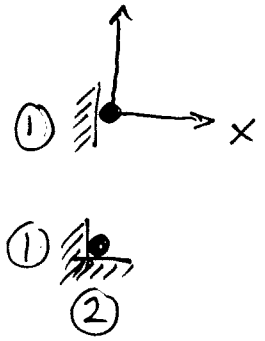
Grasping a Point in the Plane

Velocity
Kinematic Point of View



① $\dot{x} \geq 0$

② $\dot{y} \geq 0$

Can we prevent all motion with 2 contacts?

Bilateral – yes

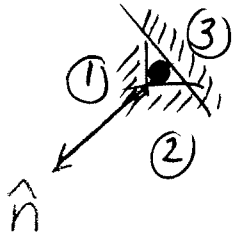
Unilateral – no

How can we fully constrain the point?

Face normal representation in twist space

$$\begin{bmatrix} \hat{n}_1^T \rightarrow & 1 & 0 \\ \hat{n}_2^T \rightarrow & 0 & 1 \\ \hat{n}_3^T \rightarrow & n_{3x} & n_{3y} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

N



$$\dot{x} \geq 0$$

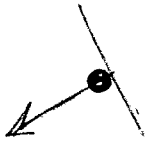
$$\dot{y} \geq 0$$

$$n_{3x}\dot{x} + n_{3y}\dot{y} \geq 0$$

How can we choose n_{3x}, n_{3y} so that the only solution is $\dot{x} = \dot{y} = 0$?

Make $n_{3x} < 0$ & $n_{3y} < 0$!

$\hat{n}_3$ must point into 3rd quadrant!

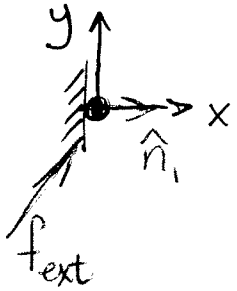


Can we draw the same conclusion from a force/~~stability~~ equilibrium analysis?

The force analysis. We want particle to have zero acceleration for all possible f_{ext}.

We want $\sum f_i = 0$, since then the point will not accelerate.

Assume no friction



$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_1 + f_{ext} = 0 \quad ; \quad \lambda_1 \geq 0$$

Is this satisfiable for arbitrary f_{ext}?

No! $f_{ext} = \begin{bmatrix} (\cdot) \\ \text{anything} \end{bmatrix}$

Add a second contact

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + f_{ext} = 0$$

$$\lambda_1, \lambda_2 \geq 0$$

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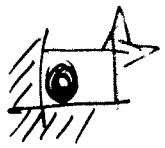
Add a third contact.

$$\begin{bmatrix} 1 & 0 & n_{3x} \\ 0 & 1 & n_{3y} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} + f_{ext} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

$$\lambda_1 + n_{3x}\lambda_3 + f_x = 0$$

$$\lambda_2 + n_{3y}\lambda_3 + f_y = 0$$

If $n_{3x} < 0$, then if $f_x < 0$, we are ok.

If $n_{3y} < 0$, then if $f_y < 0$, we are ok.

$\therefore$ $\boxed{n_{3x}, n_{3y} < 0}$

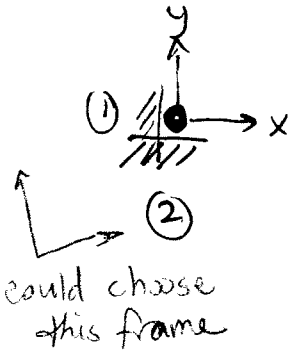
Relate to differential twists and wrenches.

Contact constraint is:

$$(C, C_0) * (\omega, v_0) \geq 0$$

$$(\hat{u}, p \times \hat{u}) * (\omega, p \times \omega + \frac{\|v\|}{\|\omega\|} \omega) \geq 0$$

Choice of frame is free, so ... →



$$\hat{u}^T v_0 + (p \times \hat{u})^T \omega \geq 0$$

$$\begin{bmatrix} \hat{u}^T & (p \times \hat{u})^T \end{bmatrix} \begin{bmatrix} v_0 \\ \omega \end{bmatrix} \geq 0$$

Note:

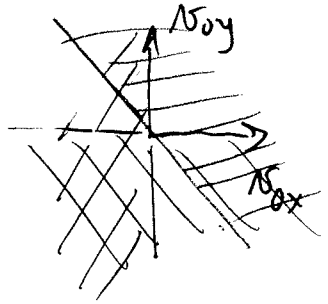
$p_i = 0$

$i = 1, 2, 3, \ldots$

W_n^T

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n_{3x} & n_{3y} & 0 \end{bmatrix} \begin{bmatrix} v_0 \\ \omega \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ n_{3x} & n_{3y} \end{bmatrix} \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

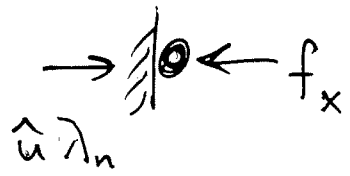


Dual Analysis in Force Space ⑥

What f_{ext} can be balanced?

1 contact: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_n = \begin{bmatrix} -f_x \\ -f_y \end{bmatrix}$

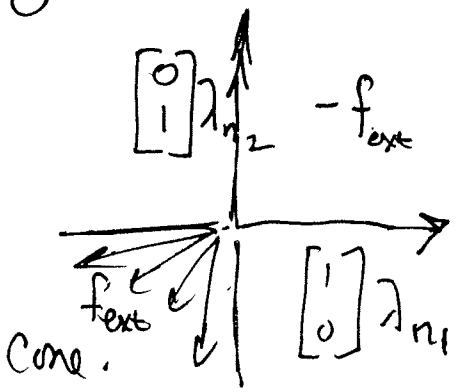
Can balance only forces opposing contact!



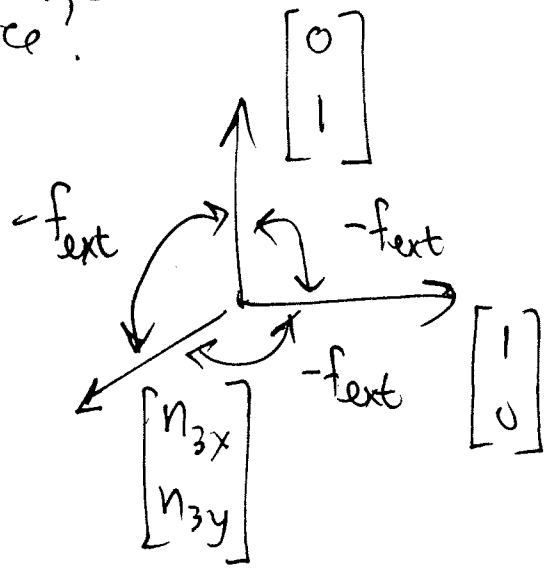
2 contacts: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -f_x \\ -f_y \end{bmatrix}$

$\lambda_1, \lambda_2 \geq 0$

Can balance everything in cone



How can we balance every possible external force?



Geometrical Test in Wrench Space

Interior of Convex hull of unit wrenches (contact screws) ~~strictly~~ contains the origin. ~~in its~~