un-normalized) signal-to-noise ratio (SNR) E_s/N_0 can be regarded as the number of signal photons at the input of the ideal high gain optical amplifier that produces the noise.

Asymmetric Gaussian Channels – Since y is the sum of $2M$ independent random variables, the application of the central limit theorem (for large M) yields a Gaussian approximation for both symbols. It is convenient and common practice in the research of this area to approximate the signals as Gaussian distributed with the same mean and variance of the Chi-square densities [1] [3]. The Gaussian approximation of ASE noise distribution is given by (see Fig. 1(A))

$$f_0(y) = \mathcal{N}(\mu_0, \sigma_0^2) = \mathcal{N}(MN_0,\ MN_0^2), \quad (5)$$
$$f_1(y) = \mathcal{N}(\mu_1, \sigma_1^2) = \mathcal{N}(MN_0+2E_s,\ MN_0^2+4E_sN_0), \quad (6)$$

where $\mathcal{N}$ denotes the Gaussian distribution. Fig. 1(A) compares the channel responses of *mark* and *space* on Chi-square and asymmetric Gaussian channels.

B. Hard Receiver (Quantized Output)

Continuous output provides the best capacity we can hope for, but requires a soft receiver with infinite precision to represent it. A simpler and more practical receiver would be a hard receiver with data quantization. With n-bit quantization, the above channel models are reduced to binary-input, 2^n-output discrete channels.

Fig. 1(B) shows the system model of a general binary-input, 2^n-output discrete channel with transition probabilities

$$p_i = \Pr(y_i|0), \quad q_i = \Pr(y_i|1), \quad i = 0, 1, \cdots, 2^n - 1. \quad (7)$$

Apparently, we have

$$\sum_{i=0}^{2^n-1} p_i = 1, \qquad \sum_{i=1}^{2^n-1} q_i = 1. \quad (8)$$

Clearly, in evaluating such a system, the transition probabilities, p_i and q_i, should be determined according to optimal thresholds. This is then a constraint optimization problem which can be solved using Lagrange Multipliers or Kuhn-Tucker Conditions.

Perceivably, as the number of quantization level increases, the channel capacity also increases, and so does the complexity of the receiver and decoder. Whereas it is desirable to exploit the full capacity of the channels (soft receiver with continuous output), the determining factor for any practical application is complexity which directly translates to speed and cost. This is particularly critical for optical fiber communications where the extraordinarily high data rate requires extremely fast signal processing. For this reason, we will primarily focus on the more practical cases of 1-bit and 2-bit quantization:

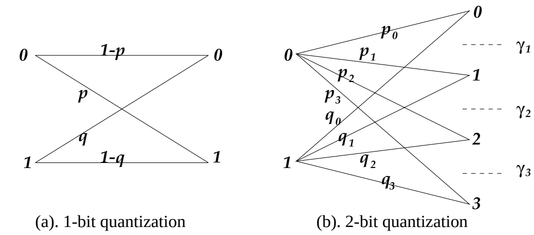

Fig. 2. Asymmetric channels with 1-bit and 2-bit Quantization.

- *1-bit Quantization* – With 1-bit hard receiver, the above channel models are reduced to asymmetric channels with binary-input and binary-output as shown in Fig. 2(a). This BAC model has been successfully used with Reed-Solomon codes to correct errors.
- *2-bit Quantization* – The increasing demand for higher data rate and higher throughput has directed research trend to look into more sophisticated coding techniques like iterative decoding and soft decoding, which in turn requires more bits to convey the reliability information. Research on turbo codes and turbo product codes have shown that internal data size of 4 bits is about necessary and sufficient to achieve near-soft-decision performance. Nonetheless, for applications like optical fiber communications where complexity is critical, the current technology seems to suggest 2 bits be a reasonable trade-off.

CHANNEL CAPACITY WITH SOFT RECEIVER

A. Computation of Channel Capacity with Soft Receiver

With soft receiver, the channel is a binary input, continuous output channel characterized by additive noise. The ultimate channel capacity (or the Shannon limit) is defined as the maximum channel mutual information (MI):

$$\begin{aligned}\mathcal{C}_{soft} &= \max_{\Pr(x)} I(X,Y) \\ &= \max_{\Pr(x)} \sum_{x=0}^{1} \int \Pr(x) p(y|x) \log \frac{p(y|x)}{p(y)} dy \\ &= \max_{0\le\Delta\le1} \Delta \int f_0(y)\Big(A\log A - (A+1)\log(A+1) \\ &\qquad - A\log(1-\Delta) - \log\Delta\Big) dy, \quad (9)\end{aligned}$$

where $A = \frac{(1-\Delta)f_1(y)}{\Delta f_0(y)}$, $\Delta = \Pr(x=0)$ is the probability of sending "0", and $f_0(\cdot)$ and $f_1(\cdot)$ are defined in (1) and (2) for Chi-square channels and in (5) and (6) for Asymmetric Gaussian channels.

The maximum value, i.e., the ultimate channel capacity, can be found by taking derivative on Δ. Since there is no simple, closed-form expression for this, numerical approach is taken to examine the capacity of Chi-square channels as well as asymmetric Gaussian channels under soft decision.

As mentioned before, the asymmetry property of the channel will lead the desired channel input to be unevenly distributed. In many practical situations, the probabilities of transmitting "0"s and "1"s are approximately the same. Hence, substituting $\Delta = 1/2$ in (9) we have the "practical capacity" as

$$\mathcal{C}^*_{soft} = \frac{1}{2} \int f_0(y)\Big(A^* \log A^* + (A^*+1) \log \frac{2}{A^*+1}\Big) dy, \quad (10)$$

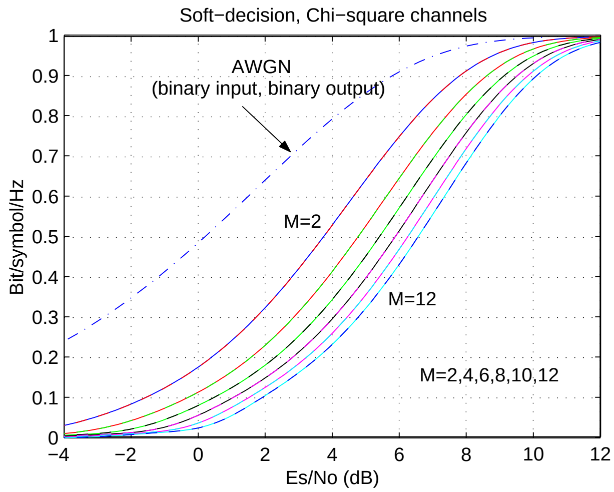

Fig. 3. Capacities of Chi-square channels with soft receiver.

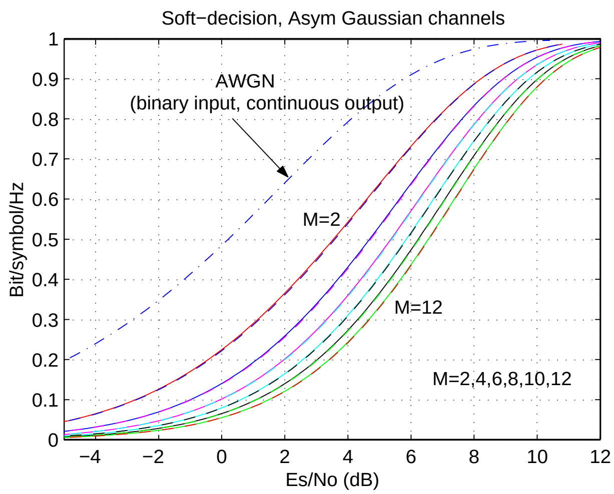

Fig. 4. Capacities of asymmetric Gaussian channels with soft receiver.

where $A^* = \frac{f_1(y)}{f_0(y)}$.

B. Numerical Results

Fig. 3 and 4 plot the capacities of Chi-square and asymmetric Gaussian channels with soft receiver for several values of the system parameter M. We observe that as M increases, capacity reduces (from left to right, $M = 2, 4, 6, 8, 10, 12$). This is understandable. Recall that $M = B_o/B_e$; hence, the increase of M implies that the bandwidth of the electronic filter is decreasing and, consequently, is prone to filter out some of the signal energy. In the plot, solid lines are the "ultimate capacity" with optimized channel input, and dashed lines which are right behind the solid line (and are hardly observable) are the "practical capacity" with equally probable of channel input. It is obvious from the plot that the difference between the ultimate capacity and the practical capacity is negligible. In fact, as shown in Fig. 5, the optimal Δ is seen to be within the range of 0.47 to 0.52 for the above values of M, which is very close to the practical case of equal probability. For comparison purpose, also presented are the capacities of continuous output AWGN channels with ASK signaling (dash-dotted lines). It should be noted that one needs to proceed with caution when interpreting the observation that optical fiber communication channels seem to have worse capacities than AWGN channels, since the quantity E_s/N_0 does not represent comparable conditions on the two channels.

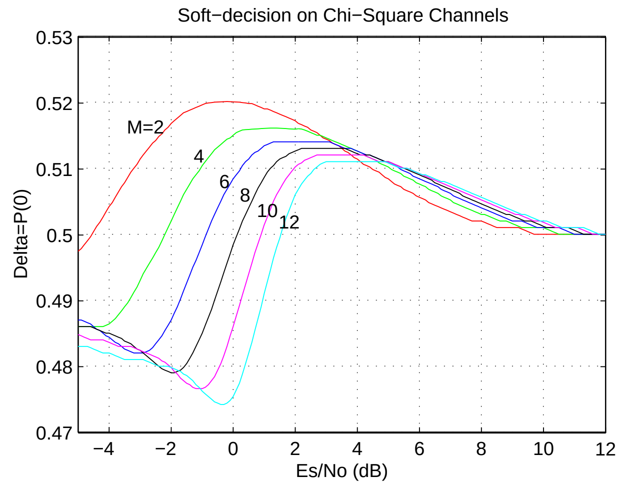

Fig. 5. Optical channel input for Chi-square channels.

Comparing the Chi-square channel model and the asymmetric Gaussian channel model (Fig. 3 and 4), we see that asymmetric Gaussian channels have a slight higher capacity than Chi-square channels. This is because when the second order statistics are the same, Gaussian pdf maximizes the entropy amount all distributions.

CHANNEL CAPACITY WITH 1-BIT HARD RECEIVER

A. Computation of Channel Capacity with 1-Bit Quantization

With 1-bit hard receiver, the channel is reduced to binary asymmetric channels as shown in Fig. 2(a). Denoting the channel cross-over probabilities as $p \triangleq \Pr(1|0)$ and $q \triangleq \Pr(0|1)$ (which will be evaluated shortly after), the ultimate channel capacity is then derived as:

$$\mathcal{C}_{1-bit} = \max_{0 \le \Delta \le 1} H_2\Big(\Delta(1-q)+(1-\Delta)p\Big) - \Delta H_2(q) - (1-\Delta)H_2(p), \quad (11)$$

where $\Delta = \Pr(x = 0)$ and $H_2(x)$ is given by $H_2(x) = -x\log(x) - (1-x)\log(1-x)$.

Notice that $I(x, y)$ is a concave function. By taking derivative on Δ, we can get that the maximum value. The ultimate channel capacity, is achieved when

$$\Delta = \frac{1 - p - pB}{(1-p-q)(1+B)}. \quad (12)$$

where $B = \exp(\frac{H_2(q)-H_2(p)}{1-p-q})$. It is worth pointing out that here we have used natural logarithm (base-e) and, hence, the unit of the capacity $\mathcal{C}$ is *nat*. If base-2 logarithm is used, the unit of $\mathcal{C}$ is then *bit* and corresponding $B = 2^{(H_2(q)-H_2(p))/(1-p-q)}$. We have $\mathcal{C}_{nat} = \ln 2 \cdot \mathcal{C}_{bit}$.

As in the soft decision case, if we assuming equally probable of input, we can compute the "practical capacity"

$$\mathcal{C}^*_{1-bit} = H_2\Big(\frac{1+p-q}{2}\Big) - \frac{1}{2}H_2(p) - \frac{1}{2}H_2(q). \quad (13)$$

B. Evaluation of Channel Cross-Over Probabilities

Chi-square Channels – Denote γ as the optimal decision threshold such that $f_0(\gamma) = f_1(\gamma)$. The channel cross-over probabilities of Chi-square channels can then be computed as

$$p \triangleq \Pr(1|0) = \int_\gamma^\infty f_0(x)dx = e^{-\frac{\gamma}{N_0}} \sum_{k=0}^{M-1} \frac{1}{k!}(\frac{\gamma}{N_0})^k, \quad (14)$$

$$q \triangleq \Pr(0|1) = \int_0^\gamma f_1(x)dx = 1 - \mathcal{Q}_M\Big(\sqrt{\frac{4E_s}{N_0}}, \sqrt{\frac{2\gamma}{N_0}}\Big), \quad (15)$$

where $\mathcal{Q}_M(a,b)$ is the generalized Marcum $\mathcal{Q}$ function of order M defined as

$$\mathcal{Q}_M(a,b) = \int_b^\infty \frac{x^M}{a^{M-1}} \exp(-\frac{x^2+a^2}{2}) I_{M-1}(ax)dx. \quad (16)$$

There is no simple, closed-form expression for calculating the the generalized Marcum $\mathcal{Q}$ function, but highly reliable and efficient numerical methods are available as discussed in [4]. Hence, the determination of the optimal threshold γ and the channel crossover probabilities can take a numerical approach.

Asymmetric Gaussian Channels – With asymmetric Gaussian channels characterized in (5) and (6), the crossover probability is given by

$$p \triangleq \Pr(1|0) = \mathrm{Q}\Big(\frac{\gamma - \mu_0}{\sigma_0}\Big), \quad (17)$$

$$q \triangleq \Pr(0|1) = 1 - \mathrm{Q}\Big(\frac{\gamma - \mu_1}{\sigma_1}\Big), \quad (18)$$

where Q-function is the tail integral of a normalized Gaussian density given by $\mathrm{Q}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$, and the optimal threshold γ is given by letting

$$\frac{(\gamma-\mu_1)^2}{\sigma_1^2} - \frac{(\gamma-\mu_0)^2}{\sigma_0^2} = 2\log\frac{\sigma_0}{\sigma_1}, \quad (19)$$

which leads to

$$\gamma = \frac{\mu_0}{2} + \sigma_0 \sqrt{\frac{E_s}{N_0} + \frac{M}{4} - \frac{\sigma_1^2}{\sigma_1^2 - \sigma_0^2} \log\frac{\sigma_0^2}{\sigma_1^2}}. \quad (20)$$

C. Numerical Results

Similar to the soft receiver case, we plot the capacities of two channel models with binary output as well as that of the binary symmetric channel (i.e. BSC, or AWGN channels with binary output). As shown in Fig. 6 and 7, many of the same observations we get from the continuous output case also prevail in the binary output case. Comparing Fig 6 and 7 to Fig. 3 and 4, we observe that converting from an infinite-precision soft receiver to a simple binary hard receiver incurs a loss of about 1 dB in channel capacity.

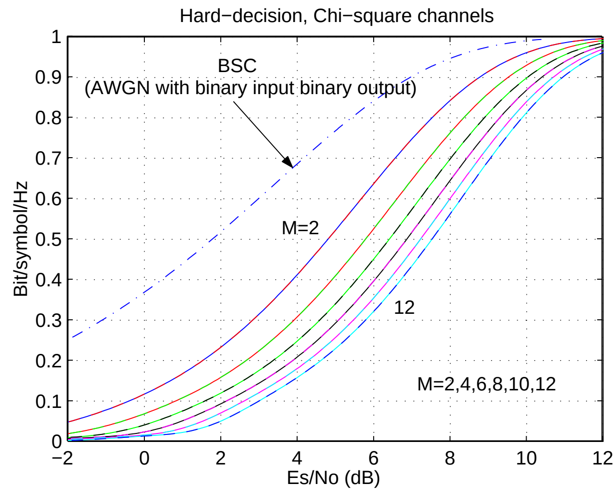

Fig. 6. Capacities of Chi-square channels with 1-bit quantization.

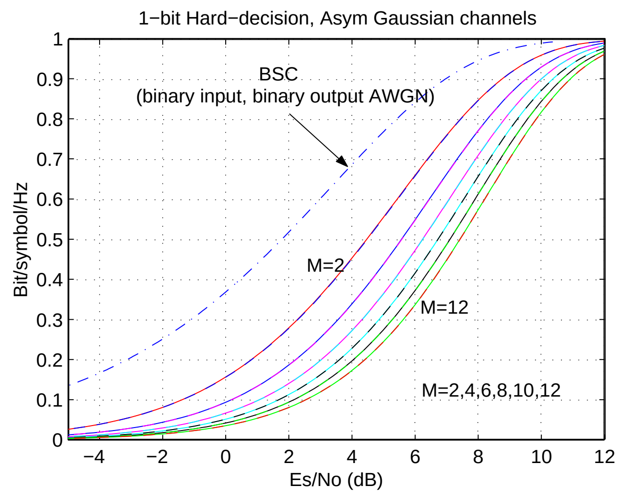

Fig. 7. Capacities of asymmetric Gaussian channels with 1-bit quantization.

CHANNEL CAPACITY WITH 2-BIT HARD RECEIVER

A. A General Case of n-Bit Quantization

For a general binary-input, 2^n-output channels with channel input probability $\Delta = \Pr(x=0)$ (see Fig. 1(B)), the channel capacity can be derived as

$$C^*_{n-bit} = \max_\Delta \sum_{i=0}^{2^n-1} \Big(H(\Delta p_i + (1-\Delta)q_i) - \Delta H(p_i) - (1-\Delta)H(q_i)\Big), \quad (21)$$

where $\Delta = \Pr(x=0)$ and $H(x) = -x\log(x)$.

This by itself is quite simple. However, complications arise when (2^n-1) optimal thresholds γ_i, $i = 1, 2, \cdots, 2^n-1$ have be to determined in order to get proper transition probabilities of p_i and q_i for $i = 0, 1, \cdots, 2^n-1$, and in the mean time Δ has to be optimized (for asymmetric channels) for maximum value of mutual information.

It is worth pointing out that the channel input distribution that is optimal for a continuous-output asymmetric channel is not necessarily optimal for its discretized counterparts with n-bit quantization, and that the optimal channel input for an n-bit quantization does not necessarily carry over for a $(n+1)$-bit quantization. However, the analysis of continuous output and binary output cases show that for both Chi-square channels and asymmetric Gaussian channels, the optimal channel input is not much different from the practical case of equal-probable channel input, and that the degradation of the "practical capacity" from the "ultimate capacity" is negligible. Hence, to make the problem more tractable, we consider equal-probable channel input for the 2-bit quantization case.

B. Computation of Channel Capacity with 2-Bit Quantization

As shown in Fig. 2(b), thresholds γ_1, γ_2 and γ_3 mark the decision regions for the four 2-bit quantized outputs, denoted as 0, 1, 2, and 3, respectively. Denote

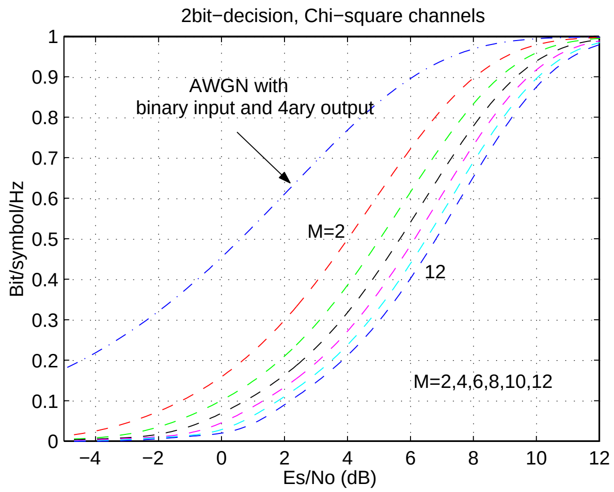

Fig. 8. Capacities of Chi-square channels with 2-bit quantization.

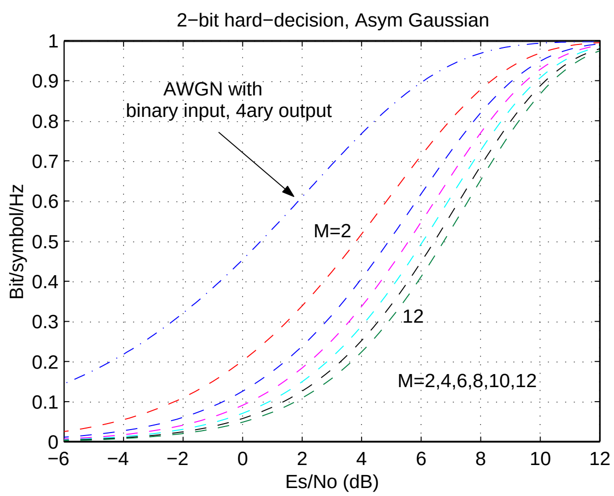

Fig. 9. Capacities of asymmetric Gaussian channels with 2-bit quantization.

$$\alpha_i \triangleq \int_{\gamma_i}^{\infty} f_0(x)\,dx, \qquad i = 1, 2, 3, \tag{22}$$

$$\beta_i \triangleq \int_{-\infty}^{\gamma_i} f_1(x)\,dx, \qquad i = 1, 2, 3, \tag{23}$$

we have the transition matrix of the equivalent channel:

$$\begin{aligned}\Pi &= \begin{pmatrix} p_0 & p_1 & p_2 & p_3 \\ q_0 & q_1 & q_2 & q_3 \end{pmatrix} \\ &= \begin{pmatrix} 1-\alpha_1 & \alpha_1-\alpha_2 & \alpha_2-\alpha_3 & \alpha_3 \\ \beta_1 & \beta_2-\beta_1 & \beta_3-\beta_2 & 1-\beta_3 \end{pmatrix} \end{aligned} \tag{24}$$

Substituting them in (21) and assuming $\Delta = 1/2$, we have

$$\begin{aligned} \mathcal{C}^*_{2-bit} =& H\left(\frac{1-\delta_1}{2}\right) + H\left(\frac{\delta_1-\delta_2}{2}\right) + H\left(\frac{\delta_2-\delta_3}{2}\right) + H\left(\frac{\delta_3+1}{2}\right) \\ &- \frac{1}{2}\Big(H(1-\alpha_1)+H(\alpha_1-\alpha_2)+H(\alpha_2-\alpha_3)+H(\alpha_3)\Big) \\ &- \frac{1}{2}\Big(H(\beta_1)+H(\beta_2-\beta_1)+H(\beta_3-\beta_2)+H(1-\beta_3)\Big). \end{aligned} \tag{25}$$

where $\delta_i = \alpha_i - \beta_i$ for $i = 1, 2, 3$.

Analytical evaluation of (25) (or (21) for $n = 2$) is possible by noting the following facts:

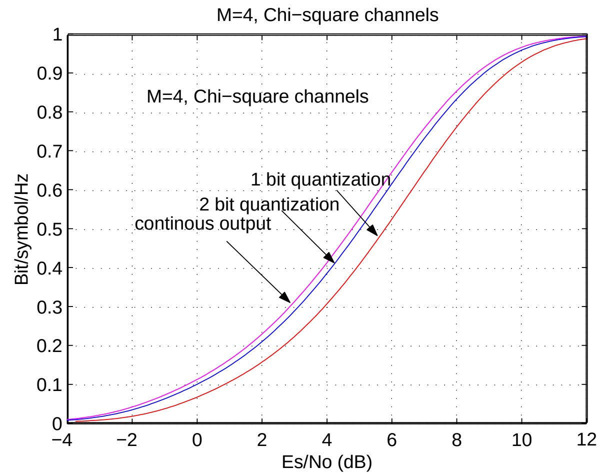

Fig. 10. Effect of quantization on the capacities of Chi-square channels ($M = 4$).

- γ_2 is the same optimized threshold in the binary output case as discussed in the previous section. In other words, $f_0(\gamma_2) = f_1(\gamma_2)$.
- Given optimized value of γ_2, the capacity of the 2-bit quantized channel, $\mathcal{C}^*_{2-bit}(\gamma_1, \gamma_3)$, is a concave plane on the normalized values of γ_1 and γ_3 for $0 < \gamma_1 < \gamma_2 < \gamma 3 < 1$.

Hence, by taking partial derivatives on γ_1 and γ_3, we can obtain the relations among γ_1, γ_3 and γ_2^* which will lead to the optimal values of γ_1^* and γ_3^* for which (25) is maximized. However, due to the integration involved, it is more convenient to use a numerical approach. A straight-forward approach is to exhaustively search the 2 dimensional space of $(0, \gamma_2^*) \times (\gamma_2^*, 1)$. Smarter approaches like newton method and the steepest descent algorithm can be used and there is no worry of getting stuck in non-optimal local maxima due to the concavity of $\mathcal{C}^*_{2-bit}(\gamma_1, \gamma_3)$. Specifically, an iterative approach can be used, where we first optimize γ_1 for an initial selection of γ_3, then optimize γ_3 on the "then-optimal" value of γ_1, then back gain to optimize γ_1, and so on. Experiments show that the convergence is quite fast.

C. Numerical Results

Fig. 8 and 9 present the capacities of binary-input, 4-ary-output channels with Chi-square and asymmetric Gaussian noise. As mentioned before, we have assumed equally probable of channel input for simplicity, but the results we obtain should be just as good as the case of optimal channel input for practical purpose. We observe that the curves are quite consistent with those of continuous output and binary output cases.

To better illustrate the effect of quantization, we plot in Fig. 10 the capacities of continuous output, binary-output, and 4-ary-output of a Chi-square channel with $M = 4$. Two things are immediately observable. First, we see that quantization has a larger impact on lower code rates than higher code rates. Second, while 1-bit quantization incurs a noticeable loss in channel capacity, 2-bit quantization leads to a much smaller degradation. For example, we see about 1 dB loss at rate 0.5 using 1-bit quantization, yet only about 0.25 dB loss for 2-bit quantization. This suggests that 2-bit quantization is, from the capacity perspective, a good trade-off for systems where complexity is critical. Although we will not show in this paper, we note that the above analytical results of the effect of quantization on the channel capacity is consistent with the simulation results of the effect of quantization on the system performance employing FEC coding.

PERFORMANCE OF FEC CODING

In this section, we report the latest simulation results of some of the best-known FEC coding schemes to give an idea of how much has been achieved and how much is yet to be achieved. We consider turbo codes using iterative soft-input and soft-output (SISO) decoding. It is instructive to point out that in evaluating the BER performance on optical fiber communication channels, it is more accepted to use Q factor to measure the channel condition rather than E_s/N_0, where Q is defined as $Q = \frac{|\mu_1-\mu_0|}{\sigma_1+\sigma_0}$. This Q factor in dB ($10\log_{10}(Q^2)$) is sometimes referred to as the *gross* Q (as opposed to *net* Q), since code rate penalty is not taken into consideration. Note that since the mean and variance of the channel output is a function of the system parameter M, Q factor depends not only on E_s/N_0, but also on system parameter M. In other words, the same E_s/N_0 value translates to different Q values for different values of M.

As a breakthrough in the coding research, turbo codes (also known as parallel concatenated convolutional codes or PCCC), have demonstrated near-capacity performance on AWGN and Rayleigh fading channels [5]. They have been listed in the standard of 3G wireless communications standards, and have been in active research for various applications, including deep space communications and data storage systems. In this work, we report the simulation results of a 16-state turbo code with generator polynomial $[1, (1+D+D^2+D^4)/(1+D^3+D^4)]$ on Chi-square and asymmetric Gaussian channels. Uniform puncturing is used to obtain high code rates and S-random interleavers are used to push down the error floors.

We use a soft decoder to iterate extrinsic information between the two convolutional subdecoders each of which implement the log-domain BCJR algorithm [6]. The BCJR algorithm involves the computation of α_k (forward path metric), β_k (backward path metric) and γ_k (transition branch metric). Whereas α_k and β_k are computed through forward and backward recursions independent of the underlying channel, the computation of the branch metric γ_k (or the evaluation of the *a priori* information from the channel) needs to incorporate the channel characteristics. Hence, the turbo decoder on Chi-square channels is slightly different from the conventional AWGN case. We assume that the channel parameters are perfectly estimated and, hence, the extrinsic information is correctly weighed and the decoder is optimized for Chi-square channels. More detailed discussion of soft-decodable codes on long-haul optical channels can be found in [7] [8] [9].
Fig. 11 shows the performance of a block size 8K, rate 0.8 and rate 0.9 turbo codes on Chi-square channels. We plot bit error rate (BER) vs gross Q^2 (in dB), and the performance is evaluated at after 8decoding iterations. As we can see, with moderate block sizes, turbo codes can perform about 0.8-0.9 dB (in Q factor) from the channel capacity, which is very impressive.

CONCLUSION

We have investigated the capacities of Chi-square channels with continuous output, binary output and 2-bit quantized output, respectively. The basic idea is actually quite simple. Complications arise from the necessity of having to evaluate several integrals. In this work, we take a numerical approach to compute and examine the capacities for several values of M. We have also reported the simulation results of some of the best-known error correction codes on optical fiber communication channel. The major results of this work are summarized as follows:

- The Chi-square and the asymmetric Gaussian channel models used in the optical fiber communications are not very asymmetric, in the sense that the optimal channel input is near equally probable and that the difference between the ultimate capacity and the practical capacity is negligible. However, it is instructive to note that they are very asymmetric in the sense that the optimal decision threshold is nowhere close to the midpoint of the two transmitted signals [1].
- For both Chi-square channel model and its asymmetric Gaussian approximation, the capacity decreases with the increase of the system parameter M.
- Whereas 2-bit quantization leads to a minor capacity loss of around 0.2 dB, the loss incurred by 1-bit quantization is as much as 1 dB. This indicates that 2-bit quantization is a good trade-off (from the capacity perspective) between performance and complexity.
- Gaussian approximation leads to a higher channel capacity than the original Chi-square channels given the same second order statistics.
- With iterative soft decoding, a block size of 8K, 16-state turbo code can perform within 1 dB (in Q factor) from the channel capacity for high rates of 0.8 and 0.9.

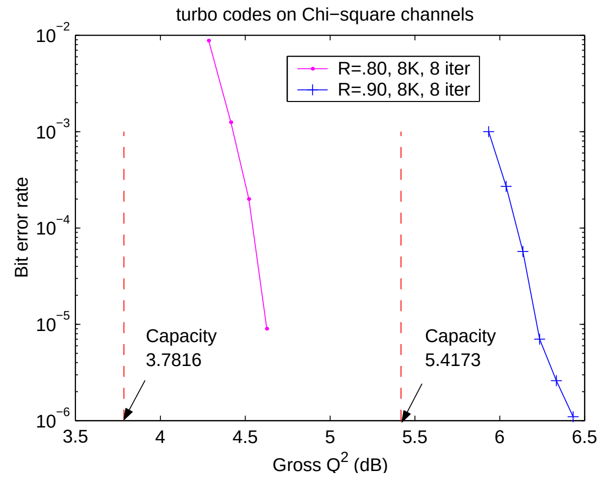

Fig. 11. Performance of high-rate turbo codes on Chi-square channels (soft decoding).

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