Decode-Amplify-Forward (DAF): A New Class of Forwarding Strategy for Wireless Relay Channels

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***Abstract*— A new class of forwarding strategy, termed *decode-amplify-forward* (DAF), is proposed for relay channels. By exploiting the coding gain on the inter-user channel and maximizing the data fidelity using soft reliability representation, the proposed DAF strategy has cleverly combined the merits of both *decode-forward* (DF) and *amplify-forward* (AF). To fully harness the power promised by the DF-extended schemes such as *coded cooperation*, we further propose a DAF-DF mixed strategy that blends DAF and DF via time sharing. The considerable gains enabled by the proposed DAF and especially the DAF-DF strategies are demonstrated using mutual information and capacity analysis as well as extensive simulations on fast and slow fading channels.**

I. INTRODUCTION

Signal relaying technology is being seriously considered in wireless networks. To gain advantage over the classical point-to-point communications, sets of single-antenna terminals in a multi-user system relay signals for each other, at the cost of power, bandwidth, and/or simplicity, so that reliable communications can be achieved at higher data rates. The study of the *relay channel*, also known as *user cooperation* or *cooperative communications*, could be traced back to the work of van der Meulen [1]. Groundbreaking work done by Cover and El Gamal in [2] proposed several relaying strategies and extensively investigated information theoretic properties based on additive white Gaussian noise (AWGN) channels. Motivated by the flourishing wireless network, recent researches have largely focused on fading channels [3]-[9].

A typical relay scenario, shown in Fig. 1, consists of a source (S), a relay (R) and a destination (D). Considerable gains can be achieved by signal relaying in wireless environments especially in fading channels, including the reduction in outage probabilities and the increase in diversity, capacity and dynamic range [4][5]. Various practical schemes have been proposed to exploit the cooperative benefits, which are usually classified into two categories: *amplifier-forward* (AF) and *decode-forward* (DF). In AF [3], the relay simply scales and retransmits the analog signal waveform received from the source. In DF [4][6][9], the signal received from the source is demodulated and decoded before retransmission. Since the erroneously decoded bits at the relay could lead to severe error propagation, the current DF strategy typically includes an option to switch to the non-cooperative mode in the case of failed cyclic redundant check (CRC).

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Compared with DF, AF requires lower implementation complexity in digital signal processing at the relay node, and is operable at all channel conditions including when the inter-user channel is at outage. On the other hand, when the inter-user channel has a good quality and the signals can be decoded correctly, DF promises a higher gain and opens the possibility for more sophisticated strategies such as *coded cooperation* (CC) [6] (in which the data are re-encoded at the relay to obtain a different set of parity bits).

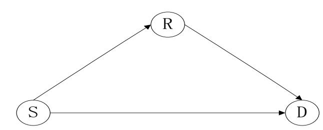

Fig. 1. The scenario of relay channel

From the coding perspective, *amplify-forward* can be viewed as a way of forwarding the signal in its soft-reliability form (will be discussed further in Section III). However, failing to exploit the channel code at the relay degrades the performance. *Decode-forward*, on the other hand, takes advantage of the channel code but the (decoded) signal is forwarded in its hard-decision form with no reliability information. Motivated by this observation, we propose in this paper a new signal relaying strategy to cleverly combine the merits of these strategies, i.e. soft signal representation in AF and channel coding gain in DF.

We discuss in detail the proposed *decode-amplify-forward* (DAF) strategy and quantify its potential gain by evaluating the source-relay mutual information on independent fading channels. Through analysis and simulations, we show that DAF outperforms AF at all times. It also exhibits a better performance than DF in the region of medium to low signal-to-noise ratios (SNR). To fully exploit the benefits enabled by DF and especially DF-extended cooperative strategies at the high SNR region, we further propose a DAF-DF mixed forwarding strategy. It is shown that DAF-DF offers considerable gains over all the other strategies under all channel conditions.

The rest of the paper is organized as follows. The system model is presented in Section II. Section III discusses the proposed DAF and DAF-DF strategy. Section IV analyzes these strategies in terms of mutual information and capacity. Simulation results in slow and fast fading scenarios are shown in Section V and Section VI concludes the paper.

II. System Model

Consider the system in Fig. 1. At the first time slot, the source broadcasts to both the destination and the relay. At the second time slot, the relay forwards the information received at the previous period to the destination. The destination then combines the signals from the source and the relay to reconstruct the original information.

We consider half-duplex systems. All the transmission channels follow a frequency-flat Rayleigh fading model:

$$y = \alpha x + n, \tag{1}$$

where α is the complex Rayleigh fading coefficient, n is the complex AWGN and $x \in \{-1, +1\}$ is transmitted signal modulated using the binary phase shift keying (BPSK). In slow fading cases, the fading coefficient is constant within one block but independent between different blocks. In fast fading cases, the fading coefficient changes independently from bit to bit. In the sequel, we will use subscripts S, R, D and SR, SD, RD to denote the quantities pertaining to the source, the relay, the destination, and the inter-user channel, the source-destination channel and the relay-destination channel. We further assume that all the transmission channels are spatially independent, and that the channel gain is known to the respective receiver but unknown to the transmitter. Hence, channel state information (CSI) can be exploited to assist decoding, but transmission power adaptation is not possible.

III. The Proposed New Forwarding Strategies

A. Decode-Amplify-Forward (DAF)

To better motivate the proposed DAF strategy, let us first take a close look at the practice of the two basic relay strategies, AF and DF.

In AF, the relay node scales (i.e. normalizes) the energy of each packet. Mathematically, this can be formulated as multiplying all the signals (bits) in the same received codeword, $y_{SR,i}$, by a common factor β, and re-transmitting them to the destination, i.e:

$$x_{RD,i} = \beta \; y_{SR,i}, \qquad i = 1, 2, \cdots, N, \tag{2}$$

where N is the length of the codeword, $x_{RD,i}$ is the ith retransmitted signal at the relay in AF, and

$$\beta = \sqrt{\frac{N}{\sum_{i=1}^{N} y_{SR,i}^2}}. \tag{3}$$

It is well-known that even without decoding, a (soft) reliability value in the form of log-likelihood ratio (LLR), thereafter referred to as *channel LLR*, can be computed from the received signals for each bit. For Rayleigh channels with known CSI, the channel LLR for bit $y_{R,i}$ is given by

$$L_{SR,i}^{ch} = \frac{2\alpha_{SR}}{\sigma_{SR}^2} y_{SR,i}. \tag{4}$$

Since the re-transmitted signal, $x_{RD,i}$, and the channel LLR, $L_{SR,i}^{ch}$, both scale linearly with $y_{SR,i}$, they are also linearly proportional to each other. Hence, amplifying a received signal is equivalent to amplifying its channel LLR:

$$x_{RD,i} = \Big(\frac{\beta \sigma_{SR}^2}{2\alpha_{SR}}\Big) L_{SR,i}^{ch} = \beta' L_{SR,i}^{ch}. \tag{5}$$

Put another way, the AF mode can be interpreted as a strategy where the error correction code (assume there is one) is not exploited, but the forwarded data are represented in their soft-reliability form.

On the other side, DF appears to constitute just the opposite elements. That is, the error correction code is fully exploited (through channel decoding at the relay), but the forwarded data are represented in their (binary) hard-decision form.

It thus becomes clear that a desirable strategy could combine the advantage of both relay modes: the error correction code in DF and the soft representation of the data in AF. This is the rational behind the proposed *decode-amplify-forward* relay mode. In the DAF mode, the relay will first soft decode the error correcting code; but instead of making the final (binary) decision for each bit, it amplifies their LLR values at the output of the decoder, thereafter referred to as *decoder LLR*, and forwards them to the destination. The destination then seeks to recover the original information by jointly decoding the source packet and the relay packet.

Since decoder LLRs can essentially be viewed as improved versions of channel LLRs, the destination can adopt the same decoding strategy used in AF. The exact algorithm depends on the actual code in use, but the input LLRs to the (soft) decoder at the destination should be computed through maximum ratio combining (MRC) of the two packets:

$$L_{D,i}^{ch} = \frac{2\alpha_{SD} y_{SD,i}}{\sigma_{SD}^2} + \frac{2\alpha_{RD} y_{RD,i}}{\sigma_{RD}^2}. \tag{6}$$

We note that DAF requires the availability of a soft-output decoder. Thanks to the advances in channeling coding research, this is no longer a problem for most codes, especially the class of capacity-approaching codes (which tend to be the choice for wireless communications). Good candidate decoders include, for example, the BCJR or the SOVA (soft-output Viterbi algorithm) decoder for convolutional codes, the message-passing decoder for low density parity check (LDPC) codes, the BCJR- or SOVA-based iterative decoder for turbo codes, and the Chase-iterative decoder for product codes.

B. DAF-DF Mixed Forwarding Strategy

Exploiting the error correcting code at the relay enables more reliable LLR values be forwarded to the destination, and DAF is therefore expected to outperform AF (confirmed by analysis and simulations in Sections IV and V). Considering that DF performs worse than AF at low SNRs, DAF is therefore the best strategy at low SNRs, or more precisely, when the inter-user channel is at outage. (By inter-user outage, we mean when the relay fails to get a clean copy of the packet.) On the other hand, when the relay decodes the packet correctly, DF would outstand others since the relay could easily exploit better coding gains by re-encoding the data using a different code (*coded cooperation* [6]), or better diversity gain by space-time transmission (*space-time cooperation* [7]), or both (*coded space-time cooperation* [9], *coded double space-time cooperation* [8]). Hence, to take full advantage of the available transmission strategies, we further propose a DAF-DF mixed forwarding mode, where the relay switches to DAF at inter-user channel outage, and DF otherwise. A simple flag bit can be piggybacked in the relay packet to notify the destination which case happened.

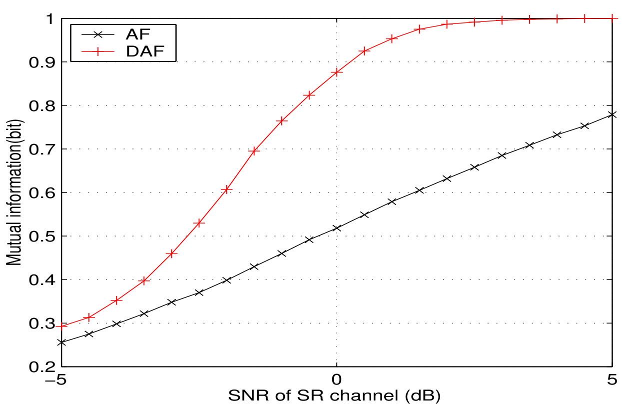

Fig. 2. Mutual information between the signal transmitted by the source node and the signal forwarded by the relay node

IV. Performance Analysis

A. Mutual Information

To see how much gain DAF obtained over AF by processing codes at the relay, we first quantify the mutual information using practical channel codes. Due to the lack of formal methods in evaluating the "capacity" of a practical channel code, here we use a combined analytical and Monte Carlo method, similar to that used in extrinsic information transfer (EXIT) charts [10]. The compound channel between the source and the destination consists of three segments and is therefore hard to characterize. To simplify the problem, we put aside the source-destination and relay-destination channel, and focus on the mutual information of in the source-relay channel, since this can be roughly taken as the "excess rate" enabled by DAF (compared to AF).

Let X be the (binary) signal transmitted by the source and A be the (analog) signal forwarded by the relay, the mutual information between them can be computed as

$$I(X;A) = \frac{1}{2}\sum_{x=-1,1}\int_{-\infty}^{+\infty} p_A(\xi|X=x) \cdot \log_2 \frac{2p_A(\xi|X=x)}{p_A(\xi|X=-1)+p_A(\xi|X=1)}d\xi, \quad (7)$$

where $p_A(\xi|X=x)$ is the conditional pdf of A, and $0 \le I(X;A) \le 1$. In the actual computation, $p_A(\xi|X=x)$ is approximated by the histogram collected from the Monte Carlo simulation using large block sizes. Since ergodicity is required, i.i.d fading coefficients are used, which represent the best possible performance.

Figure 2 illustrates the mutual information of the AF and DAF assuming that the SR link is protected using a rate 1/2 recursive systematic convolutional (RSC) code with generator polynomial $(1, 35/23)_{oct}$ and length of 10^7. Even though convolutional codes are not powerful codes, we see that DAF outperforms AF by a considerable margin.

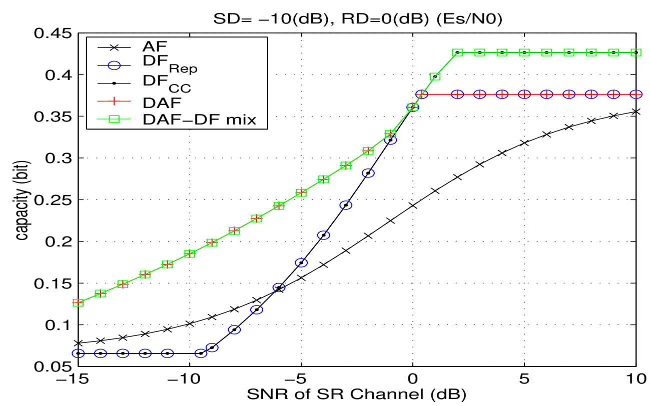

Fig. 3. Theoretical capacity bounds for different forwarding strategies

B. Capacity

For a complete and exact analysis, we further evaluate the theoretical ergodic capacities for the above forwarding strategies. The capacity of the AF scheme can be obtained using the same formula for a 1-by-2 multi-input multi-output system. The capacity of the DF scheme can be computed using the *min-cut max-flow* theory. The evaluation of the DAF scheme becomes more involved, due to the intermediate process at the relay. The idea here is to consider the channel between the source and the soft output of the decoder at the relay as a better inter-user channel. For Gaussian channels (fading channels can be viewed as time-varying Gaussian channels), theory of message passing and density evolution assures that the soft decoder outputs (for any linear channel code) follow (approximated) Gaussian distributions [10]. In this sense, the impact of the soft-decodable channel code (exploited at the relay) can be interpreted as transferring a Gaussian channel to one with a higher signal-to-noise ratio (SNR); or equivalently, a DAF scheme can be taken as an AF scheme operated on a better inter-user channel. Hence, by computing the effective SNR of the better source-relay channel using the rate distortion theory [11] and subsequently adopting the treatment of AF, we can obtain the capacity of DAF. Due to the space limitation, we omit the detailed discussion. Interested users pleaser refer to [12].

The the capacity results of Gaussian relay channels are provided in Fig. 3. For the DF strategy, we evaluated both the *repetition* and the *coded cooperation* scheme, whose capacities differ only at high SNRs (due to the better coding gain available at *coded cooperation*). The DAF-DF mixed scheme shown in the plot is in fact the *DAF-coded cooperation* mixed scheme, whose capacities are obtained by taking the maximum of DAF and *coded cooperation*. We observe a significant capacity improvement enabled by DAF. At low SNRs in particular, DAF has more than doubled the capacity promised by either AF or DF (be it repetition or coded cooperation)! At high SNRs, the DAF-DF mixed scheme can further boost the achievable information rate by exploiting coding gains.

V. SIMULATIONS

To evaluate the true performance of the proposed scheme, computer simulation is conducted. As an example, this paper considers the above transmission modes using a distributed turbo coding strategy similar to that in *turbo coded cooperation* [9]. At the source node, a $(2048, 1024)$ RSC code with generator polynomial $(1, 35/23)$ is used to protect the data. In the AF and DAF modes, the relay will amplify and forward the channel LLRs and the decoder LLRs of the 1024 systematic bits, respectively. In the DF and DAF-DF modes, upon decoding success, the relay will scramble the systematic bits, re-encode them using the same rate 1/2 RSC code and transmit the new set of 1024 parity bits, thus completing a rate 1/3 turbo code for the destination; in the case of decoding failure, the relay will revert to the non-cooperative mode or the DAF mode. In other words, the DF strategy we simulated is in fact *coded cooperation* and the DAF-DF strategy is DAF-CC.

We consider both slow (block) fading and fast (independent) fading. In each case, we test two scenarios: (I) fixing the two user channels and evaluating the performance as a function of the inter-user channel quality, and (II) fixing the inter-user channel and evaluating the performance as a function of the user channel quality. Block error rate (BLER) is used as a figure of merit.

I - Slow Fading: Fig.4 presents a slow fading case, where the SD and RD channels are both fixed to 20 dB and the SR channel changes from 0 dB to 30 dB. Notice that the BLERs of all the forwarding strategies decrease as the SNR increases, but the rate slows down at high inter-user SNRs as the quality of the two user channels starts to become the limiting factor. We see that DAF outperforms AF by more than 3 dB over the entire SNR region. It also saturates to a slightly lower BLER floor than AF. DF initially has a worse BLER than DAF due to the severe inter-user outage, but the performance picks up at a faster rate, and eventually outperforms DAF and reaches a lower error floor (due to the better coding gain offered by the turbo code). As expected, the DAF-DF mixed mode delivers the best performance of all. Compared with DF, DAF-DF offers a consistent 2-3 dB gain over the entire SNR region. Compared with DAF, DF performs from marginally better at low SNRs to more than 8 dB gain at high SNRs.

I - Fast Fading: Fig. 5 presents the same scenario but on a fast fading channel. The two user channels are fixed to 5 dB and the inter-user channel varies from -2 to 5 dB. The four modes exhibit the same relative qualities as we observed in the slow fading case, but the slopes of the curves and the gaps between the curves are different. Again, DAF offers a consistent gain over AF at all times (≥ 0.5 dB in this case), and the DAF-DF mixed mode provides the best performance, with 2 dB gain over DF at high SNRs and a BLER floor of more than a magnitude lower than DAF and DF. It is also interesting to observe that, in this setup, the curve of DAF looks like a horizontal shift of that of AF, and the curve of DAF-AF looks like a horizontal shift of that of DF. This phenomenon is also present in the slow fading case in Fig.4, but less noticeable.

II - Slow Fading: In Fig. 6, the inter-user SNR is fixed to 10 dB; the SNRs of the two user channels are set equal and they both change from 0 to 30 dB. We can see that the BLER of all the forwarding strategies decreases with the increase of the SNR with no noticeable satiation effect. Furthermore, we find that AF results in the worst performance in all the SNRs tested. DAF starts with a performance on par with AF when the SNR is low, but its BLER drops faster than AF (and DF) as the SNR increases. It also starts to outperform DF at the SNR of around 16 dB, and the performance eventually gets very close DAF-DF at high SNRs. As expected, DAF-DF performs best; but this time, instead of dropping at the same rate as DF, the BLER curve drops at a rate comparable to that of DAF, which is faster than that of DF. Putting together the observations made in this scenario (fixed inter-user channel and varying user channels) and that in the above (fixed user channels and varying inter-user channels), we can conclude that not only does DAF-DF always perform the best, the slope of its BLER curve also adopts the best slope offered by the other three modes.

II - Fast Fading: The advantage of DAF-DF is most obvious in Fig.7. In this testing case, the inter-user channel is fixed to a relatively high SNR value of 5 dB and the two user channels have SNRs from 0 to 5 dB. Since the inter-user channel has a quite good quality, considerable coding gains are achieved by successful coded cooperation in DF

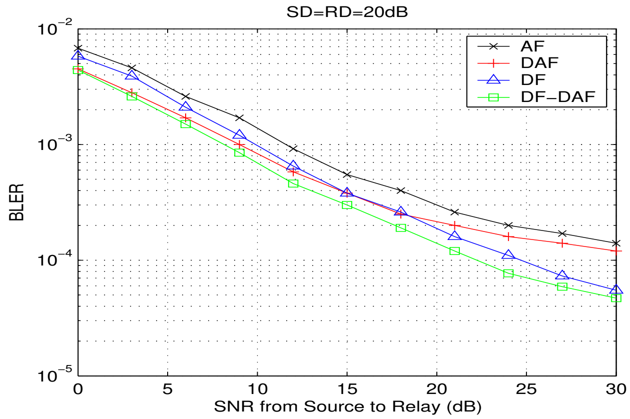

Fig. 4. Performance on slow fading channels. (Scenarios I: $SNR_{SD} = SNR_{RD} = 20$ dB, $0 \leq SNR_{SR} \leq 30$ dB.)

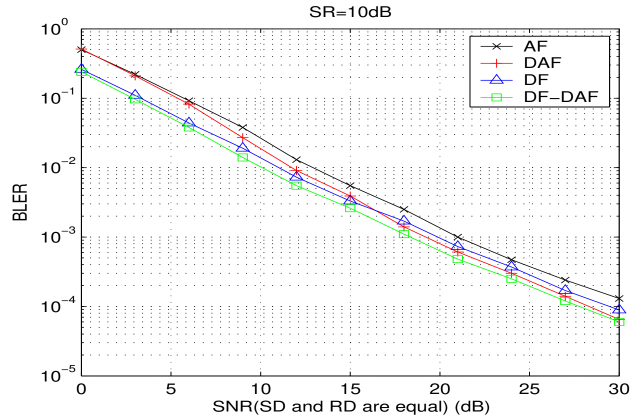

Fig. 6. Performance on slow fading channels. (Scenarios II: $SNR_{SR} = 10$ dB $0 \leq SNR_{SD} = SNR_{RD} \leq 30$ dB.)

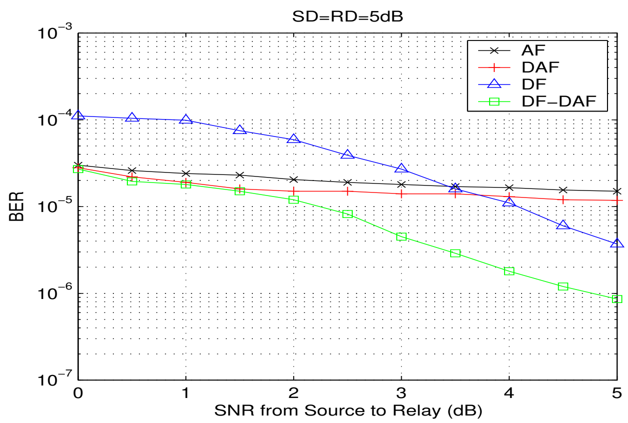

Fig. 5. Performance on fast fading channels. (Scenarios I: $SNR_{SD} = SNR_{RD} = 5$ dB, $-2 \leq SNR_{SR} \leq 5$ dB.)

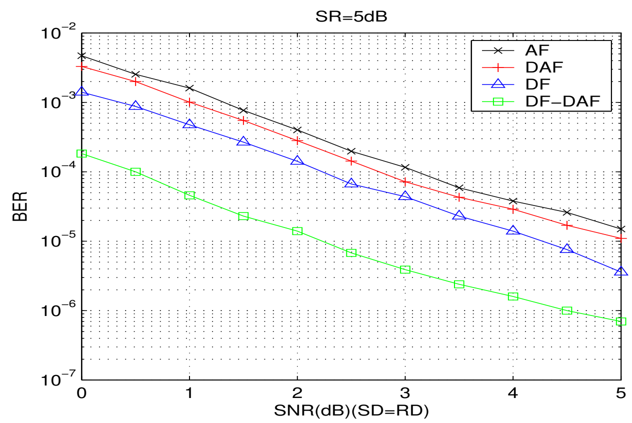

Fig. 7. Performance on fast fading channels. (Scenarios II: $SNR_{SR} = 5$ dB $0 \leq SNR_{SD} = SNR_{RD} \leq 5$ dB.)

and DAF-DF, making them better than DAF and AF. It is encouraging to see that DAF-DF can outperform DF by 2 dB and AF by 3 dB at all times.

VI. Conclusion

By combining the merits of *amplify-forward* and *decode-forward* strategies, we have proposed a new class of forwarding strategy, namely, *decode-amplify-forward*. The idea is to take advantage of the coding gain by performing decoding at the relay, and at the same time take advantage of the soft signal representation by forwarding the decoder LLR values to the destination. To fully harness the power promised by the DF-extended schemes such as *coded cooperation*, we further proposed to combine DAF and DF using time sharing. Analysis and simulations show that the DAF-DF mixed forwarding strategies offers considerable gains over the existing strategies at all times.

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