15. Moment Labeling

Mechanics of Manipulation

Matt Mason

matt.mason@cs.cmu.edu

http://www.cs.cmu.edu/~mason

Carnegie Mellon

Outline.

Review: where are we? Where are we going?

Moment labeling: practical introduction.

Moment labeling: a more formal approach.

Relation to PCC's in wrench space.

Examples.

Where are we?

Wrench and twist spaces.

- Chasles' theorem: rigid body motion is a twist.
- Poinsot's theorem: forces on rigid body are a wrench

Polyhedral convex cones.

- Edge representation: $\mathrm{pos}(\{\mathbf{e}_i\})$
- Face representation: $\cap\{\mathrm{half}(\mathbf{n}_i)\}$
- Supplementary cone: $\mathrm{pos}(\{\mathbf{n}_i\})$ or $\cap\{\mathrm{half}(\mathbf{e}_i)\}$

Oriented plane.

- Rays in planar homogeneous coordinate space (x, y, w).
- Project to $w = 1$ plane, with $+$ or $-$ to remember sign of w.
- Convexity: interior and exterior line segments.

Where are we going?

Graphical techniques for planar contact problems:

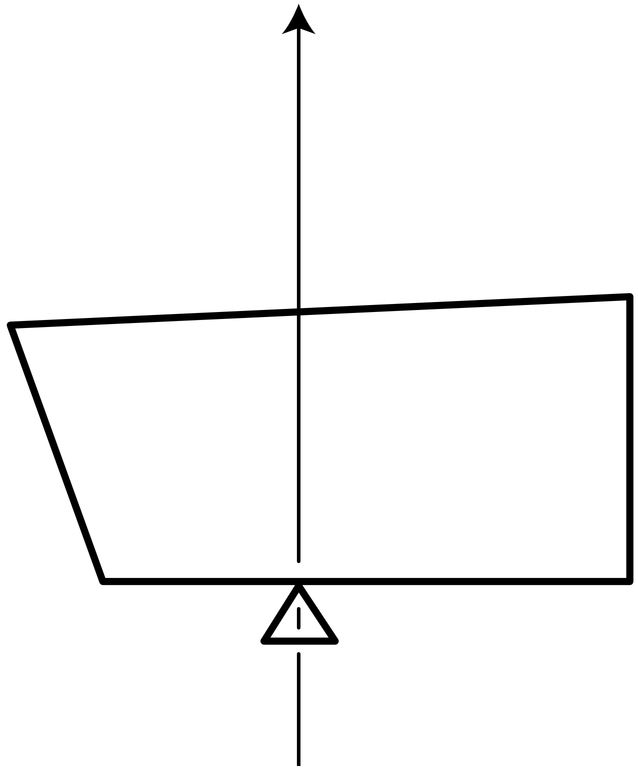
	Project cone to oriented plane	**Project supplementary cone to oriented plane**
single wrench	(acc'n center)	line of action
single diff'l twist	IC	?
wrench cone	force dual	moment labeling
diff'l twist cone	Reuleaux	?

Possible resultants for one contact.

For a frictionless contact, force is along contact normal, magnitude is indeterminate.

I.e., the (directed) line of force is determined.

Another definition of line of force: locus of zero moment points.



Another way of drawing the line of force.

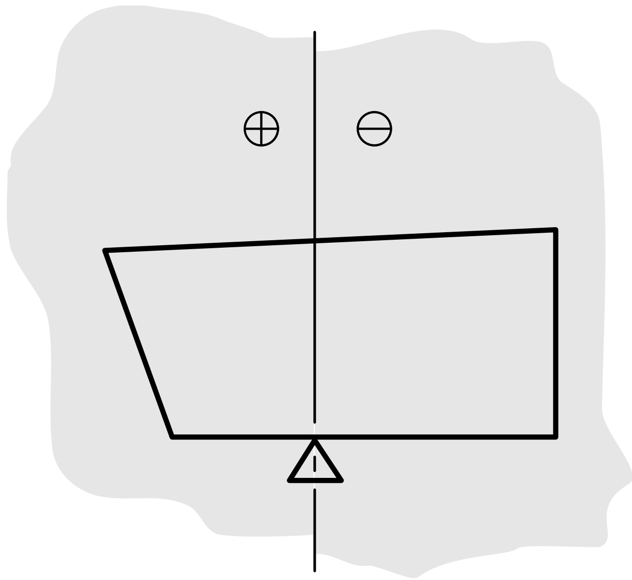
If we indicate sign of moment at each point . . .

. . . half plane of $\oplus$ to the left;

. . . half plane of $\ominus$ to the right;

zero moment points are the boundary.

Gives line of force *and* its direction.

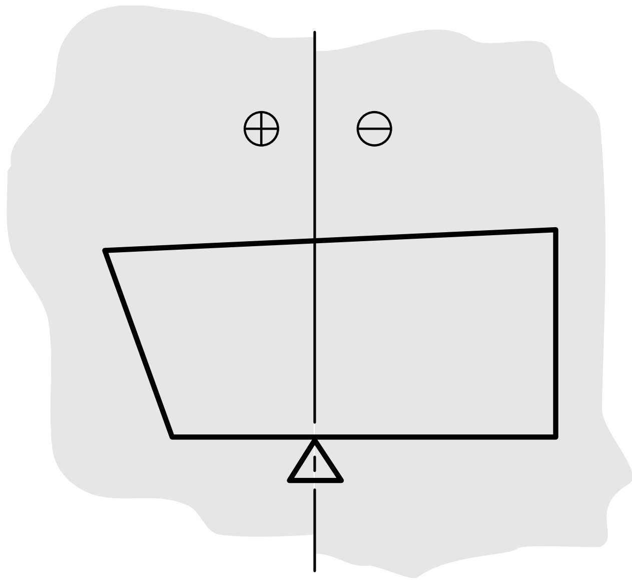


Reuleaux's method has *two* interpretations!

The cone of differential twists reciprocal or repelling to a given wrench.

The cone of wrenches in the positive linear span of a given wrench.

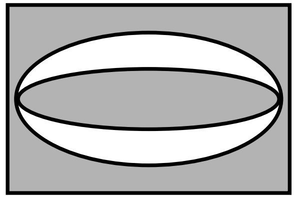
Reciprocal or repelling turns any wrench-PCC rep'n into a twist-PCC rep'n.

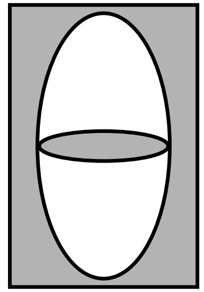


Superiority of wrench interpretation.

Actually the wrench interpretation is more useful.

> Differential twist cone is only first order approximation.
>
> Wrench interpretation extends to include friction.
>
> Wrench interpretation extends beyond convex cones.

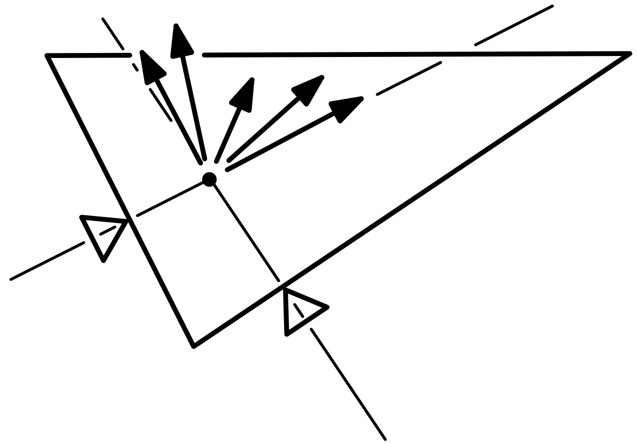




Possible resultants of *two* contacts.

Remember how to construct resultant of two forces in the plane?

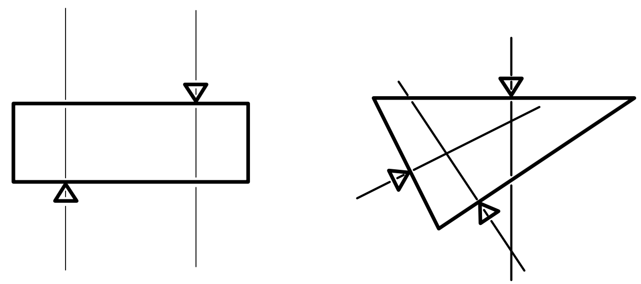
We can represent this set of resultants by a cone drawn in the plane.



Some more challenging problems

How do we deal with these cases:

- Possible resultants of two *parallel* contacts?
- Possible resultants of three contacts?

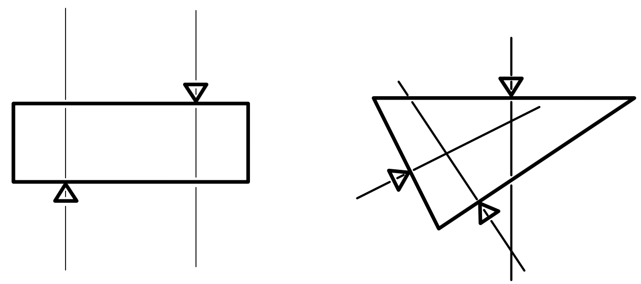


Some more challenging problems

How do we deal with these cases:

- Possible resultants of two *parallel* contacts?
- Possible resultants of three contacts?

Let's try Reuleaux's method and reinterpret.

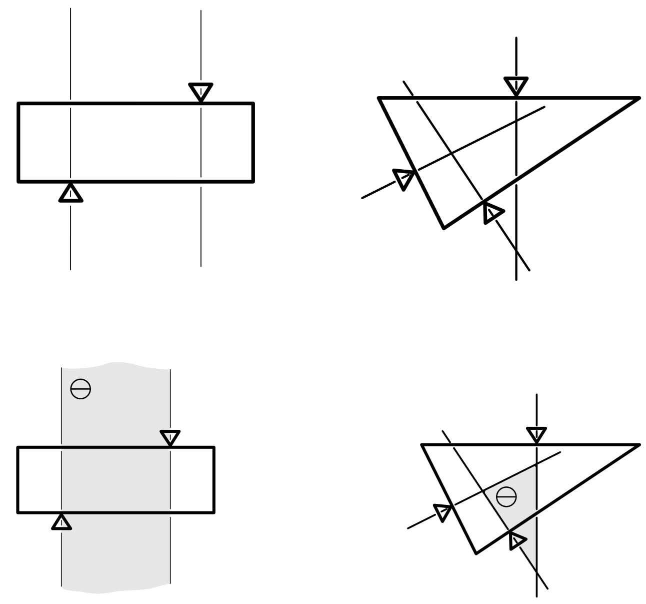


Some more challenging problems

How do we deal with these cases:

- Possible resultants of two *parallel* contacts?
- Possible resultants of three contacts?

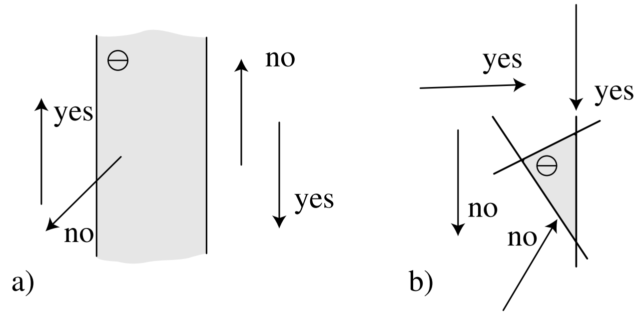
Let's try Reuleaux's method and reinterpret.



Interpreting the labeled regions.

The set of all forces that go between ⊕ and ⊖, in the right direction.

They can graze, but cannot go through the interior.



More formally

Given a set of contact wrenches $\{w_i\}$,

Let $\oplus_i$ and $\ominus_i$ be the points of nonnegative and nonpositive moments, respectively, for contact w_i,

Let $\oplus$ be the intersection of all nonnegative regions

$$\oplus = \cap \oplus_i$$

and let $\ominus$ be the intersection of all nonpositive regions

$$\ominus = \cap \ominus_i$$

Then the possible resultants $\mathrm{pos}(\{w_i\})$ is the set of all wrenches making nonnegative moments with all points in $\oplus$ and nonpositive moments with all points in $\ominus$.

Why does it work?

If, for example, all w_i give positive moments with respect to some point,

Then so does any wrench of the form

$$\sum k_i w_i$$

if all the k_i are nonnegative.

So, represent every w_i by where it *can't* go,

Intersect to determine where *none* of the w_i can go,

That's where wrenches $\mathrm{pos}(\{w_i\})$ cannot go.

Examples

Sliding friction (preview).

Two facing cones.

Disk in concrete.

GOAT against step.