Papers summery on: R.Zamir and S.Shamai and U.Erez, ”Nested linear/lattice codes for structured multiterminal binning”

Meng Yu

***Abstract*— In [1] nested codes are defined and some fundamental properties are introduced. We review the usage of nested codes in wyner’s noiseless binning scheme,noisy side information problems and dirty paper problems. Some related topic such as multiterminal coding of correlated sources and coordinated encoding over mutually interfering channels are also touched.**

I. Nested Codes: Preliminaries

Nested code is a pair of linear or lattice codes (C_1, C_2) satisfying

$$C_2 \in C_1.$$

C_1, C_2 are called “fine code” and “coarse code” respectively.

A. Nested Parity-Check Codes

The check matrices of the parity-check codes of a nested code $\{(n, k_1), (n, k_2)\}$can be represent by

$$H_2 = \begin{bmatrix} H_1 \\ \cdots \\ \Delta H \end{bmatrix}$$

The syndromes $\mathbf{s_1} = \mathbf{H_1 x}$ and $\mathbf{s_2} = \mathbf{H_2 x}$ satisfy $\mathbf{s_2^t} = [\mathbf{s_1^t}, \mathbf{\Delta s^t}]$. When $\mathbf{x}$ is a codeword of C_1, $\mathbf{s_2^t} = [\mathbf{0}, \cdots, \mathbf{0}, \mathbf{\Delta s^t}]$. We can partition C_1 into $2^{k_1 - k_2}$ cosets of C_2 by choose different $\mathbf{\Delta s^t}$, we will use $C_{2,\mathbf{s_2}}$ to denote them.

B. Lattices and Nested Lattice Codes

A n-dimensional lattice Λ is defined by all combination of a set of n real basis vectors $\mathbf{g_1}, \cdots, \mathbf{g_1}$ in R^n

$$\Lambda = \{\mathbf{l} = \mathbf{G} \cdot \mathbf{i} : \mathbf{i} \in \mathbf{Z^n}\}$$

where $Z = \{0, \pm 1, \pm 2, \cdots\}$ and $G = [\mathbf{g_1}, \cdots, \mathbf{g_1}]$ is generator matrix.

The nearest neighbor quantizer $Q(\cdot)$ is

$$Q(\mathbf{x}) = \mathbf{l} \in \mathbf{\Lambda}\ \mathbf{if} \|\mathbf{x} - \mathbf{l}\| \le \|\mathbf{x} - \mathbf{l}'\|$$

The basic cell of Λ is

$$V_0 = \{\mathbf{x} : \mathbf{Q}(\mathbf{x}) = \mathbf{0}\}$$

The volume of Λ is

$$V = \int_{V_0} d\mathbf{x}$$

In [1], there are definition of good δ-codes. They have asymptotically property

$$\frac{1}{n} logV \simeq \frac{1}{2} log(2\pi e \delta)$$

In [1], some fundamental prosperities are given.

The generator matrices of the lattice of a nest lattice $\{\Lambda_1, \Lambda_2\}$ can be represent by

$$G_2 = G_1 \cdot \mathbf{J}$$

where $\mathbf{J}$ is a n-by-n matrix whose determinate is greater than one.

II. Wyner’s noiseless binning scheme and Noisy Side Information Problems

A. Wyner-Ziv and Slepain-Wolf Problem

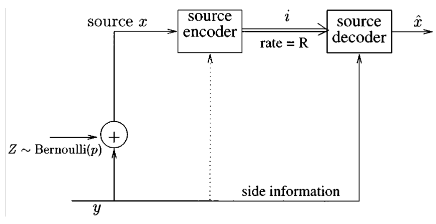

Fig. 1. Wyner-Ziv and Slepain-Wolf problem

Fig II-A describes a case of the problem of source coding with side information at the decoder. X_i is a memoryless source. It is encoded as a rate R and send out. At the decoder, a correlated source Y_i is used to jointly decode X. In continuous case, if X and Y are jointly Gaussian, we use σ_z^2 to denote $Var(X|Y)$. In discrete case, the difference between X and Y can be described as a Bernoulli p random process. Denote the distortion between the source and the reconstruction as

$$\frac{1}{n}Ed(\mathbf{X},\hat{\mathbf{X}}) \leq \mathbf{D}$$

When D=0, it is lossless case, else it is lossy case.

In discrete case, nested linear code $\{C_1, C_2\}$ can be used to encode and decode source X.

Encoding: quantize $\mathbf{x}$ as a codeword $\mathbf{x_q}$ of C_1, then transmit $\Delta\mathbf{s} = \mathbf{\Delta H}\cdot\mathbf{x}$, which requires $k_1 - k_2 \simeq n\cdot[H(p*D) - H(D)]$.

Decoding: compute $\mathbf{s_2} = \mathbf{H_2 x}$ by zero padding, i.e., $\mathbf{s_2} = (\mathbf{0}, \mathbf{\Delta s})$; then reconstruct $\mathbf{x}$ by the codeword in the coset $C_{2,\mathbf{s_2}}$ which is equal to $\mathbf{y}$. The operation can be written as

$$\hat{\mathbf{x}} = \mathbf{y} \oplus \hat{\mathbf{w}}, \mathbf{where}\ \hat{\mathbf{w}} = \mathbf{f_2}(\mathbf{s_2} \oplus \mathbf{H_2 y})$$

Here the function $f(a)$ means to find the vector with minimum hamming weight in the vectors with syndrome a. In lossless case C_1 is $(n, 0)$.

In continuous case, nested lattice code $\{\Lambda_1, \Lambda_2\}$ can be used to encode and decode source X.

Encoding: find out $\mathbf{x_q} = \mathbf{Q_1}(\alpha\mathbf{x} + \mathbf{u})$ in Λ_1, then transmit index of $\mathbf{v_2} = \mathbf{x_1}\ \mathbf{mod}\ \mathbf{\Lambda_2}$. Here requires $log(V_2/V_1) \simeq \frac{n}{2}log(\sigma_z^2/D)$ and $\alpha = \sqrt{1 - D/\sigma_z^2}$.

Decoding: reconstruct $\mathbf{x}$ as

$$\hat{\mathbf{x}} = \mathbf{y} + \alpha\hat{\mathbf{w}}, \mathbf{where}\ \hat{\mathbf{w}} = [\mathbf{v_2} - \mathbf{u} - \alpha\mathbf{y}]\mathbf{mod}\mathbf{\Lambda_2}$$

B. Dirty Paper Problem

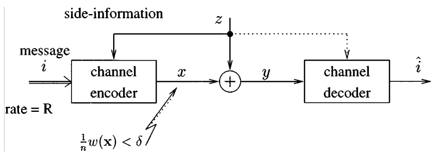

Fig. 2. Wyner-Ziv and Slepain-Wolf problem

Fig II-B describes a case of channel coding with perfect channel information at the encoder. The channel is

$$Y = X + S + N$$

X, Y, Z and N is the channel input and output, interference signal, additive noise respectively. In discrete case, $\mathbf{Z}$ is a bernoulli-δ process. And the average hamming weight of $\mathbf{X}$ should be less than a constraint $\delta \leq 1/2$. In continuous case, generally, $\mathbf{X}$ will have a average power constraint P.

In discrete case, N is a bernoulli-p process, the capacity is

$$C(\delta) = u.c.e\{H(\delta) - H(p), (0,0)\}, 0 \leq \delta \leq 0.5$$

The coding procedure will be

Message selection: Identify each syndrome $\mathbf{s}$ with a unique message.

Encoding: transmit

$$\mathbf{x} = \mathbf{z}\ \mathbf{mod}\ \mathbf{C_{2,s}}$$

Decoding: reconstruct the message as

$$\hat{\mathbf{s}} = H\mathbf{y}$$

In continues, N is a Gaussian noise with variance σ_N^2. The coding procedure will be,

Message selection: Identify each coset $\Lambda_{2,v}$ with a unique message. Here requires $log(V_2/V_1) \simeq \frac{n}{2}log(1 + P/\sigma_N^2)$

Encoding: transmit

$$\mathbf{x} = [\mathbf{v} - \alpha\mathbf{z} - \mathbf{u}]\mathbf{mod}\mathbf{\Lambda_2}$$

Decoding: reconstruct the message as

$$\hat{\mathbf{v}} = Q_1(\alpha\mathbf{y} + \mathbf{u})\mathbf{mod}\mathbf{\Lambda_2}$$

III. Multiterminal Coding of Correlated Sources

Using nested code, we can achieve the theoretic multiterminal rate region

$$R_1 \geq H(X|Y)$$
$$R_2 \geq H(Y|X)$$
$$R_1 + R_2 \geq H(X,Y)$$

The strategy is to transmit one source first, then use the first source as side information to encode the other source. The procedure is similar in previous section.

IV. Coordinated Encoding over Mutually Interfering Channels

The ”dirty paper” method can be used to achieve the capacity region of broadcast channel

$$R_i \leq \frac{1}{2} log \left((1 + \frac{h_i^2 P_i}{1 + h_i^2 \sum_{j=i+1}^{M} P_j} \right)$$

The strategy is to encode message for user 1 first. Then treat the message for user 1 as known interference signal to encode user 2 using dirty paper method. Then encode user 3 and so on. At the decoder, all user’s message can be decode independently.

References

[1] R.Zamir and S.Shamai and U.Erez, ”Nested linear/lattice codes for structured multiterminal binning”*IEEE Trans. on Inform. Theory*, pp. 1250-1276, June.2002.