

11. Kinematic models of contact

Mechanics of Manipulation

Matt Mason

`matt.mason@cs.cmu.edu`

`http://www.cs.cmu.edu/~mason`

Carnegie Mellon

Chapter 1 Manipulation 1

- 1.1 Case 1: Manipulation by a human 1
- 1.2 Case 2: An automated assembly system 3
- 1.3 Issues in manipulation 5
- 1.4 A taxonomy of manipulation techniques 7
- 1.5 Bibliographic notes 8
- Exercises 8

Chapter 2 Kinematics 11

- 2.1 Preliminaries 11
- 2.2 Planar kinematics 15
- 2.3 Spherical kinematics 20
- 2.4 Spatial kinematics 22
- 2.5 Kinematic constraint 25
- 2.6 Kinematic mechanisms 34
- 2.7 Bibliographic notes 36
- Exercises 37

Chapter 3 Kinematic Representation 41

- 3.1 Representation of spatial rotations 41
- 3.2 Representation of spatial displacements 58
- 3.3 Kinematic constraints 68
- 3.4 Bibliographic notes 72
- Exercises 72

Chapter 4 Kinematic Manipulation 77

- 4.1 Path planning 77
- 4.2 Path planning for nonholonomic systems 84
- 4.3 Kinematic models of contact 86
- 4.4 Bibliographic notes 88
- Exercises 88

Chapter 5 Rigid Body Statics 93

- 5.1 Forces acting on rigid bodies 93
- 5.2 Polyhedral convex cones 99
- 5.3 Contact wrenches and wrench cones 102
- 5.4 Cones in velocity twist space 104
- 5.5 The oriented plane 105
- 5.6 Instantaneous centers and Reuleaux's method 109
- 5.7 Line of force; moment labeling 110
- 5.8 Force dual 112
- 5.9 Summary 117
- 5.10 Bibliographic notes 117
- Exercises 118

Chapter 6 Friction 121

- 6.1 Coulomb's Law 121
- 6.2 Single degree-of-freedom problems 123
- 6.3 Planar single contact problems 126
- 6.4 Graphical representation of friction cones 127
- 6.5 Static equilibrium problems 128
- 6.6 Planar sliding 130
- 6.7 Bibliographic notes 139
- Exercises 139

Chapter 7 Quasistatic Manipulation 143

- 7.1 Grasping and fixturing 143
- 7.2 Pushing 147
- 7.3 Stable pushing 153
- 7.4 Parts orienting 162
- 7.5 Assembly 168
- 7.6 Bibliographic notes 173
- Exercises 175

Chapter 8 Dynamics 181

- 8.1 Newton's laws 181
- 8.2 A particle in three dimensions 181
- 8.3 Moment of force; moment of momentum 183
- 8.4 Dynamics of a system of particles 184
- 8.5 Rigid body dynamics 186
- 8.6 The angular inertia matrix 189
- 8.7 Motion of a freely rotating body 195
- 8.8 Planar single contact problems 197
- 8.9 Graphical methods for the plane 203
- 8.10 Planar multiple-contact problems 205
- 8.11 Bibliographic notes 207
- Exercises 208

Chapter 9 Impact 211

- 9.1 A particle 211
- 9.2 Rigid body impact 217
- 9.3 Bibliographic notes 223
- Exercises 223

Chapter 10 Dynamic Manipulation 225

- 10.1 Quasidynamic manipulation 225
- 10.2 Brie y dynamic manipulation 229
- 10.3 Continuously dynamic manipulation 230
- 10.4 Bibliographic notes 232
- Exercises 235

Appendix A Infinity 237

Outline.

Grübler

- Review of kinematic mechanisms
- Mobility and connectivity
- Grübler's formula

Salisbury

- Taxonomy of contacts
- Mobility and connectivity of grasp

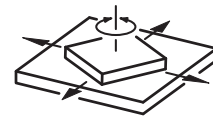
Kinematic mechanisms

Link: a rigid body;

Joint: imposes one or more constraints on the relative motion of two links;

Kinematic mechanism: a bunch of links joined by joints;

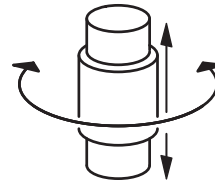
lower pairs joints involving positive contact area.



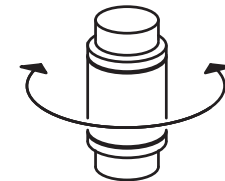
Planar
3 freedoms



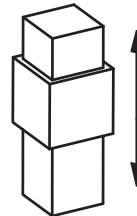
Spherical
3 freedoms



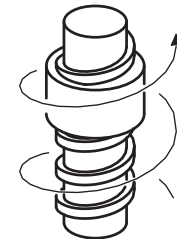
Cylindrical
2 freedoms



Revolute
1 freedom



Prismatic
1 freedom



Helical
1 freedom

Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

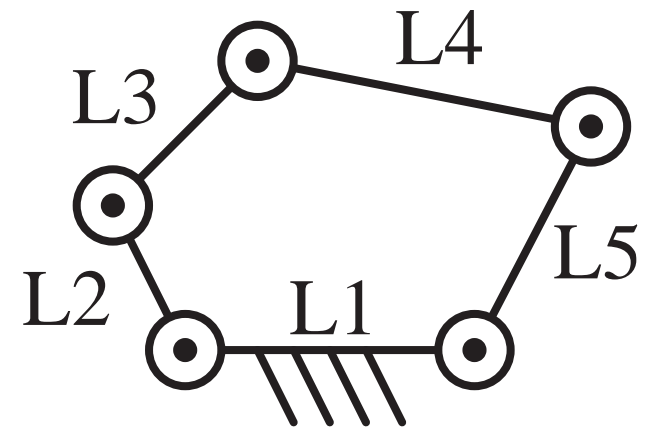
What is the mobility of the five bar linkage at right?

What is the connectivity of

Link 1 relative to link two?

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

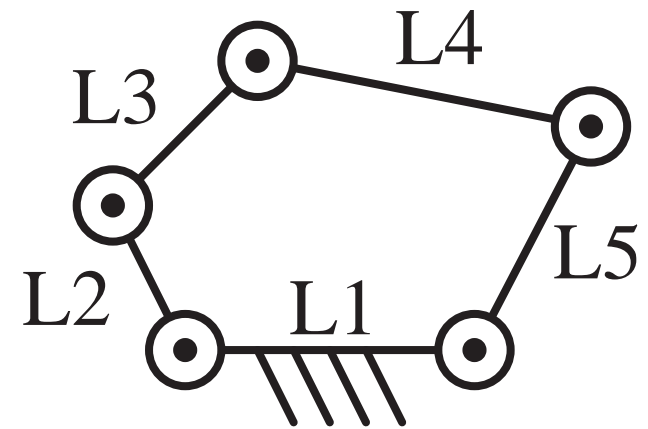
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two?

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

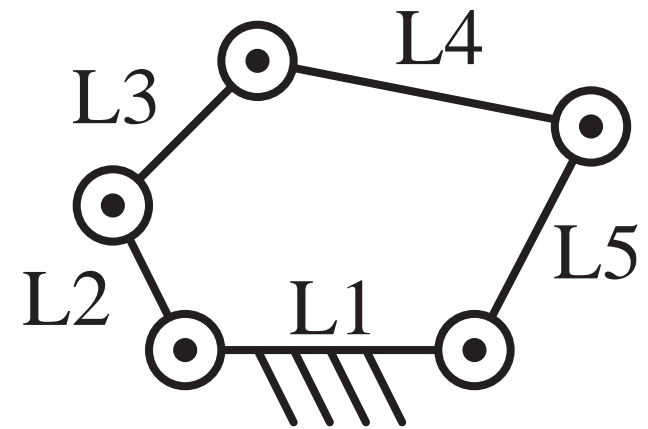
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two? One.

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

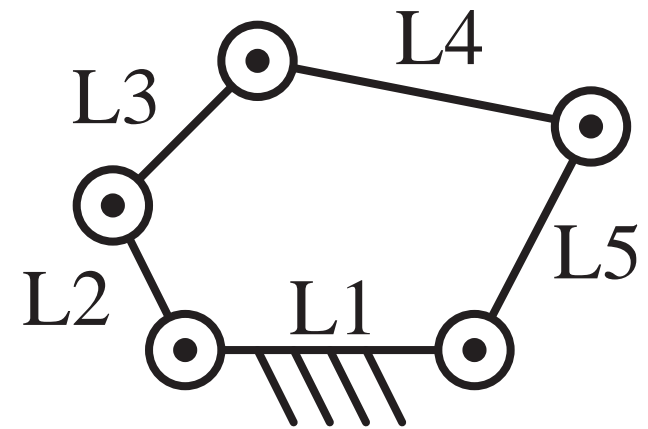
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two? One.

Link 3 relative to link 1? Two.

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

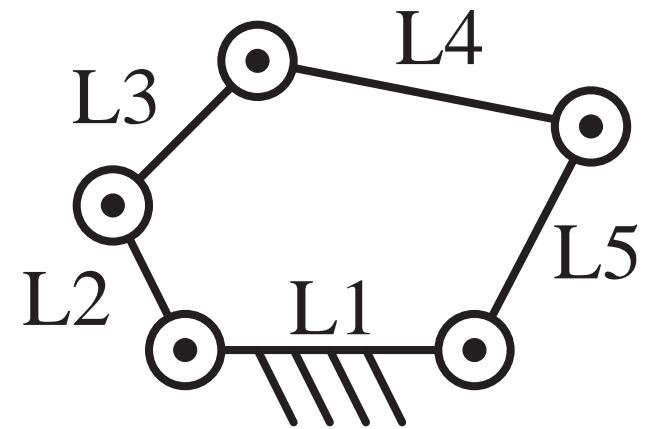
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two? One.

Link 3 relative to link 1? Two.

Link 3 relative to link 4? One.



Grübler's formula

Given n links joined by g joints,

with u_i constraints and f_i freedoms at joint i . (Note that $u_i + f_i = 6$.)

Assume one link is fixed and constraints are all independent.

The mobility M is

$$\begin{aligned}M &= 6(n - 1) - \sum u_i \\ &= 6(n - 1) - \sum (6 - f_i) \\ &= 6(n - g - 1) + \sum f_i\end{aligned}$$

Or, for a planar mechanism:

$$\begin{aligned}M &= 3(n - 1) - \sum u_i \\ &= 3(n - g - 1) + \sum f_i\end{aligned}$$

Grübler: special case for loops

The previous formula works (sort of) for all mechanisms.

For loops there is a variant.

One loop: $n = g$, so

$$M = \sum f_i + 6(-1)$$

Two loops: make a second loop by adding k links and $k + 1$ joints:

$$M = \sum f_i + 6(-2)$$

Every loop increases excess of joints over links by 1. For l loops:

$$M = \sum f_i - 6l$$

for a spatial linkage, and

$$M = \sum f_i - 3l$$

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i =$$

$$M = 3(n - g - 1) + \sum f_i =$$

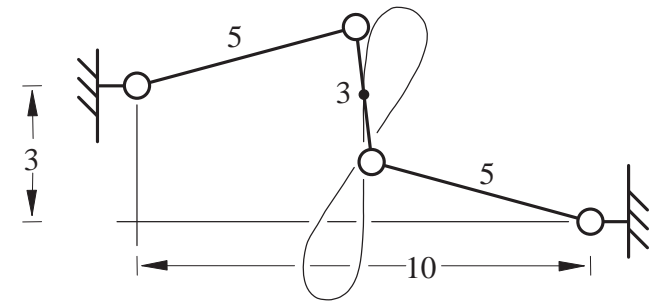
$$M = \sum f_i - 3l =$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i =$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i =$$

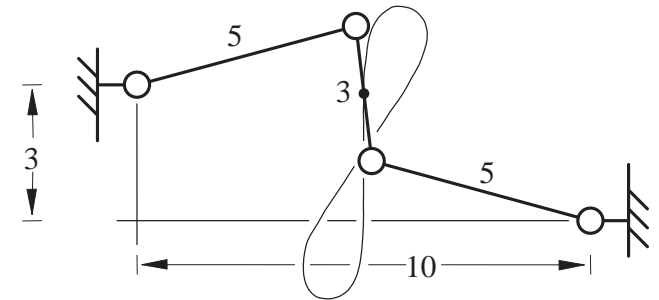
$$M = \sum f_i - 3l =$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i =$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

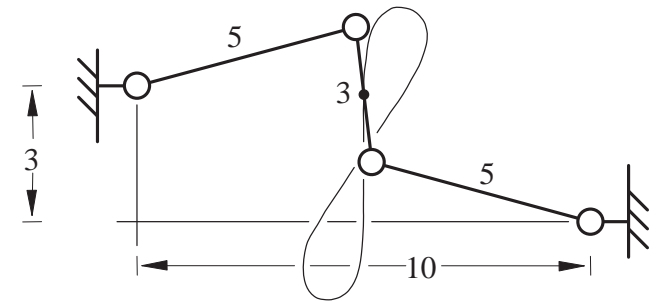
$$M = \sum f_i - 3l =$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i =$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gröbler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

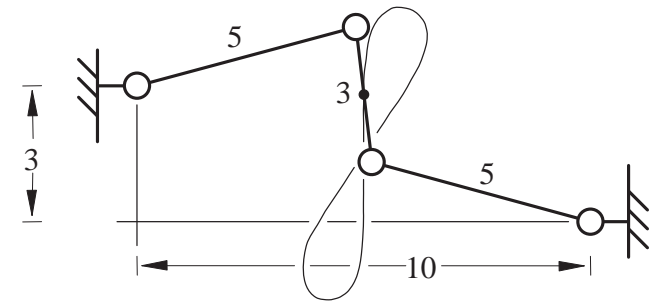
$$M = \sum f_i - 3l = 1$$

Spatial Gröbler's formula:

$$M = 6(n - 1) - \sum u_i =$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

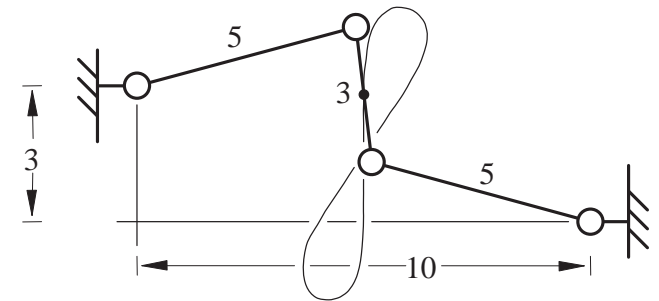
$$M = \sum f_i - 3l = 1$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i = -2$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

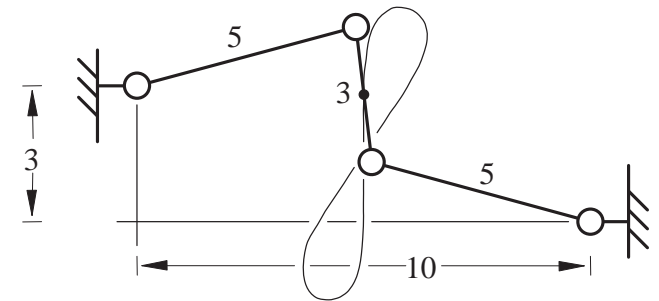
$$M = \sum f_i - 3l = 1$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i = -2$$

$$M = 6(n - g - 1) + \sum f_i = -2$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gr ubler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

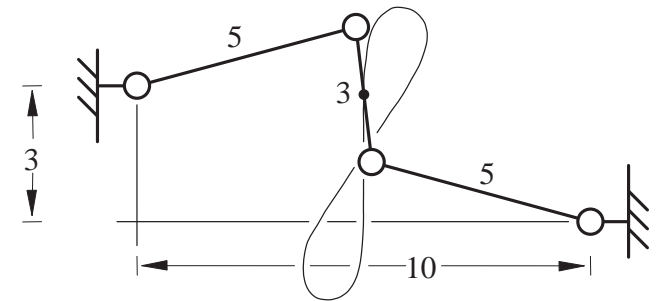
$$M = \sum f_i - 3l = 1$$

Spatial Gr ubler's formula:

$$M = 6(n - 1) - \sum u_i = -2$$

$$M = 6(n - g - 1) + \sum f_i = -2$$

$$M = \sum f_i - 6l = -2$$



Independent constraints is a very strong assumption.

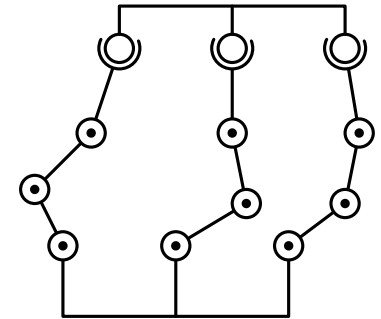
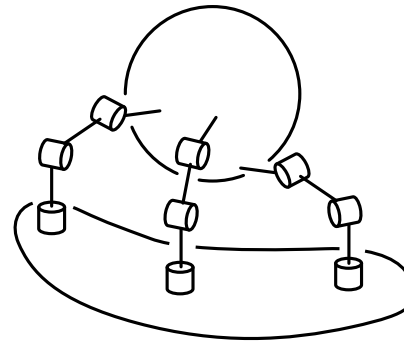
Kinematic models of contact

A grasp is like a kinematic mechanism.

Assume fingers do not lift or slip.

Model each contact as a joint

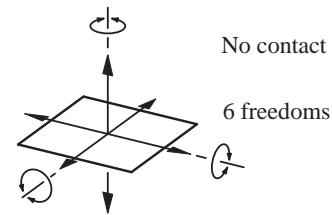
Apply Grübler's formula!



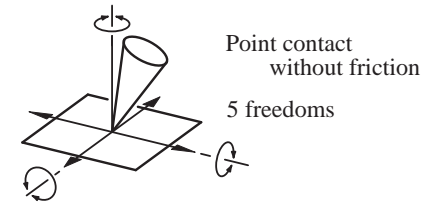
Taxonomy of contact types

In previous slide, contact was modeled as spherical joint. Are there other possibilities?

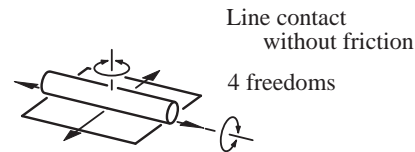
Salisbury's PhD thesis, 1982, included a taxonomy.



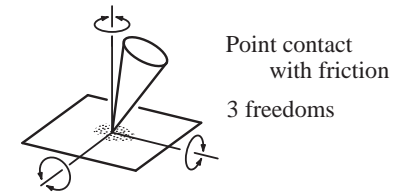
No contact
6 freedoms



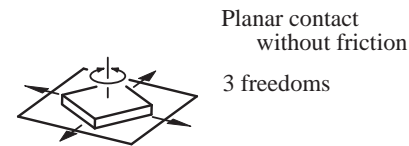
Point contact
without friction
5 freedoms



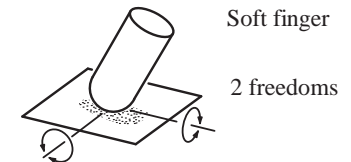
Line contact
without friction
4 freedoms



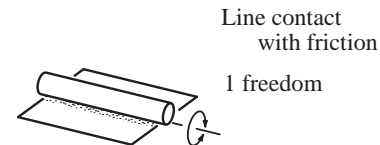
Point contact
with friction
3 freedoms



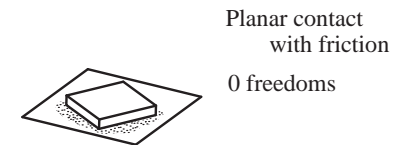
Planar contact
without friction
3 freedoms



Soft finger
2 freedoms



Line contact
with friction
1 freedom



Planar contact
with friction
0 freedoms

Mobility and connectivity of grasp

Salisbury suggests four measures:

M Mobility of the entire system with the finger joints free.

M' Mobility of the entire system, with the finger joints locked.

C Connectivity of the object relative to a fixed palm, with the finger joints free.

C' Connectivity of the object relative to a fixed palm, with the finger joints locked.

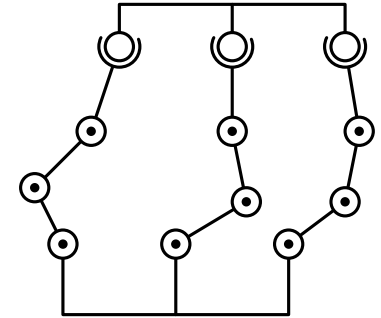
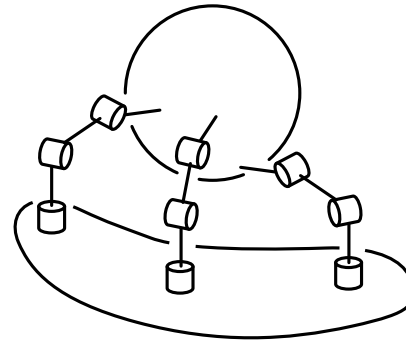
If $C = 6$ then object can make general motions.

If $C' \leq 0$ then hand can immobilize object.

Example: the Salisbury hand

What is C ?

What is C' ?



Chapter 1 Manipulation 1

- 1.1 Case 1: Manipulation by a human 1
- 1.2 Case 2: An automated assembly system 3
- 1.3 Issues in manipulation 5
- 1.4 A taxonomy of manipulation techniques 7
- 1.5 Bibliographic notes 8
- Exercises 8

Chapter 2 Kinematics 11

- 2.1 Preliminaries 11
- 2.2 Planar kinematics 15
- 2.3 Spherical kinematics 20
- 2.4 Spatial kinematics 22
- 2.5 Kinematic constraint 25
- 2.6 Kinematic mechanisms 34
- 2.7 Bibliographic notes 36
- Exercises 37

Chapter 3 Kinematic Representation 41

- 3.1 Representation of spatial rotations 41
- 3.2 Representation of spatial displacements 58
- 3.3 Kinematic constraints 68
- 3.4 Bibliographic notes 72
- Exercises 72

Chapter 4 Kinematic Manipulation 77

- 4.1 Path planning 77
- 4.2 Path planning for nonholonomic systems 84
- 4.3 Kinematic models of contact 86
- 4.4 Bibliographic notes 88
- Exercises 88

Chapter 5 Rigid Body Statics 93

- 5.1 Forces acting on rigid bodies 93
- 5.2 Polyhedral convex cones 99
- 5.3 Contact wrenches and wrench cones 102
- 5.4 Cones in velocity twist space 104
- 5.5 The oriented plane 105
- 5.6 Instantaneous centers and Reuleaux's method 109
- 5.7 Line of force; moment labeling 110
- 5.8 Force dual 112
- 5.9 Summary 117
- 5.10 Bibliographic notes 117
- Exercises 118

Chapter 6 Friction 121

- 6.1 Coulomb's Law 121
- 6.2 Single degree-of-freedom problems 123
- 6.3 Planar single contact problems 126
- 6.4 Graphical representation of friction cones 127
- 6.5 Static equilibrium problems 128
- 6.6 Planar sliding 130
- 6.7 Bibliographic notes 139
- Exercises 139

Chapter 7 Quasistatic Manipulation 143

- 7.1 Grasping and fixturing 143
- 7.2 Pushing 147
- 7.3 Stable pushing 153
- 7.4 Parts orienting 162
- 7.5 Assembly 168
- 7.6 Bibliographic notes 173
- Exercises 175

Chapter 8 Dynamics 181

- 8.1 Newton's laws 181
- 8.2 A particle in three dimensions 181
- 8.3 Moment of force; moment of momentum 183
- 8.4 Dynamics of a system of particles 184
- 8.5 Rigid body dynamics 186
- 8.6 The angular inertia matrix 189
- 8.7 Motion of a freely rotating body 195
- 8.8 Planar single contact problems 197
- 8.9 Graphical methods for the plane 203
- 8.10 Planar multiple-contact problems 205
- 8.11 Bibliographic notes 207
- Exercises 208

Chapter 9 Impact 211

- 9.1 A particle 211
- 9.2 Rigid body impact 217
- 9.3 Bibliographic notes 223
- Exercises 223

Chapter 10 Dynamic Manipulation 225

- 10.1 Quasidynamic manipulation 225
- 10.2 Brie y dynamic manipulation 229
- 10.3 Continuously dynamic manipulation 230
- 10.4 Bibliographic notes 232
- Exercises 235

Appendix A Infinity 237