11. Kinematic models of contact

Mechanics of Manipulation

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Outline.

Grübler

- Review of kinematic mechanisms
- Mobility and connectivity
- Grübler's formula

Salisbury

- Taxonomy of contacts
- Mobility and connectivity of grasp

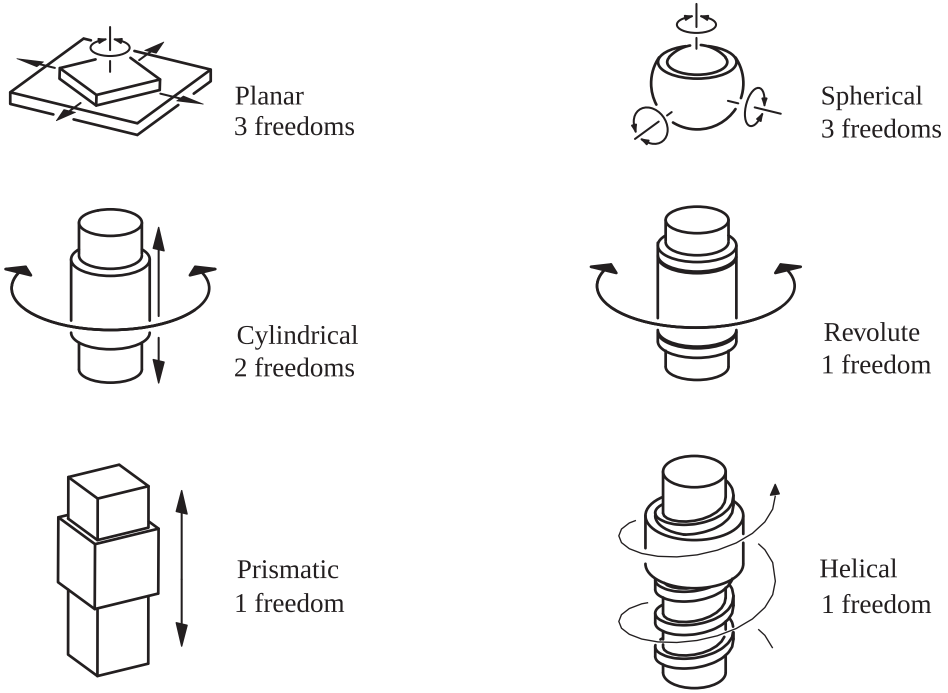
Kinematic mechanisms

Link: a rigid body;

Joint: imposes one or more constraints on the relative motion of two links;

Kinematic mechanism: a bunch of links joined by joints;

lower pairs joints involving positive contact area.



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

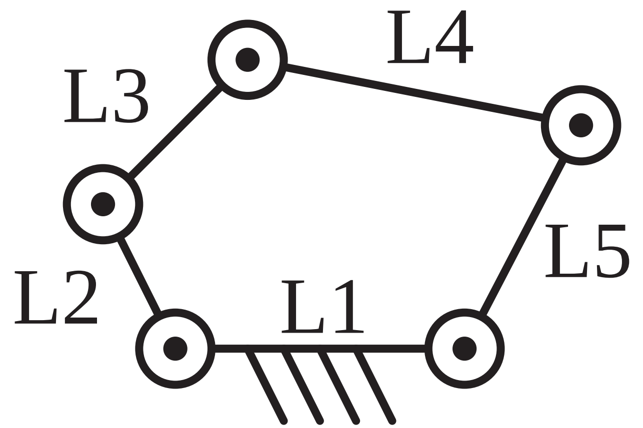
What is the mobility of the five bar linkage at right?

What is the connectivity of

Link 1 relative to link two?

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

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What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two?

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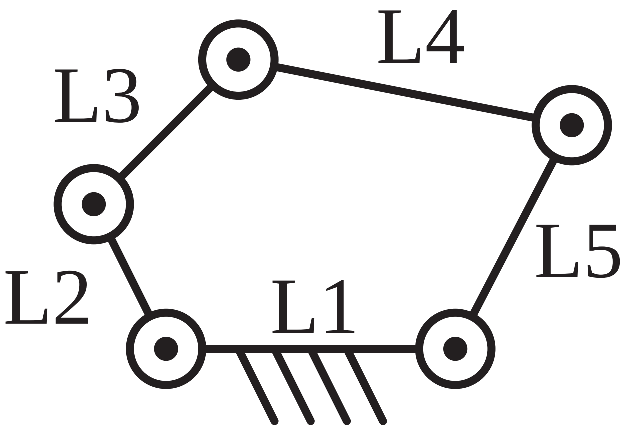
Link 3 relative to link 4?



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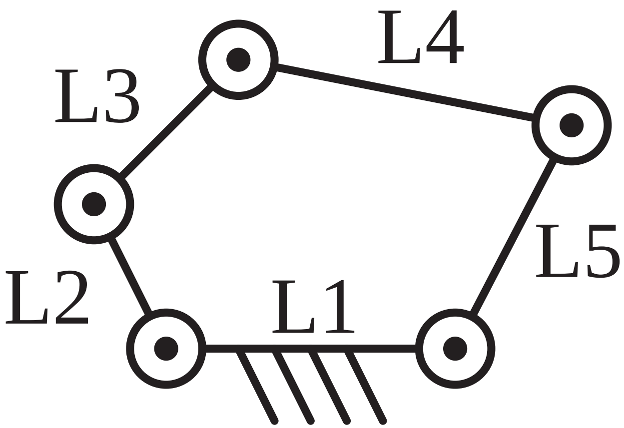

What is the connectivity of

Link 1 relative to link two? One.

Link 3 relative to link 1?

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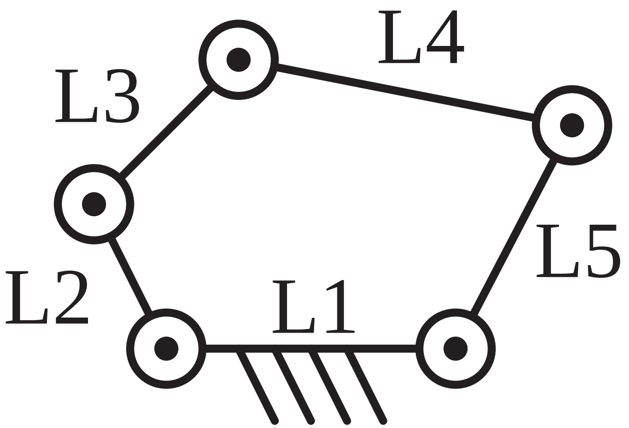

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> Link 1 relative to link two? One.
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> Link 3 relative to link 1? Two.
>
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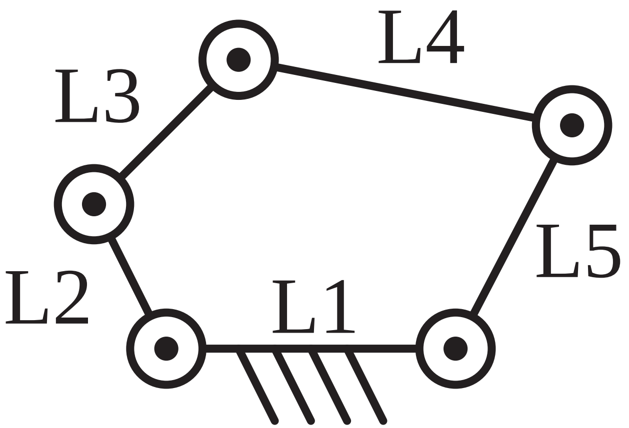

Grübler's formula

Given n links joined by g joints,

with u_i constraints and f_i freedoms at joint i. (Note that $u_i + f_i = 6$.)

Assume one link is fixed and constraints are all independent.

The mobility M is

$$
\begin{aligned}
M &= 6(n-1) - \sum u_i \\
&= 6(n-1) - \sum (6 - f_i) \\
&= 6(n-g-1) + \sum f_i
\end{aligned}
$$

Or, for a planar mechanism:

$$
\begin{aligned}
M &= 3(n-1) - \sum u_i \\
&= 3(n-g-1) + \sum f_i
\end{aligned}
$$

Grübler: special case for loops

The previous formula works (sort of) for all mechanisms.

For loops there is a variant.

One loop: n = g, so

$$M = \sum f_i + 6(-1)$$

Two loops: make a second loop by adding k links and $k+1$ joints:

$$M = \sum f_i + 6(-2)$$

Every loop increases excess of joints over links by 1. For l loops:

$$M = \sum f_i - 6l$$

for a spatial linkage, and

$$M = \sum f_i - 3l$$

Common sense

Example: what is the mobility of Watt's linkage?

Planar Grübler's formula:

$$M = 3(n-1) - \sum u_i =$$

$$M = 3(n-g-1) + \sum f_i =$$

$$M = \sum f_i - 3l =$$

Spatial Grübler's formula:

$$M = 6(n-1) - \sum u_i =$$

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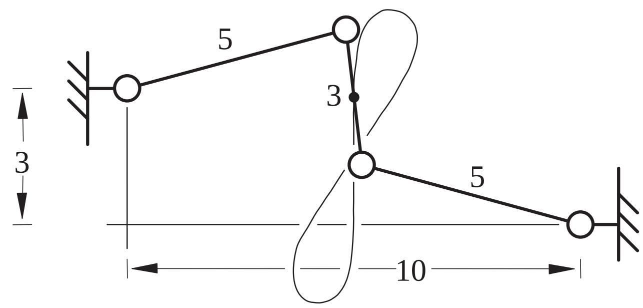
Why?



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

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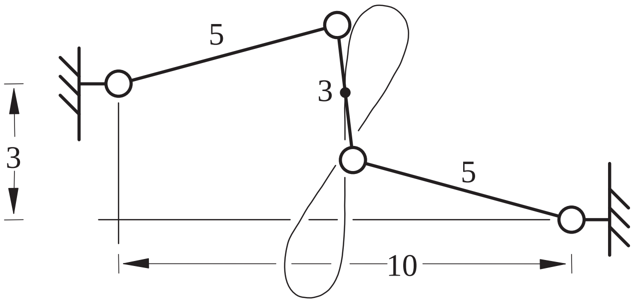

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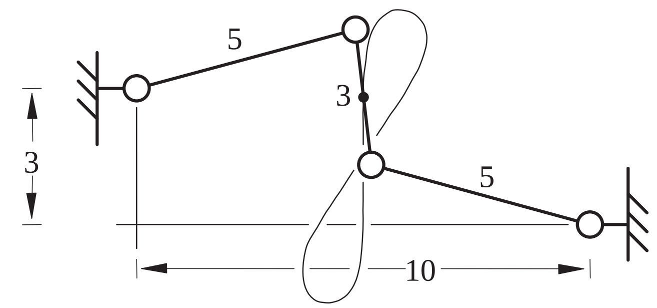

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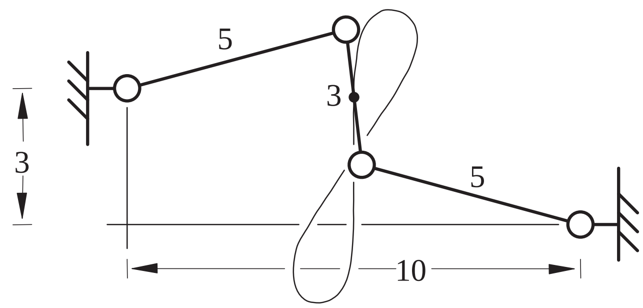

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Spatial Grübler's formula:

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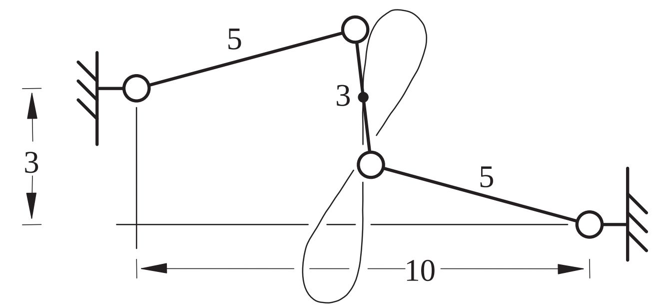
Why?



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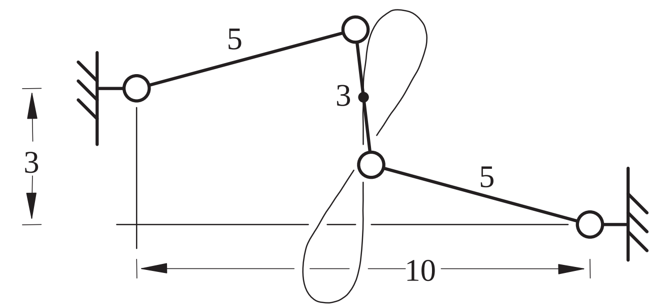

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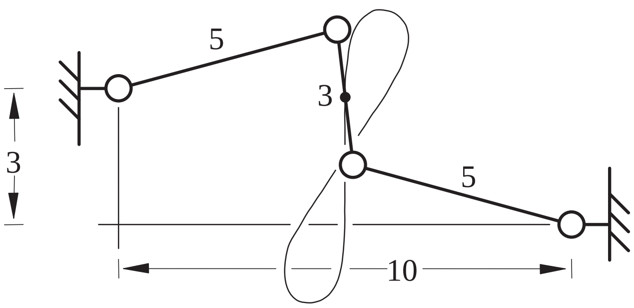

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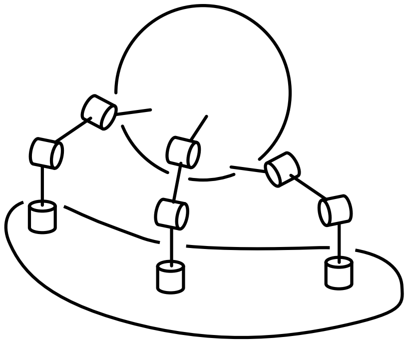
Kinematic models of contact

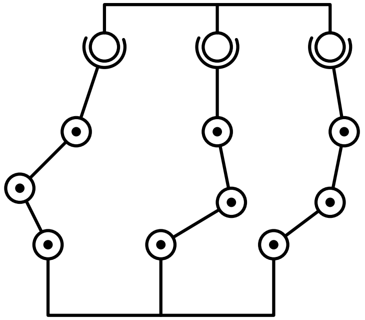
A grasp is like a kinematic mechanism.

Assume fingers do not lift or slip.

Model each contact as a joint

Apply Grübler's formula!

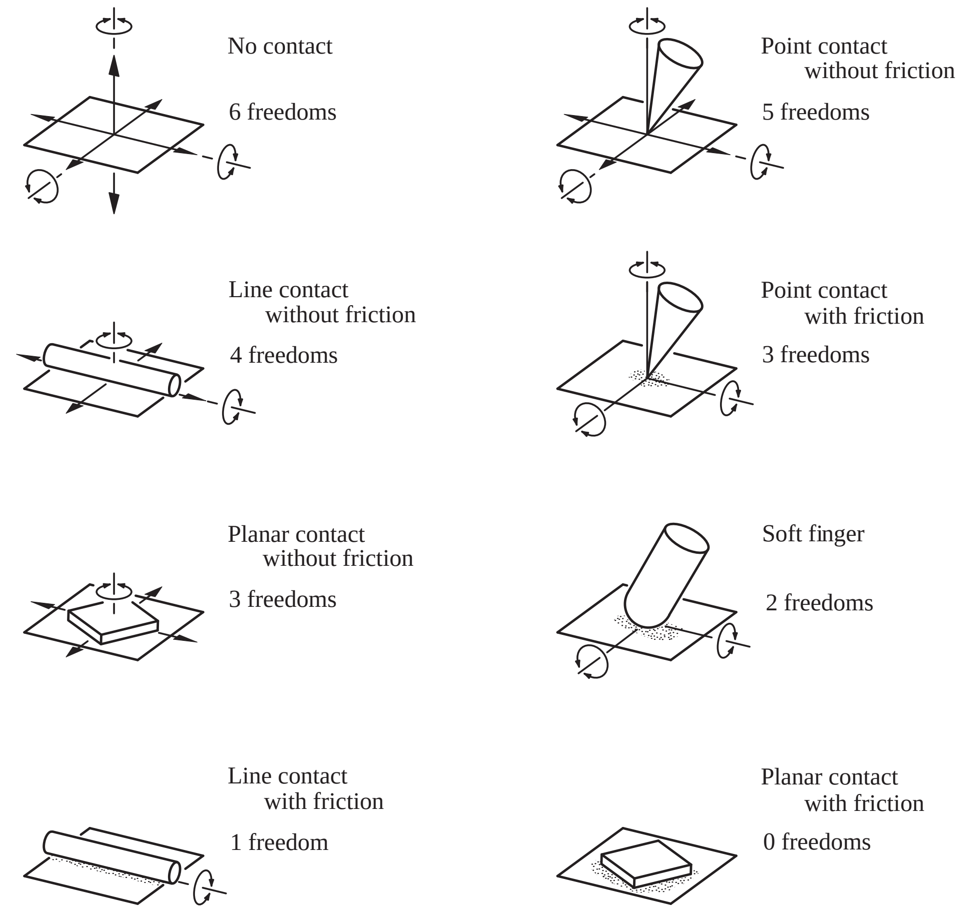




Taxonomy of contact types

In previous slide, contact was modeled as spherical joint. Are there other possibilities?

Salisbury's PhD thesis, 1982, included a taxonomy.



Mobility and connectivity of grasp

Salisbury suggests four measures:

- M Mobility of the entire system with the finger joints free.
- M' Mobility of the entire system, with the finger joints locked.
- C Connectivity of the object relative to a fixed palm, with the finger joints free.
- C' Connectivity of the object relative to a fixed palm, with the finger joints locked.

If $C = 6$ then object can make general motions.

If $C' \leq 0$ then hand can immobilize object.

Example: the Salisbury hand

What is C?

What is C'?