1 Optimization

First we focus on the one dimensional, unconstrained cases, and discuss the finding of the root of \( g'(x) \) where \( g(x) \) is our objective function. The problem boils down to the finding of the root of \( g' \).

- Bisection search: can find only one of the roots if there are more than one; need initialize the algorithm properly such that \( g'(a)g'(b) < 0 \) where \( a \) and \( b \) are two end points of the search.

- Newton-Raphson method: use Taylor expansion of \( g' \) at the current solution \( x^{(t)} \): \( g'(x^*) \approx g'(x^{(t)}) + g''(x^{(t)})(x^* - x^{(t)}) \). Then let \( g'(x^*) = 0, \)
  \( x^* = x^{(t)} - g'(x^{(t)})/g''(x^{(t)}) \). \( g \) needs to be second order differentiable and \( g'' \neq 0 \).

- Fisher scoring: mainly used in MLE where one wants to maximize the log-likelihood \( \ell \). In the Newton-Raphson, replace \(-g''\) by the Fisher information \( I = E(-\ell'') \) and we get: \( x^* = x^{(t)} + \ell'(x^{(t)})/E(-\ell''(x^{(t)})) \).

- Secant method: replace \( g'' \) by the approximation \( (g'(x^{(t)}) - g'(x^{(t-1)}))/(x^{(t)} - x^{(t-1)}) \).

- Fixed point iteration

- Golden section search

2 EM algorithm