21. Pushing

Mechanics of Manipulation

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Outline.

- Finish the “voting theorem”.
  - We’ve proven that line of motion dictates rotation direction.
  - Prove that line of force dictates rotation direction.
  - Prove the voting theorem.
- Application to stable pushing.

Pushing

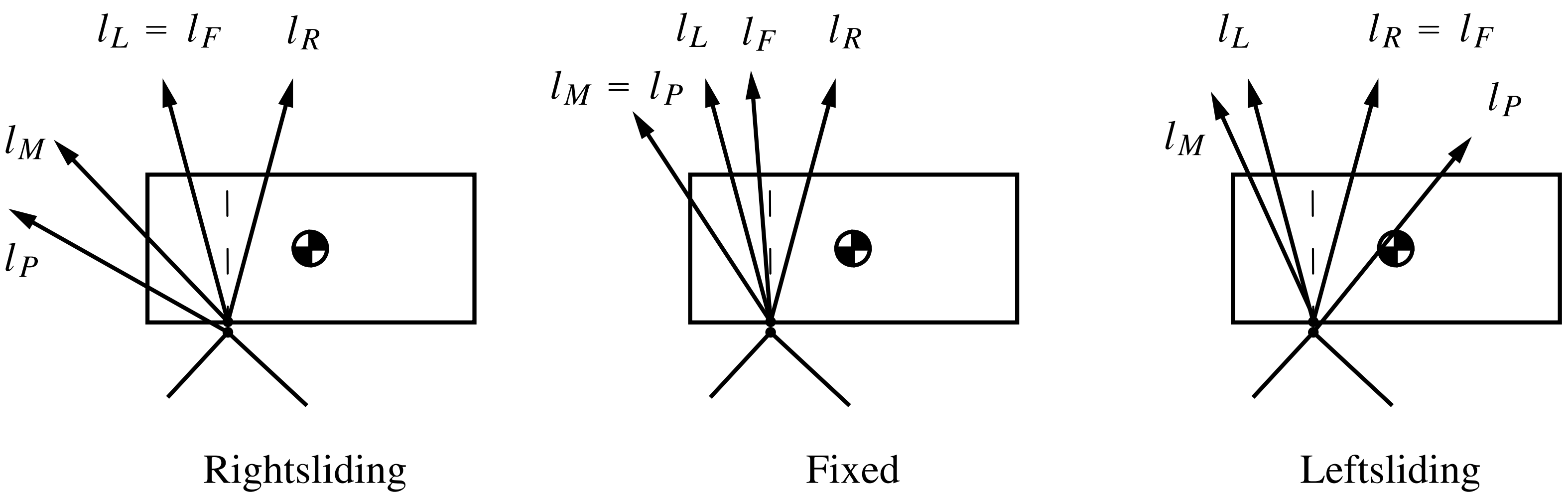
Can we predict direction of rotation?

Line of pushing l_P defined along vel of point in pusher.

Line of motion l_M defined along vel of point in slider.

Line of force l_F defined as usual.

Two edges of friction cone l_L and l_R.



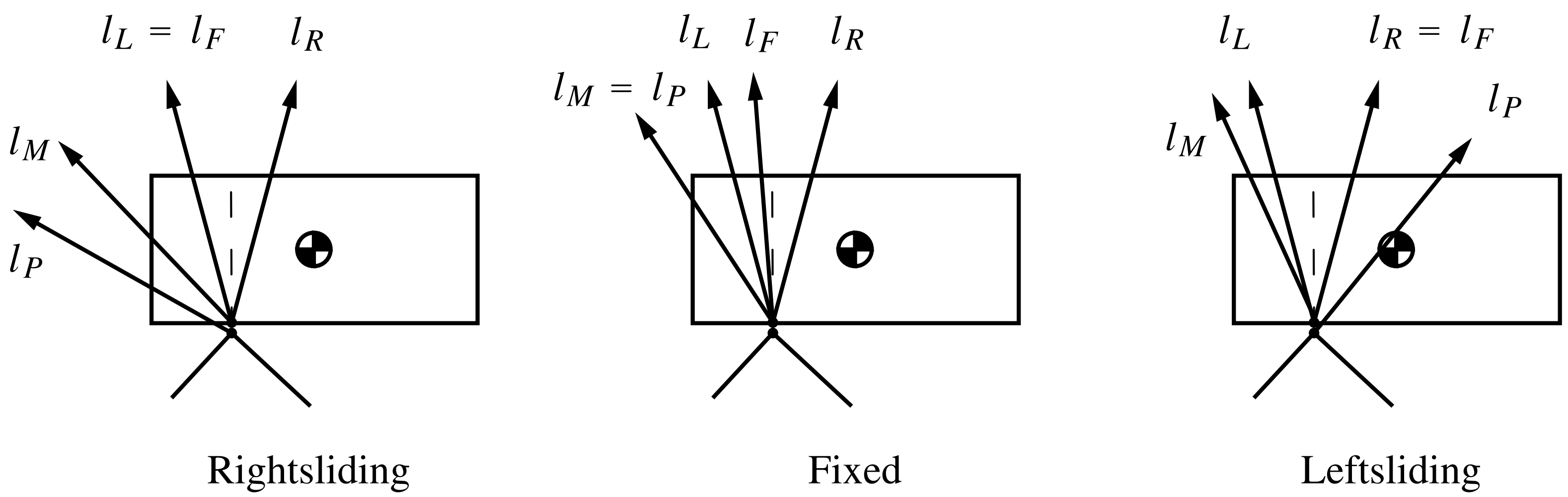
Which way will it turn?

Easy to predict from l_M or from l_F, but what you *know* is l_L, l_R, and l_P.

Main result: l_L, l_R, and l_P vote on rotation direction.

First: l_M dictates rotation direction.

Second: l_F dictates rotation direction.



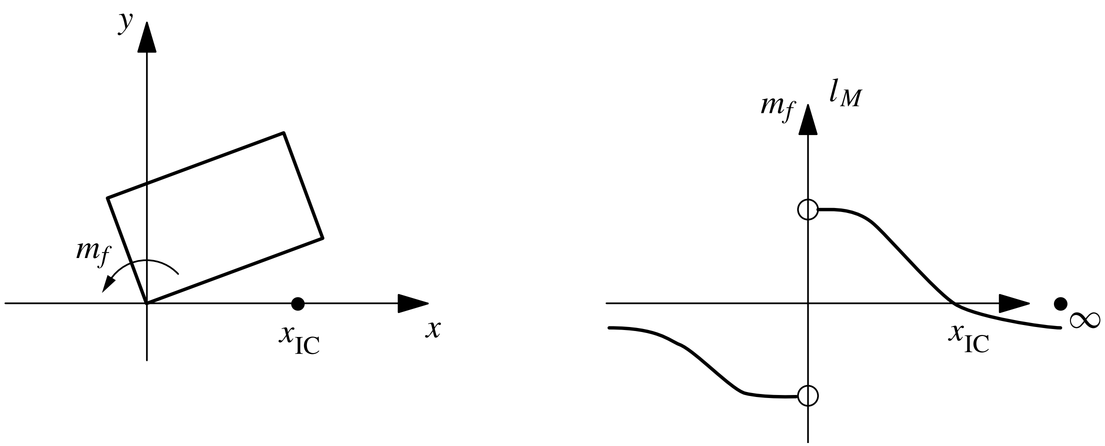
Line of motion dictates

Theorem: For quasistatic pushing of a rigid body in the plane, with uniform coefficient of friction, the line of motion dictates the rotation direction.

Let y-axis be line of motion, let origin be contact point, let x_{IC} be IC coordinate, let $m_f(x_{\mathrm{IC}})$ be frictional moment as function of IC.

Show $m_f(x_{\mathrm{IC}})$ is monotone decreasing.

Look at values at 0^+, 0^-, ∞, apply intermediate value theorem.



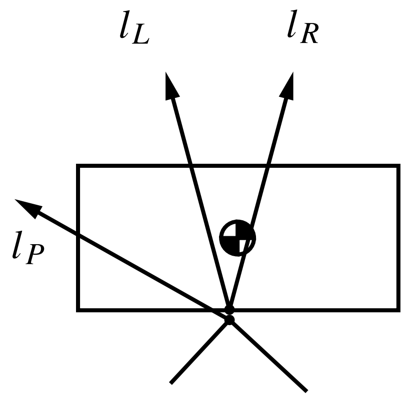
Line of force dictates . . .

Theorem: For quasistatic pushing of a rigid body in the plane, with uniform coefficient of friction, the line of force dictates the rotation direction.
Proof:

> Choose origin at center of friction, construct limit surface.
>
> Normals at f_x-f_y plane are horizontal.
>
> By convexity, normals in upper half point up, in lower half point down.

Voting theorem

Theorem: For quasistatic pushing of a planar rigid body with uniform coefficient of friction, rotation direction is determined by a vote l_P, l_L, and l_R.



Construct voting tree.

If edges of friction agree, then so does line of force, and theorem follows.

Consider case where edges do not agree.

l_L votes $-$, l_R votes $+$, and l_P votes $-$. The majority is $-$.

Assume positive rotation. So l_F and l_M would vote $+$ by previous theorems. If l_M is right of $\mathbf{r}_0$ then it is right of l_P, so we have right sliding. So $l_F = l_L$: a contradiction.

The voting theorem really works.

Demo on overhead.

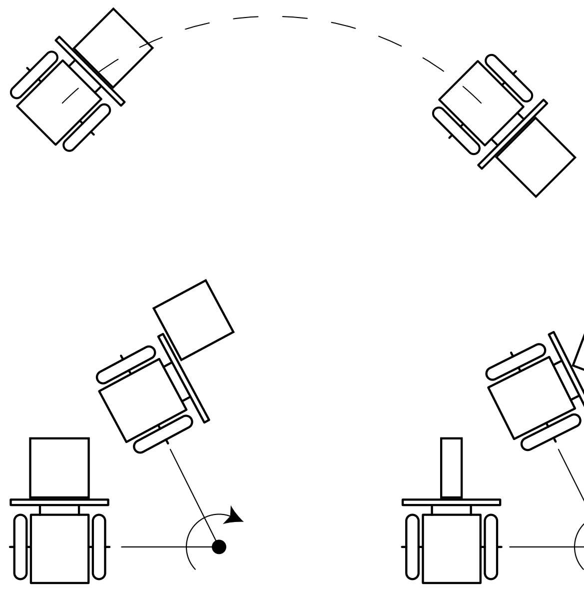
It tells you which way it turns but

> *not* how fast, and
>
> *not* about what IC.

Very useful when pushing with a translating edge.

Stable pushing

Sometimes we want to turn while pushing!



How can we achieve a stable push?

No slip of slider along pusher.

No rolling of slider on pusher.

Voting theorem by itself is not enough. We need more constraints on the IC.

Peshkin's bound

The voting theorem is a bound on IC's. It tells you whether the IC is in the positive plane, the negative plane, or the line at infinity. We need tighter bounds!

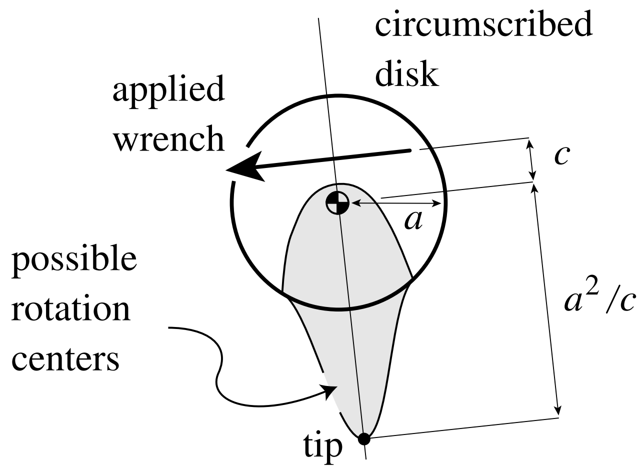
Circumscribe slider support R by a circle centered at center of friction.

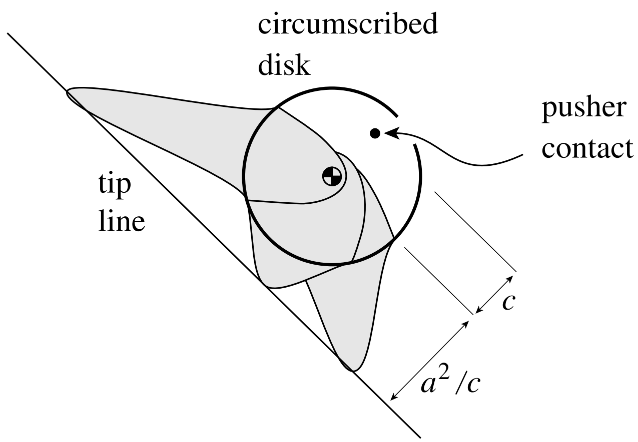
Construct IC for every possible support dipod.

Conjecture: resulting locus includes every possible support, not just dipods.

If we allow line of force to vary, locus sweeps out "tip line".

Note duality of tip line to contact point!





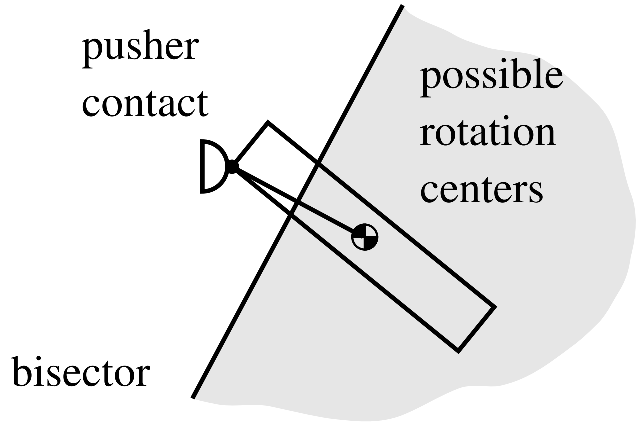
The “bisector bound”

Construct line from contact to center of friction.

Construct perpendicular bisector.

IC is on c.o.f. side of perp bisector.

Proof never published.

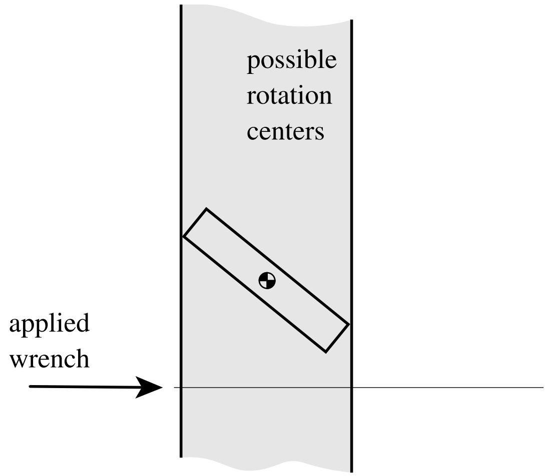


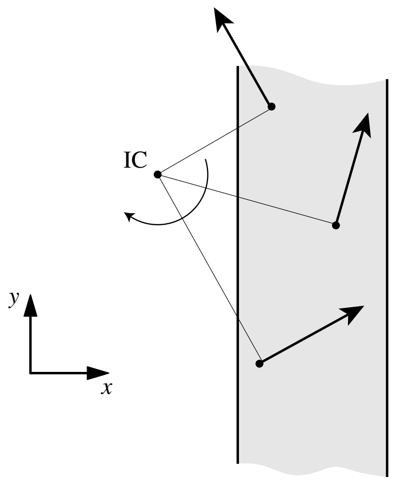
The vertical strip bound

Project support region R onto pushing line of force.

IC must fall in inverse projection.

Proof: Force balance impossible otherwise.

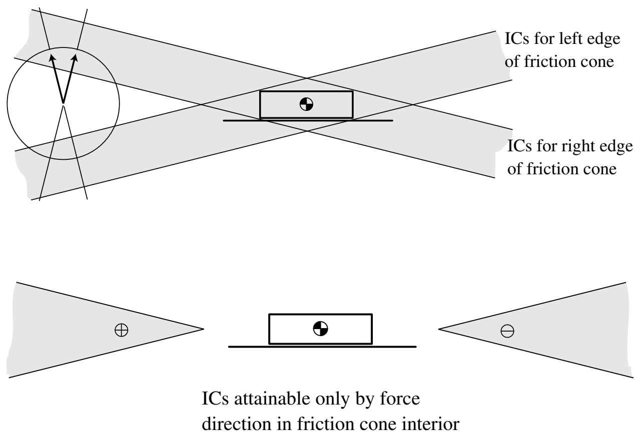




Not slipping off the pusher

Slipping of slider on pusher corresponds to left or right edge of FC.

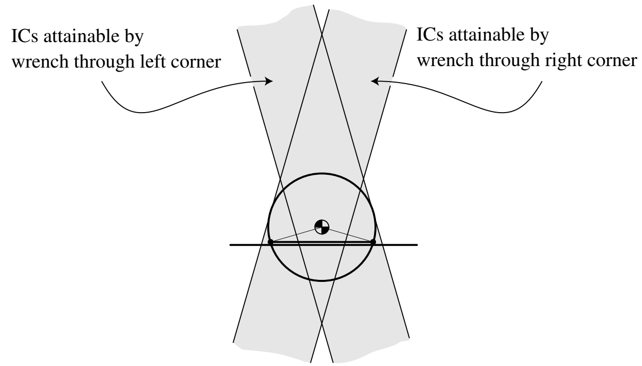
No slipping: interior of FC.

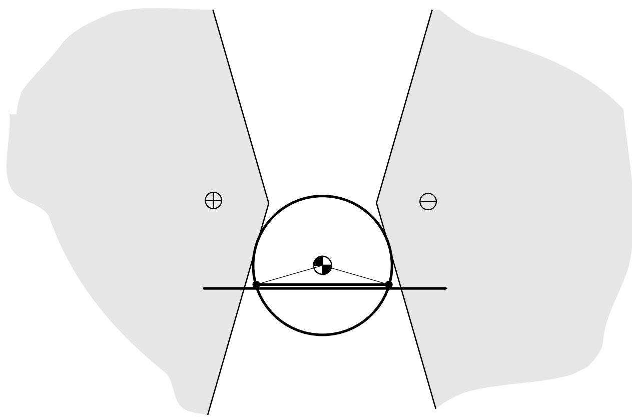


Not rolling off the pusher

Rolling corresponds to force through left or right corner of block.

Not rolling: line of force between corners.





ICs attainable only by wrench
between the two corners

Combining constraints, planning a path

We eliminate all failure modes;

we can also incorporate nonholo constraints of the pusher;

and we plan a path using NHP.