Exploiting Slepian-Wolf Codes in Wireless User Cooperation

Ruiyuan Hu and Jing (Tiffany) Li
Department of Electrical and Computer Engineering
Lehigh University, Bethlehem, PA 18105

***Abstract*— A novel scheme, termed *Slepian-Wolf cooperation*, is proposed, which exploits distributed source coding technologies in wireless cooperative communication to improve the inter-user outage performance. After motivating the idea, we discuss in detail a simple estimation mechanism, a combined data-syndrome transmission strategy and an optimal joint decoding algorithm. Using low-density parity-check codes as an example, we show that encouraging gains of up to 13 dB for the outage performance and 3.8 dB for the overall performance can be achieved on block Rayleigh fading channels.**

I. Introduction

Cooperative communication for wireless networks has seen a flurry of research in recent years. It allows single-antenna users in a multi-user scenario to share antennas with each other to form a virtual multiple-antenna system. A basic cooperative communication model (also called *relay channel*) contains three parts: the source (S), the relay (R), and the destination (D), as shown in Fig. 1(A). From the information-theoretic aspect, user cooperation can effectively reduce the outage probability and increase the sum rate [1][2].

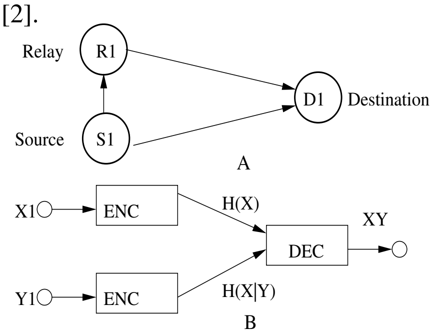

Fig. 1. (A) Relay channel model; (B) Slepian-Wolf system model.

This paper considers two-user cooperation using half-duplex terminals on block fading channels (time-limited channels where user cooperation is most useful). *Practical* user cooperation modes can be roughly grouped in two categories: *amplify-forward* (AF), where the relay rescales and retransmits the received signal waveform, and *decode-forward* (DF), where the relay demodulates and decodes the packet and forwards part or all of the data possibly using a different code. Existing schemes extending the decoder-and-forward (DF) mode (e.g. [3]-[6]) have demonstrated encouraging gains, but depend heavily on the good quality of the inter-user channel. An inter-user outage, although infrequent, could drastically degrade the performance. When the relay does not get a correct copy of the data, DF-based strategies can not be used. However, with the AF strategy, since the packet has been badly corrupted in the inter-user transmission and further distorted in the relay, it tends to be extremely noisy or close to useless [7].

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It has been shown that, in the case of inter-user outage, the asymptotic error probability of AF scales almost linearly with that of DF by a factor of $1/2$ or so [7]. This indicates that AF does not perform much differently from DF in this worst-case scenario. Since the average performance of a system tends to be dominated by the worst case, the finding in [7] also suggests that the performances of AF and DF are on par with each [7], provided that worst cases are not rare. In fact, our simulations show that, even protected with a $(3000, 2000)$ low-density parity-check (LDPC) code, inter-user outage happens at a probability of 10.4%-1.06% at an inter-user SNR of 10-22 dB, which is certainly non-negligible.

The above suggests that the results obtained by looking at only the successful cooperation case, as in most existing papers, are partial and optimistic. In the case where user cooperation is much needed but inter-user link is bad (e.g. emote areas with weak wireless links and sparse users), solving the inter-user outage problem is particularly crucial.

This paper attacks the inter-user outage problem from a non-conventional perspective. An important notion developed here is that *cooperative communication need not be treated as a solely channel coding or transmission strategy problem*. As we will show, the ideas in source coding, and particularly distributed source coding (DSC), can also be exploited in relay channels to considerably improve the performance.

The idea is motivated by the observation that, in the case of inter-user outage, although the relay fails to get the packet entirely correct, it gets most of the bits right most of the time. Take the case of a (3000,2000) LDPC code for example (figure omitted), even at a low SNR of 7 dB, around 85% of the failed blocks contain less than 5% of errors. Hence, instead of having the original data X, the relay now has data Y, which is closely related to X. This then opens the possibility for DSC to pitch in.

Figure 1(B) illustrates the concept of lossless DSC.Also known as Slepian-Wolf coding, lossless DSC refers to the problem of separate encoding and joint decoding of multiple statistically correlated i.i.d. sources [8]. For the two-source case, the famous Slepian-Wolf theorem states that the achievable rate pairs are given by $R_x \geq H(X|Y)$, $R_y \geq H(Y|X)$ and $R_x + R_y \geq H(X, Y)$ [8].

Clearly, user cooperation and DSC are intrinsically related even though the two problems have hardly been placed together before. As shown in Fig. 1, they both have three terminals: S, R and D in user cooperation, and X, Y and Z in DSC. High correlation between data recovered at R and data at S is exactly like the correlation between X and Y. The relay packet, available at the destination, can also be viewed as decoding with side information in DSC. Further, encoding of DSC is performed *separately* at each source, obviating the need for inter-user coordination in cooperative communications. Hence, it seems natural that DSC can be exploited in user cooperation, whose purpose here is to *maximally exploit the spatial diversity offered by the relay channel and at the same time minimizing the bandwidth expansion*. In such a context, the asymmetric compression strategy is especially relevant, where the relay sends data Y at rate $H(Y)$ and the source sends an additional amount $H(X|Y)$ to complete a Slepian-Wolf code. The destination thus has two spatially diversified copies, one from the joint decoding of $H(Y)$ and $H(X|Y)$ and the other from the initial transmission of $H(X)$.

In the remainder of this paper, we will discuss in detail the proposed *Slepian-Wolf cooperation* scheme. We first introduce a simple but effective mechanism for estimating the decoding quality, so that the relay knows when to invoke Slepian-Wolf coding. We then discuss in detail the encoding and transmission strategies, followed by the proposition of an efficient and optimal joint decoding algorithm. Gains of 13 dB and 3.8 dB are demonstrated using the proposed *Slepian-Wolf cooperation* scheme with (short) LDPC codes.

We mention that the notion of compress-forward (CF), where the relay estimates, compresses or quantizes the observation and forwards it to the destination, was put forth back in the seventies [1]. Recent work by Kramer *et al* established the *random* coding framework for compress-forward, and demonstrated the it achieves the gain associated with multi-antenna reception. The proposed SW-cooperation is, to the best of our knowledge, the first *practical* compress-forward scheme. It targets solving the practical bottleneck *when the relay does not get a correct copy of the packet*, and hence exploits *distributed* compression, rather than conventional single-source compression, in the cooperation.

II. Preliminaries

A. Cooperative System

We consider two-user cooperation (Fig. 1(A)) on block Rayleigh fading channels. Let subscripts sd, sr and rd denote the quantities pertaining to the source-destination, source-relay (inter-user) and relay-destination channel. Perfect channel state information (CSI) is assumed known to the receivers but not to the respective transmitter. We consider binary i.i.d. sources. The binary phase shift keying (BPSK) modulated (and possibly encoded) source signal $x \in \{\pm 1\}$ is received by the destination and the relay as,

$$r_{sd} = \sqrt{E_{sd}} h_{sd} x + n_{sd}, \quad r_{sr} = \sqrt{E_{sr}} h_{sr} x + n_{sr}. \quad (1)$$

After processing the received signal, the relay transmits the decoded and possibly re-encoded signal y to the destination through the relay channel,

$$r_{rd} = \sqrt{E_{rd}} h_{rd} y + n_{rd}, \quad (2)$$

where $h \sim \mathcal{N}(0,1)$ denotes the complex-valued channel gain, and $n \sim \mathcal{N}(0,N)$ denotes the additive white Gaussian noise (AWGN). E denotes the average energy over one symbol period and is assumed unity for all channels. h_{sr}, h_{sd} and h_{rd} are independent to each other and constant over a complete round of cooperation.

B. Slepian-Wolf System

In a typical two-source Slepian-Wolf system as shown in Fig. 1(B), two correlated sources, X and Y, generate and send output sequences to a common destination without inter-source communication. We assume that X and Y are memoryless binary symmetric sources that are correlated at the same time instant with $\Pr(X \neq Y) = p < 0.5$ (i.e. the virtual channel of DSC is a binary symmetric channel (BSC)). In the asymmetric compression scenario, one of the sources, say Y, is assumed to be compressed using entropy coding, sent at a rate close to $H(Y)$, and (losslessly) available at the common decoder. The other source, X, needs to be compressed as much as possible (i.e. using a rate close to $H(X|Y)$) and sent to the common decoder where joint decoding will be performed to recover X.

In practice, the compression of X to $H(X|Y)$ is achieved via algebraic binning using linear channel codes [8]. The basic idea is to view X^n, a sequence of X with length n, as a virtual codeword of some (n,k) linear channel code. Compression is performed by mapping X^n to its syndrome S^{n-k}. If the channel code is capacity approaching on BSC(p), then the rate of the channel code $k/n \rightarrow 1-H(p)$ (capacity of BSC), and a compression rate of $(n-k)/n \rightarrow H(p) = H(X|Y)$ is thus achieved.

Details on the principle and practice of Slepian-Wolf coding can be found, for example, in [8]-[9]. Below we briefly discuss an LDPC formulation that will be used in this paper. Let $\mathbf{H}_{m\times n}$ be the parity check matrix of an (n,k) LDPC code, where $m = n-k$. At the encoder, mapping X^n to S^m is accomplished by matrix multiplicity: $S^m = X^n \times \mathbf{H}^T$ (assuming X^n and S^m are represented in row vectors). At the common decoder, the side information Y^n is treated as a noisy version of X^n due to their correlation. Note that although X^n may not be a valid codeword for $\mathbf{H}$, the combination of X^n and its syndrome S^m, i.e. $[X^n, S^m]$, forms a valid codeword of a *super* code with parity check matrix $[\mathbf{H}_{m\times n}, \mathbf{I}_m]$, where $\mathbf{I}_m$ is an identity matrix. Hence, feeding $[Y^n, S^m]$ to the message-passing decoder of $[\mathbf{H}, \mathbf{I}]$ could foreseeably recover $[X^n, S^{n-k}]$ (if the original LDPC code is sufficiently powerful). Fig. 3(A) illustrates this idea. More discussion can be found, for example, in [9].

III. A Simple Estimation Mechanism

That the relay tends to get most bits correct suggests that there is a good chance for *Slepian-Wolf cooperation* in user cooperation, but how does the relay know? More specifically, how does the relay decide when to use Slepian-Wolf code and/or what Slepian-Wolf code to use?

For ease of exposition, let p_e denote the percentage of errors in the data block at the output of the relay decoder. Note that to benefit from (practical) Slepian-Wolf processing, the decoded data Y should differ only marginally from the original data X (i.e. small p_e). Otherwise, the relay data Y do not provide much "essential" information or diversity for X, and the induced bandwidth expansion (due to the additional transmission of $H(X|Y)$) would be nontrivial. Further, practical Slepian-Wolf codes are typically designed for a fixed correlation threshold p_{th}, where error-free recovery is guaranteed only when $p_e = P(X \neq Y) \leq p_{th}$. Using Slepian-Wolf codes on a data block whose p_e far exceed p_{th} could lead to disastrous error propagation. Hence, a simple and effective mechanism is needed for the relay to estimate the relative quality of the decoded data, or to at least identify the cases when Y is too distorted from X.

While other metrics exist, here we propose to use the mean of the absolute log-likelihood ratio (LLR) of the decoded frame, denoted as $\mu_{|LLR|}$, as a figure of merit (to judge the decoding quality). Figure 2 demonstrates the relation between p_e and $\mu_{|LLR|}$ for a $(3000, 2000)$ LDPC code. Each dot represents an experiment. We see that $\mu_{|LLR|(p_e)}$ are closely centered around the average value, $E[\mu_{|LLR|}]$. We therefore use the single quantity, $E[\mu_{|LLR|}(p_e)]$, as the estimation threshold. Aside from simplicity and tractability, a particularly desirable feature about $E[\mu_{|LLR|}]$ is that it is only a function of p_e and the channel code in use, and is *independent of the underlying SNR* for block fading channels (can be easily verified). This thus allows a single estimation rule to serve for all channel conditions. Table I lists the estimation thresholds for several targeted values of p_{th}. To see the accuracy of this simple estimation rule, we listed in Table II the normalized estimation distortion collected over millions of blocks. The normalized estimation distortion is computed as $D = \frac{1}{M}\sum_{i=1}^{M} |\hat{p}_e - p_e|$, where p_e and $\hat{p}_e$ are the exact and the estimated error rate of a block, respectively. We see that the estimation distortion is consistently small for all the SNRs tested.

In the proposed *Slepian-Wolf cooperation*, $E[\mu_{|LLR|}]$ is combined with the cyclic redundant check (CRC) code, widely available in practical systems, to determine which action to take: when the CRC is not satisfied, the relay will invoke *Slepian-Wolf cooperation* if $\mu_{|LLR|} \geq E[\mu_{|LLR|}(p_{th})]$ (where p_{th} is the correlation threshold for which the Slepian-Wolf code is designed), and will revert to the non-cooperative mode otherwise. In the case of decoding success (CRC satisfied), a DF-based cooperative scheme can be adopted. A simple flag bit can be piggybacked to the relay's packet to denote which case happened (note that both the destination and the source hear the relay).

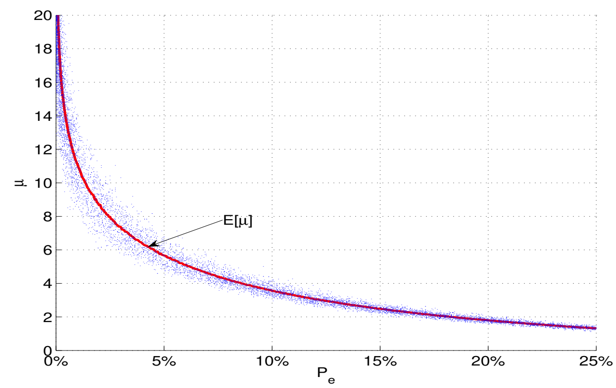

Fig. 2. Relation between $\mu_{|LLR|}$ and p_e in a (3000, 2000) LDPC code.

IV. Slepian-Wolf Cooperation

A. The Base Strategy

In the basic approach of the proposed *Slepian-Wolf cooperation*, the source and the relay will transmit alternatively in three consecutive time slots. As illustrated in Fig. 4(A), in the first time slot, the source sends data X, possibly protected by an error correcting code, to the destination and the relay simultaneously. The relay, upon obtaining a lightly distorted version Y, would invoke *Slepian-Wolf cooperation* by transmitting Y (or $H(Y)$) in the second time slot. Notified by the flag bit, the source would then transmit an additional packet containing $H(X|Y)$ to complete the Slepian-Wolf code.

The above three stages are straight-forward but not necessary. As will be discussed in the next subsection, they can be consolidated in two stages, where the relay transmits after the source as in a conventional cooperative scheme. Before that, let us first look into the decoding strategy.

The destination now collects three packets that contain information $H(X)$ (denoted as $packet_X$), $H(Y)$ (denoted as $packet_Y$) and $H(X|Y)$ (denoted as $packet_{X|Y}$). How does it efficiently recover X? A natural way is to first (soft) decode $packet_Y$ and $packet_{X|Y}$ as in a conventional Slepian-Wolf setup, and then send the (soft) results about X as *a priori* information to $packet_X$. A second possibility is to embark on an iterative strategy, i.e. after the above sequential treatment, pass the output from $packet_X$ back to the Slepian-Wolf decoder for another round of decoding. Yet a third choice is to combine all three packets altogether, treat them as different parts of a single *super* code, and perform joint decoding.

This last strategy is made possible by two facts. First, since $packet_Y$ is a noisy version of $packet_X$, it can be

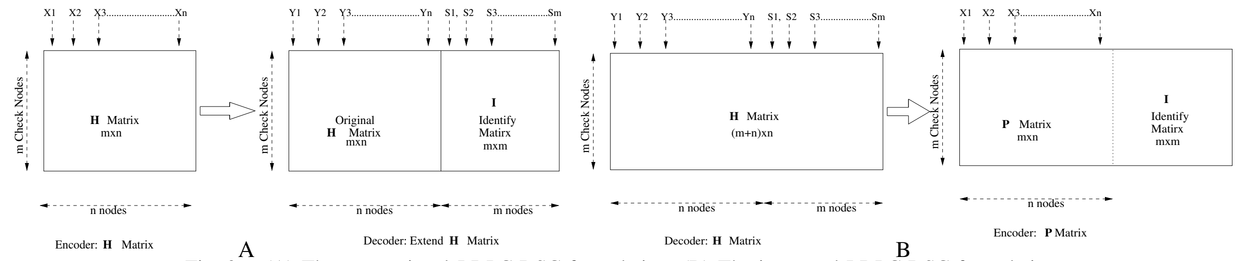

Fig. 3. (A) The conventional LDPC-DSC formulation; (B) The improved LDPC-DSC formulation.

naturally subsumed in the decoding of $packet_X$ via a maximum ratio combining (MRC) of the channel LLRs. Since $packet_Y$ is essentially $packet_X$ passed through a cascade of a BSC(p) and a block fading channel, the corresponding channel LLRs (about X, obtained from $packet_Y$) can be computed as

$$L_{ch,2}(x) = \ln \frac{p + (1-p)\exp\left(\frac{2\mathrm{h}_{\mathrm{rd}}\tilde{\mathrm{y}}}{\sigma^2_{\mathrm{rd}}}\right)}{(1-p) + p\exp\left(\frac{2\mathrm{h}_{\mathrm{rd}}\tilde{\mathrm{y}}}{\sigma^2_{\mathrm{rd}}}\right)}, \quad (3)$$

where $\tilde{y}$ denotes the received signal of $packet_Y$. These LLRs can be added to the channel LLRs computed from the received signals of $packet_X$, $L_{ch,1} = 2h_{sd}\tilde{x}/\sigma^2_{sd}$, and serve together as "channel LLRs" in decoding $packet_X$. Second, as discussed in Section II, under the existing algebraic binning practice, $packet_{X|Y}$ or $H(X|Y)$ is essentially a syndrome sequence for the virtual codeword $packet_{X|Y}$ or $H(X)$. Hence it can be attached to $packet_{X|Y}$ to form a valid codeword of the super code and be *jointly* decoded according. This strategy is particularly easy to implement for LDPC codes, since a single massage-passing decoder, pertaining to the super code $[\mathbf{H}, \mathbf{I}]$, can be used to decode all three packets of $packet_X$, $packet_Y$ and $packet_{X|Y}$. It is easy to see that such a joint decoding strategy is both efficient and optimal.

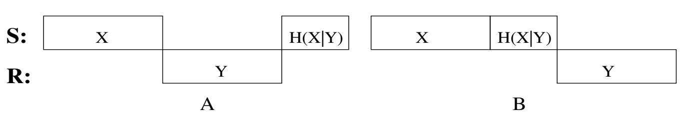

Fig. 4. *Slepian-Wolf cooperation*: (A) The base strategy; (B) the improved strategy.

B. The Improved Strategy

We have shown that $packet_X$ and $packet_{X|Y}$ can combinedly form *one* valid super codeword and be decoded together. Then why should not this one codeword be transmitted together? Figure 4(B) illustrates this idea where the source sends X and $H(X|Y)$ combinedly in the first time slot. This obviates the need for the source to re-switch to the transmit mode later on, but does it cause a bandwidth waste(since $H(X|Y)$ is needed only when *Slepian-Wolf cooperation* is invoked)? The answer is no; and the trick lies in the fact that $H(X|Y)$ can be interpreted in two ways: the syndrome bits of the original code and the parity bits of the super code. Such a view point helps to unify the technologies of source coding and channel coding.

Take an LDPC code for example. As depicted in Fig. 3, $packet_{X|Y}$, denoted as $s_1 \cdots s_m$ in the plot, was initially computed as the syndrome for X^n (with respect to code $\mathbf{H}_{m\times n}$). Since these syndrome bits $s_1 \cdots s_m$ have completed all the checks induced by X^n, they can also be viewed as the parity check bits in the supper code $[\mathbf{H}_{m\times n}, \mathbf{I}_m]$ (where X^n can be treated as the systematic bits). This concept holds regardless of whether X^n is uncoded or already coded by another linear channel code.

It should be noted that the combined transmission of $packet_X$ and $packet_{X|Y}$ also enables the integration of *Slepian-Wolf cooperation* with an exiting DF-based cooperative scheme in a way that is efficient and transparent to the source. Upon successful decoding, the relay could transmit X using a DF-based strategy, and the destination could treat $packet_{X|Y}$ as parity bits for X. When the decoded data Y is lightly distorted from X, the relay could transmit $H(Y)$ (or its coded version), and the destination could use $packet_{X|Y}$ to complete a Slepian-Wolf code. Finally, when Y is too distorted, the relay could revert to the non-cooperative mode or transmit a packet of its own. In either of these cases, the source simply transmits the joint packet of X and $H(X|Y)$.

We consider LDPC codes for *Slepian-Wolf cooperation*. Before proceeding to simulation results, let us discuss a small but useful improvement to the conventional LDPC-DSC formulation shown in Fig. 3(A). Although conceptually simple and optimal, we note that the conventional model is not robust in *noisy transmissions*. This is because the extended matrix, $[\mathbf{H}, \mathbf{I}]_{m\times(n+m)}$, contains lots of weight-1 columns and is therefore unfit for message-passing decoding. To see this, consider a syndrome bit s_i that arrives at the destination in error. Since s_i is connected to the rest of the bits via only one check, the erroneous message it passes out through this one check can never get updated or corrected. To improve this situation, a natural thought would be to convert $[\mathbf{H}, \mathbf{I}]$ to an equivalent matrix that has all the desirable features (e.g. randomness and column weights of at least 2), but the task is nontrivial. The solution we propose here is to reverse the construction procedure: i.e. start with a good *super* LDPC code with a parity check matrix $\mathbf{H}^*_{m\times(n+m)}$ that is fitful for message-passing decoding, and then use Gaussian elimination to diagonalize it to $[\mathbf{P}_{m\times n}, \mathbf{I}_m]$, where $\mathbf{P}$ can be used to generating syndromes (see Fig. 3(B) for illustration). We note that $\mathbf{P}$ tends to be dense and may not be "LDPC-like", but this does not matter since its sole purpose is to generate syndromes using binary matrix multiplication.

V. SIMULATIONS

This section evaluates the performance of the proposed scheme using computer simulations. Considering that the syndrome bits in $packet_{X|Y}$ also serve the dual purpose of parity check for $packet_X$, CRC-protected but otherwise uncoded $packet_X$ is used to simplify the system. Since the block size is typically limited to a few thousand bits in practical systems, we consider length $n = 2000$ for $packet_X$ and $packet_Y$, and length $m = 1000$ for $packet_{X|Y}$. Using the strategy illustrated in Fig. 3(B), a $(3000, 2000)$ *super* LDPC code is adopted, whose codewords (3000 bits) form the initial transmission from the source, $packet_{X,X|Y}$.

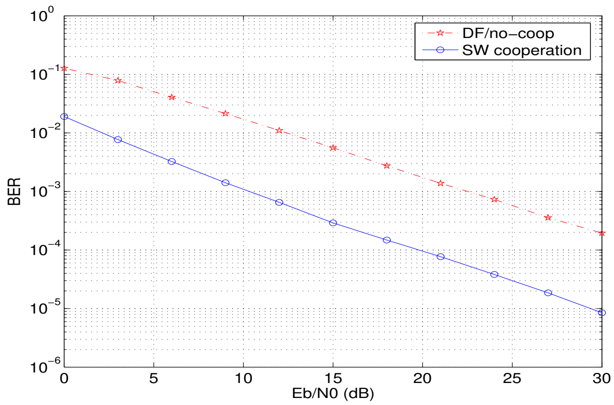

Fig. 5. Performance of *Slepian-Wolf cooperation* at its favorable situation.

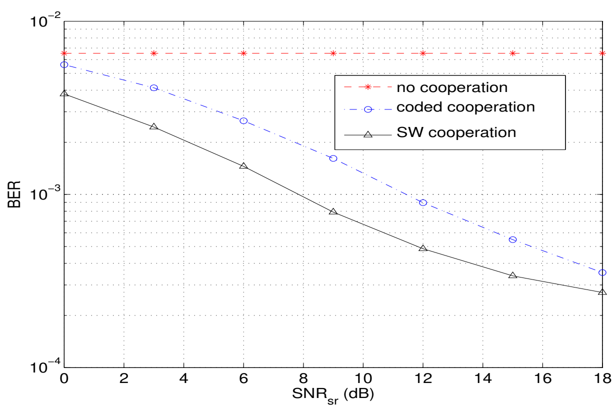

Fig. 6. Comparison of *Slepian-Wolf cooperation*, *coded cooperation* and *no cooperation*.

Figure 5 shows the performance of the above *Slepian-Wolf cooperation* at its favorable situation: when the inter-user is at outage (DF would fail) and when the decoded data contain less than $p_{th} = 5\%$ errors (Slepian-Wolf code could help positively). In the plot, the X-axis denotes the SNR of the two user channels, and the inter-user channel is always 10dB better. The solid line represents *Slepian-Wolf cooperation*, and the dashed line represents the DF-based strategy which, due to inter-user outage, essentially reduces to *no cooperation*. The plot clearly demonstrates the considerable advantage *Slepian-Wolf cooperation* has over conventional schemes in the case of inter-user outage, with a gain of as much as 13 dB! Note that this gain is evaluated after penalizing the additional energy spent in transmitting $packet_Y$ in *Slepian-Wolf cooperation*.

To see the overall gain offered by *Slepian-Wolf cooperation*, we blend in the two other cases (i.e. successful decoding and severe errors at the relay) and plot in Fig. 6 the average performances. The two user channels are fixed to an SNR of 14 dB, and the inter-user channel changes from 0 to 18 dB. We compare between *no cooperation*, *coded cooperation* and *Slepian-Wolf cooperation*. In *coded cooperation*, upon successful decoding, the relay re-encodes the data using a different $(4000, 2000)$ LDPC code and passes on the new parity bits; it reverts to *no cooperation* otherwise. In *Slepian-Wolf cooperation*, the relay switches between *coded-cooperation*, Slepian-Wolf coding, and the non-cooperative mode depending on the estimated decoding quality. Again the energy consumption has been normalized, and we observe an encouraging gain of close to 4 dB enabled by *Slepian-Wolf cooperation*.

VI. CONCLUSION AND FUTURE WORK

A *Slepian-Wolf cooperation* scheme is proposed to improve the inter-user outage performance in wireless cooperative communications. The gains achieved clearly demonstrate the feasibility and benefits of exploiting the ideas of source coding in channel coding. The proposed scheme can be extended and enriched in many different ways, including the exploitation of joint source-channel Slepian-Wolf coding strategies and adaptive Slepian-Wolf coding strategies.

TABLE I

ESTIMATION RULE BASED ON $E[\mu_{|LLR|}(p_e)]$

$E[\mu_{\|LLR\|}]$	10.99	8.68	7.29	6.35	5.66	5.10	4.61
p_e	1%	2%	3%	4%	5%	6%	7%

TABLE II

NORMALIZED ESTIMATION DISTORTION

E_s/N_0	0 dB	6 dB	12 dB	18 dB	24 dB
D	$5.0E-3$	$1.8E-3$	$5.1E-4$	$1.4E-4$	$3.0E-5$

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