Abstract – The success of the wideband communication has recently revived considerable research effort in the low-power wideband regime of the communication system. Pioneering work by Verdu in [1], [2] formulate and study the fundamental tradeoff between bandwidth and implementation complexity in the wideband, low-power region. Unlike the conventional infinite bandwidth analysis, which often leads to an asymptotic approximation lack of practical insight, the works in [1], [2] concentrate on the ‘wide but still finite’ scenario and analyze the practical implications. New design criteria and signaling format to approach capacity with optimal bandwidth usage are proposed.

I. BACKGROUNDS AND DEFINITIONS

A. Backgrounds

The capacity of an AWGN channel has been formulated by Shannon in 1948 as

$$C = W \log_2(1 + \frac{P}{WN_0}) \text{ (b/s)}.$$  (1)

As the bandwidth $W \to \infty$, we have

$$\lim_{W \to \infty} C = \frac{P}{N_0} \log_2 e$$  (2)

Notice that $C$ is monotonically increasing with $W$, hence (2) gives the maximum $C$ achievable by the a given power $P$. When communicating at the rate $R$, the energy per information bit can be denoted as

$$E_b = \frac{P}{R}$$  (3)

As $C = \max(R)$, we have

$$\min(E_b) = \frac{P}{\max(R)} = \frac{P}{C},$$  (4)

where the minimum is achieved when the capacity is achieved. Plug (2) into (4) gives

$$\left(\frac{E_b}{N_0}\right)_{\min} = \log_e 2 = -1.59dB$$  (5)

From (5), it is clear that achieving the capacity of an infinite-bandwidth system is equivalent to finding the minimum energy per bit required for reliable communication. It also follows from (2) that to achieve a given rate $R$, a minimum power of $P = N_0 \log_e 2$ is required. Conventionally, the criterion of achieving $\frac{E_b}{N_0} = -1.59dB$ has been widely used as a synonym of asymptotic optimality in the low-power wideband regime. Also, the flash signaling (on-off keying with vanishing duty cycle and infinite peak-to-average ratio) has been proposed to obtain such optimality. It was also indicated that under such criterion, capacity of wideband system is not affected by fading or presence channel state information (CSI). Those conclusions, again, are based on the infinite bandwidth assumption such that $\frac{E_b}{N_0} = -1.59dB$ is achieved, and roots in the belief that the study of the performance at the “wide yet finite” regime can be guided by the study of the “infinite bandwidth” system. Unrealistic as those conclusions sound, Verdu prove in [1], [2] that the “wideband” characteristic indeed differ from theoretical “infinite bandwidth” system. The works in [1], [2] also propose new criteria for the system evaluation in the low-power wideband regime.
B. Spectral Efficiency

The spectral efficiency of a channel can be written as:

$$ C = \frac{C}{W} = \log_2(1 + \frac{P}{WN_0}) \text{ (b/s/Hz)} \quad (6) $$

When the capacity in (6) is achieved, we have $P = E_b\bar{C}W$, substituting back into (6) gives

$$ \bar{C}(\frac{E_b}{N_0}) = \log_2(1 + \frac{E_b}{N_0}\bar{C}) \quad (7) $$

Solve for $\frac{E_b}{N_0}$ gives

$$ \frac{E_b}{N_0} = \frac{2^\bar{C} - 1}{\bar{C}} \quad (8) $$

Equation (8) can also be evaluated as $\bar{C} \to 0$, such that

$$ (\frac{E_b}{N_0})_{\min} = \lim_{\bar{C} \to 0} \frac{2^\bar{C} - 1}{\bar{C}} = \log_e 2 = -1.59(dB), \quad (9) $$

which is identical to the result obtained in (5).

While the above computations are all based on the AWGN assumption, study in [1] also shows that (5) and (9) actually hold for any fading channel as long as the background noise is Gaussian. Yet lower bound $\frac{E_b}{N_0}$ is possible under non-Gaussian background noise.

C. Approaching $(\frac{E_b}{N_0})_{\min}$

As was mentioned in subsection I-A, the criterion of simply seeking to achieve $(\frac{E_b}{N_0})_{\min}$ results in many unpractical guidance for the design of system operating in the low-power wideband region, such as:

- Capacity of wideband system is not affected by fading;
- Assistance of Channel State Information (CSI) would not increase capacity of wideband system;

The misleading conclusion result from the assumption that indeed infinite bandwidth is available. The drawback of such assumption can be illustrated by Figure. 1 [2], in which the spectral efficiency of different channel conditions (fading v.s. no fading; full CSI v.s. no CSI) is shown, when the power and data rate are fixed. All schemes indeed approach $(\frac{E_b}{N_0})_{\min}$ when the spectral efficiency $\bar{C} \to 0$ (or equivalently, $W \to \infty$), as is indicated by (9), yet “approaching paths” are quite different. This phenomenon can be interpreted as in a “theory v.s. practice” plot. Given any of the two parameters from the set {power, bandwidth, rate}, Shannon theory determines the third, hence whether faded or not, with or without CSI, same $(\frac{E_b}{N_0})_{\min}$ is reached at $\bar{C} = 0$. On the other hand, it is necessary to access the complexity-performance tradeoff (a little more complexity may result a “better” path to reach $(\frac{E_b}{N_0})_{\min}$). For example, considered a particular $\frac{E_b}{N_0}$ that is slightly larger than $(\frac{E_b}{N_0})_{\min}$, † such that

$$ \frac{E_b}{N_0} = 1.25dB > -1.59dB. \quad (10) $$

As is shown in Fig. (1), for the non-coherent fading Rayleigh channel, a bandwidth efficiency of $\bar{C}_{NC} = 0.0011$ b/s/Hz is resulted. On the other hand, $\frac{E_b}{N_0} = 1.25dB$ turns out to offer a bandwidth efficiency that’s too large to be read from Fig. 1, with the assistance of CSI. Numerical analysis gives that $\bar{C}_C = 1.1$ in this case. Given a fixed power and data rate, the difference $\frac{\bar{C}_C}{\bar{C}_{NC}} = 1000$ indicates that a factor of 1000 in bandwidth can be bought by the CSI.

†In fact, when the peak-to-average ratio is constrained to be less than infinity, system can not operate at the point of $(\frac{E_b}{N_0})_{\min}$ with 0 spectral efficiency, study has shown that the lower the input burstiness is allowed, the further away the achievable $(\frac{E_b}{N_0})$ is from $(\frac{E_b}{N_0})_{\min}$.
II. Wideband Slope $b/s/Hz/3dB$

As was demonstrated in the previous section, the achievable $\frac{E_b}{N_0}$ by itself is not an abundant parameter to describe the system performance in the wideband region. The concept of wideband slope $S_0$ is proposed in [1], defined as:

$$S_0 \equiv \lim_{\frac{E_b}{N_0} \to (\frac{E_b}{N_0})_{\min}} \frac{C(\frac{E_b}{N_0})}{(\frac{E_b}{N_0})_{\min}} \cdot 10 \log_{10} 2, \quad (11)$$

in which $\frac{E_b}{N_0}$ and $(\frac{E_b}{N_0})_{\min}$ are all in dB terms.

The spectral efficiency can actually also be written as a function or SNR, that is, $C = \tilde{C}(SNR)$.

$$\frac{E_b}{N_0} \tilde{C}(SNR) = SNR. \quad (12)$$

It can be proved by Taylor expansion that the wideband slope defined in (11) can also be written as:

$$S_0 = \frac{2[\tilde{C}(0)]^2}{\tilde{C}(0)}, \quad (13)$$

in which the differentiation is taken over SNR.

On-off signaling has been used as the optimal signaling strategy in conventional wideband system. Should we computer the wideband slope, however, it’s not that optimal if we also seek to minimize the bandwidth requirement. Given (13) and that $\tilde{C} = \log_2(1 + SNR)$, we can compute the optimal wideband slope for AWGN is

$$(S_0)_{Optimal} = 2(b/s/Hz/3dB). \quad (14)$$

On the other hand, when on-off keying is adopted with duty cycle $\alpha$, the wideband slope can be expressed as:

$$S_0 = S_0(\alpha^2) = \frac{2|\alpha|^4}{N_0 \exp\left(\frac{2|\alpha|^2}{N_0}\right) - 1}, \quad (15)$$

maximize (15) over $\alpha$ yields

$$S_0 \leq 0.3238(b/s/Hz/3dB), \quad (16)$$

compare (16) with (14), we can see the upper bound of $S_0$ achievable by on-off keying is not even close to optimal. In fact, on-off signaling requires at least 618% of the minimum bandwidth. To this end, a modified modulation scheme which encodes the information bit in phase as well as in time is proposed in [1], and gives $S_0 \leq 2$ with a tight upper bound. Of the many possible implementations, QPSK was shown to be able to achieve the optimum upper bound in [1].

III. Achieving Minimum Bandwidth

A. Perfect Channel State Information at the Receiver

1) Perfect Channel State Information at both ends: For an $m$-dimensional complex channel

$$y = Hx + n, \quad (17)$$

Suppose both transmitters and the receivers know the channel matrix $H$, but no power control is performed, it can be proved that:

$$S_0 = \frac{2\ell}{m\mathcal{K}(\sigma_{\max}(H))} \quad (18)$$

where $\sigma_{\max}(H)$ is the maximal singular value of the channel matrix $H$, $\ell$ is the multiplicity of $\sigma_{\max}(H)$. The kurtosis of a random variable $Z$ is defined as:

$$\mathcal{K}(Z) = \frac{\mathbb{E}[Z^4]}{\mathbb{E}[Z^2]^2}. \quad (19)$$

Equation (18) can be proved by the water-filling formula and (13).

Should we revisit (18), we can write $S_0 \sim \frac{1}{\mathcal{K}}$, also, as $S_0 \sim \frac{1}{W_{\min}}$, we then have $W_{\min} \sim \mathcal{K}$, i.e. the minimum bandwidth of the fading channel, with CSI at both ends of the communication, is proportional to the kurtosis of the fading amplitude distribution.

2) Perfect Channel State Information at Receiver Only: If the transmitter does not know the channel matrix $H$, the wideband slope can be written as:

$$S_0 = \frac{2(\text{Tr}(E[H^H]))^2}{m(\text{Tr}(E[H^H]^2])}, \quad (20)$$

where $\text{Tr}(A)$ is the trace of a matrix $A$. If the entries of $H$ are independent zero-mean r.v., and assume there are $m$ receive antennas and $n$ transmit antennas, then (20) is reduced to

$$S_0 = \frac{2n}{\mathcal{K}(|H_{ij}|) + m + n - 2}, \quad (21)$$

\footnote{While $C$ and $\tilde{C}$ both denote the spectral efficiency and they are actually equal to each other, different notations are used when the spectral efficiency is presented by functions of different variables ($\frac{E_b}{N_0}$ and SNR).}
also, we can derive that $\mathcal{K}(|H_{ij}|) = 2$ in face of the Rayleigh flat fading, then we have:

$$S_0 = \frac{2nm}{n + m} \text{ (b/s/Hz/3dB).}$$  (22)

There is a lot to say about the neat presentation of (22), one of the most important result is that unlike the misconception that the capacity of low-SNR wideband regime is not affected by the number of transmit antennas $n$, the result in (22) shows that the bandwidth needed to achieve the capacity varies with the number of transmit antennas $n$. A times of bandwidth is required to achieve the capacity in single-antenna system compared to the system with $n$ transmit antennas.

B. Imperfect Channel State Information at the Receiver

If neither the receiver nor the transmitter has the channel information, which often indicates that

$$\lambda_{\max}(E[H]^\dagger E[H]) < \lambda_{\max}(E[H^\dagger H]),$$  (23)

the wideband slope is then $S_0 = 0$. That means to achieve wideband low-power capacity in absence of CSI at the receiver is prohibitively bandwidth expensive.

IV. MULTIPLE ACCESS

As was established in the previous sections, the wideband slope $S_0$ is a more valid and practical performance measure in the wideband region, if the required bandwidth to achieve capacity is taken into consideration. Previous study in information theory shows that TDMA can approach $\frac{E_b}{N_0}$ of min and a “single user” capacity in both MAC and broadcast channels. Those facts seem to leave little room of improvement for the superposition strategy such as CDMA. It is, however, quite a different story if we consider bandwidth expenditure. Instead of using the rate pairs, wideband slope pairs are considered in [3],[4] to evaluate system performance.

Take the broadcast channel for example such that

$$Y_1 = X + N_1, \quad Y_2 = X + N_2,$$  (24)

where $\mathbb{E}[|N_1|^2] \leq \sigma_1^2$. Let the rate vanish while keeping $R_1/R_2 = \theta$. The wideband slope region achievable by TDMA is:

$$\{(S_1, S_2): 0 \leq S_1 \leq \frac{2\theta}{1+\theta}, 0 \leq S_2 \leq \frac{2}{1+\theta}\},$$  (25)

in comparison to the optimum wideband slope region achievable by superposition (such as CDMA):

$$\{(S_1, S_2): 0 \leq S_1 \leq \frac{2(\theta+\sigma_2^2/\sigma_1^2)}{\theta^2+2\theta+\sigma_2^2/\sigma_1^2},$$

$$0 \leq S_2 \leq \frac{2(\theta+\sigma_2^2/\sigma_1^2)}{\theta^2+2\theta+\sigma_2^2/\sigma_1^2}\}.$$  (26)

In the trivial case of $\sigma_1^2 = \sigma_2^2$, (25) is identical to (26), and TDMA achieve the optimum as the superposition strategy. However, in the more general case, when $\sigma_1^2 \neq \sigma_2^2$, TDMA can be very wasteful of bandwidth.

For example, when $\sigma_1^2 = 10\sigma_2^2$, given $\theta = 3$, we can show that

$$\{(S_1, S_2)_{TDMA}: 0 \leq S_1 \leq \frac{3}{2}, 0 \leq S_2 \leq \frac{1}{2}\},$$  (27)

and

$$\{(S_1, S_2)_{CDMA}: 0 \leq S_1 \leq \frac{78}{25}, 0 \leq S_2 \leq \frac{260}{151}\},$$  (28)

compare (27) and (28) indicates that TDMA would requires more than twice the bandwidth to achieve certain capacity.

V. CLOSING REMARKS

The conventional information-theoretic study in the wideband low-power region has been conducted in the fashion of treating “infinite bandwidth” as a special case of “wideband” region, and has assumed that the results and criteria obtained by infinite bandwidth analysis can be been treated as the guidelines of designing wideband system. The works by [1]~[4] reveal the inaccuracy of those guidelines, and that caution should be taken when making such assumptions. New analysis criterion, such as the wideband slope has emerged to give more insights into designing the systems of “wide-yet-still-precious bandwidth”.

REFERENCES


