CSCI 2200 - Spring 2017

Exam 1

Name: ______________________________

Instructions:

- Write your name on the **front and back** of this cover sheet and **sign the bottom of this page**.
- You have **100 minutes** to complete this exam. The exam is worth a total of 100 points.
- Put away laptop computers and other electronic devices. Calculators are NOT allowed. Cheating on an exam will result in an **immediate F** for the entire course.
- A one-page, two-sided crib sheet is allowed. Rulers are also allowed.
- **Write your answers clearly and completely.**
- Logical equivalences and rules of inference are given on the last page.
- Please read each question carefully several times before beginning to work and especially before asking questions. We generally will not answer questions except when there is a glaring mistake or ambiguity in the statement of a question.

I will not discuss the contents of this exam in the presence of anyone who has not yet taken the exam.

Signature: ______________________________ **Date:** ____________

Name: ______________________________

Problem 1 (8)	
Problem 2 (12)	
Problem 3 (12)	
Problem 4 (4)	
Problem 5 (12)	
Problem 6 (12)	
Problem 7 (12)	
Problem 8 (12)	
Problem 9 (16)	
Total (100)	

1. (4×2 pts $= 8$ pts) Mark the following statements TRUE or FALSE.

(a) T $(1+1=3)$ if and only if $(2+2=3)$.

(b) F The inverse of 3 **mod** 7 is $-3/7$.

(c) T 4 is congruent to 18 modulo 7.

(d) F -10 **mod** $7 = -3$.

2. (6×2 pts $= 12$ pts) Let $A = \{1, 2, 3\}$ and $V = \{a, e, i, o, u\}$. Mark the following statements TRUE or FALSE.

(a) F $A \subset A$

(b) T $A \subseteq A$

(c) F $|A \times B| = 8$

(d) T $(1, a) \in A \times B$

(e) T $\varnothing \subseteq \mathcal{P}(A)$

(f) T $|\mathcal{P}(B)| = 32$

3. (12 pts) Using propositional logic, write a statement that contains the propositions p, q, and r that is true when $(q \to p)$ and $(q \leftrightarrow \neg r)$ are true and is false otherwise. The only logical connectives your statement should contain are $\neg$, $\vee$ and $\wedge$.

p	q	r	$\overbrace{q \to p}^{a}$	$\overbrace{q \leftrightarrow \neg r}^{b}$	$a \wedge b$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	F	F

DNF: $(p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$

4. (4 pts) Write a proposition that is equivalent to $q \to r$ that uses only the logical connectives $\neg$ and $\wedge$.

$$q \to r \equiv \neg q \vee r$$
$$\equiv \neg\neg(\neg q \vee r)$$
$$\equiv \neg(q \wedge \neg r)$$

5. (2 × 6 pts = 12 pts) Let $x \in \mathbb{Z}$, $y \in \mathbb{Z}$, $L(x,y) := x < y$, $G(x) := x > 0$, and $P(x) :=$ "x is a prime number." Write a statement in English prose (complete sentences, no math symbols or variables) that is equivalent to the following:

(a) $\forall x \exists y\ L(x,y)$

For every integer, there is a larger one.

(b) $\forall x \exists y\ [G(x) \rightarrow (P(y) \wedge L(x,y))]$

For every positive integer, there is a prime number larger than it.

6. (12 pts) Formally analyze the following argument to show whether it is valid or not. Explain your result in English, that is, what part of your analysis indicates that the argument is valid or not.

Rainy days make gardens grow.	$r \rightarrow g$	premise 1
Gardens don't grow if it is not hot.	$\neg h \rightarrow \neg g$	premise 2
Whenever it is not hot, it rains.	$\neg h \rightarrow r$	premise 3
Therefore, if it is not hot, then it is hot.	$\therefore \neg h \rightarrow h$	conclusion

h:= "hot day"

r:= "rainy day"

g:= "garden grows"

(1)	$r \rightarrow g$	premise 1
(2)	$\neg h \rightarrow \neg g$	premise 2
(3)	$g \rightarrow h$	contrapositive of (2)
(4)	$r \rightarrow h$	hypo. syll. of (1) & (3)
(5)	$\neg h \rightarrow r$	premise
(6)	$\neg h \rightarrow h$	hypo. syll of (4) & (5)

The argument is valid, because the conclusion was derived from known rules of inference.

The argument is valid!

7. (12 pts) Let n be any positive integer. Prove that the following three statements are equivalent: (a) $\underbrace{n \text{ is even}}_{a}$, (b) $\underbrace{n^3+1 \text{ is odd}}_{b}$, (c) $\underbrace{n^2-1 \text{ is odd}}_{c}$.

Two approaches: ① Prove $a \to b \wedge b \to c \wedge c \to a$

② Prove $a \leftrightarrow b \wedge a \leftrightarrow c$

Allow use of odd and even results as axioms.

Approach ①: $a \to b$. $n = 2k, k \in \mathbb{Z}$. $n^3+1 = 2(4k^3)+1$. Done.

$b \to c$. n^3+1, odd $\Rightarrow n^3$, even. Then n even. ~~So~~ Apply to prop. c. n even. Then n^2 even, n^2-1 odd. Done

$c \to a$. n^2-1 is odd. then n^2 is even. then n is even. Apply to a. a is true. Done

q.e.d.

8. (2×6 pts $= 12$ pts) For the integers 126 and 210:

(a) use the Euclidean Algorithm to find their gcd.

$$210 = 126 \cdot 1 + 84$$
$$126 = 84 \cdot 1 + \boxed{42} \text{ gcd}$$
$$84 = 42 \cdot 2 + 0$$

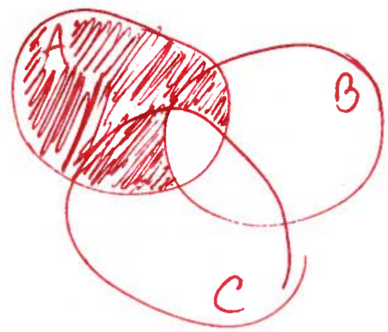
(b) find their Bezóut coefficients. find $s, t \in \mathbb{Z}$ such that $s210 + t126 = 42$

$$42 = 126 - 84$$
$$= 126 - (210 - 126)$$
$$= 2 \cdot 126 - 210$$

Thus $(s,t) = (-1, 2)$

9. (4pts for (a), 12 pts for (b)) Let A, B, and C be sets. For the expressions, $A-(B\cap C)$ and $(A-B)\cup(A-C)$, do the following:

(a) Draw a Venn Diagram highlighting the region identified by $A-(B\cap C)$.



(b) Using definitions from set theory and set identities, prove or disprove the statement $A-(B\cap C)=(A-B)\cup(A-C)$.

$A-(B\cap C)=\{x\mid x\in(A-(B\cap C))\}$ by def. of set builder notation

$=\{x\mid (x\in A)\wedge(x\notin(B\cap C))\}$ def. of set difference

$=\{x\mid (x\in A)\wedge\neg(x\in(B\cap C))\}$ def. of $\notin$

$=\{x\mid (x\in A)\wedge\neg((x\in B)\wedge(x\in C))\}$ def of set intersection

$=\{x\mid (x\in A)\wedge(\neg(x\in B)\vee\neg(x\in C))\}$ DeM. Law of p.l.

$=\{x\mid (x\in A)\wedge((x\notin B)\vee(x\notin C))\}$ def. of $\notin$

~~$=\{x\mid (x\in A)$ A~~

$=\{x\mid ((x\in A)\wedge(x\notin B))\vee((x\in A)\wedge(x\notin C))\}$ distrib. law of p.l.

$=\{x\mid (x\in(A-B))\vee(x\in(A-C))\}$ def. of set difference

$=\{x\mid x\in(A-B)\cup(A-C)\}$ def. of set union.

[SCRATCH PAPER]