ASSIGNMENT 1

1 Warm Up (do before recitation lab)

(1) p and q are two propositions that are either T or F. How many rows are there in the truth table of the compound proposition $\neg(p \vee q) \wedge \neg p$.

(2) Give the truth table for the compound proposition above.

(3) **Theorem:** If integer $n \in \mathbb{N}$ is even then n^2 is even". What is wrong with the following proof:

1. Suppose n^2 is even.
2. Let n have a prime factorization $p_1^{q_1} p_2^{q_2} \cdots$.
3. So in the prime factorization of n^2, each prime appears an even number of times, $n^2 = p_1^{2q_1} p_2^{2q_2} \cdots$.
4. Since n^2 is even, 2 is a prime factor of n^2, so $n^2 = 2^{2q} \cdots$, with $q > 0$.
5. So $n = 2^q \cdots$, with $q > 0$ and so n has 2 as a factor and is therefore even as stated in the theorem.

What does the above proof actually prove?

(4) Is $n^2 + n + 41$ prime for $n = 1, 2, 3, 4, \ldots, 10$. Does that mean that $n^2 + n + 41$ is prime for all $n \in \mathbb{N}$?

(5) Let $F(x)$ denote 'x is a freshman' and $M(x)$ denote 'x is a math major'. Translate into sensible english:

(a) $\forall x(M(x) \to \neg F(x))$

(b) $\neg \exists x(M(x) \wedge \neg F(x))$

2 Recitation Lab (TA will work these out in lab)

(1) Using a truth table, determine whether these two compound propositions are logically equivalent:

$$(p \to q) \to r \qquad\qquad p \to (q \to r).$$

(2) What is the negation of "Jan is rich and happy"

(3) **[DNF]** Using only $\neg, \wedge, \vee$, give a compound proposition which has the following truth table:

p	q	r	
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

(4) What is the set $\mathbb{Z} \cap \overline{\mathbb{N}} \cap \mathcal{S}$, where the set S is the set of integers which are perfect squares.

(5) Prove that there are infinitely many primes.

3 Problems (hand these in)

(1) Use a direct proof to show that the product of two odd numbers is odd.

(2) Using only $\neg, \wedge, \vee$, give a compound proposition which has the following truth table:

q	r	
T	T	F
T	F	T
F	T	F
F	F	F

(3) Compute the *number* of positive divisors of the following integers:

$$6, 8, 12, 18, 30,$$
$$4, 9, 16, 25, 36$$

Formulate a conjecture that relates a property of the number of divisors of n to a property of n. Your conjecture should at the very least agree with your data. State your conjecture as a theorem. You do not have to prove your theorem but be **precise** in your statement of the theorem. (You may want to define some convenient notation, for example let $\phi(n)$ be the number of positive divisors of n).

(4) For the Ebola spreading model, a square gets infected if at least two (non-diagonal) squares are infected. Show the final state of the grid (who is infected) when the shaded in squares are initially infected.

FIGURE NOT INCLUDED

Can you find an arrangement of 5 initial infections that can eventually infect the whole 6×6 square. What about with 6 initial infections? Try the same game with a 4×4 square and a 5×5 square.

Suppose the square is $n \times n$ where $n \in \mathbb{N}$. Formulate a conjecture that asserts the minimum number of initial infections required to eventually infect the whole square.

State your conjecture as a theorem. You do not have to prove your theorem but be **precise** in your statement of the theorem.

(5) Prove or Disprove each of the following theorems:

Theorem. If x and y are both *irrational*, then x^y is irrational.
[Hint: We proved in class that $\sqrt{2}$ is irrational.]

Theorem. If n is an integer and n^2 is divisible by 3, then n is divisible by 3.