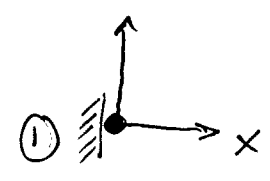


3/4/04
①

Grasping a Point in the Plane

Velocity
Kinematic Point of View

① $\dot{x} \geq 0$



② $\dot{y} \geq 0$



Can we prevent all motion
with 2 contacts?

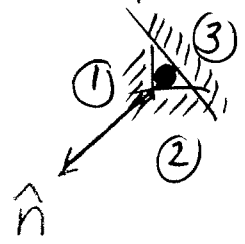
Bilateral - yes

Unilateral - no

How can we fully constrain the point?

Face normal representation in twist space

$$\underbrace{\begin{matrix} \hat{n}_1^T \rightarrow \\ \hat{n}_2^T \rightarrow \\ \hat{n}_3^T \rightarrow \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ n_{3x} & n_{3y} \end{bmatrix}}_N \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



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(2)

$$\dot{x} \geq 0$$

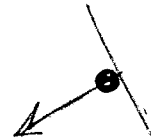
$$\dot{y} \geq 0$$

$$n_{3x}\dot{x} + n_{3y}\dot{y} \geq 0$$

How can we choose n_{3x}, n_{3y} so that
the only solution is $\dot{x} = \dot{y} = 0$?

Make $n_{3x} < 0$ & $n_{3y} < 0$!

\hat{n}_3 must point into
3rd quadrant!

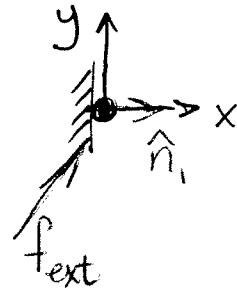


Can we draw the same conclusion from
a force/~~stability~~ equilibrium analysis?

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The force analysis. We want particle to $\textcircled{3}$
 have zero acceleration for all possible f_{ext} .
 We want $\sum f_i = 0$, since then
 the point will not accelerate.

Assume no friction



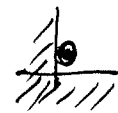
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_1 + f_{\text{ext}} = 0 ; \lambda_1 \geq 0$$

Is this satisfiable for arbitrary f_{ext} ?

No! $f_{\text{ext}} = \begin{bmatrix} (\cdot) \\ \text{anything} \end{bmatrix}$

Add a second contact

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + f_{\text{ext}} = 0$$



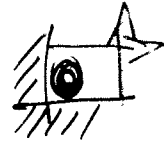
$$\lambda_1, \lambda_2 \geq 0$$

Add a third contact.

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(4)

$$\begin{bmatrix} 1 & 0 & n_{3x} \\ 0 & 1 & n_{3y} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} + f_{\text{ext}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

$$\lambda_1 + n_{3x} \lambda_3 = f_x = 0$$

$$\lambda_2 + n_{3y} \lambda_3 = f_y = 0$$

If $n_{3x} < 0$, then if $f_x < 0$, we are ok.

If $n_{3y} < 0$, then if $f_y < 0$, we are ok.

$$\therefore \boxed{n_{3x}, n_{3y} < 0}$$



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Relate to differential twists
and wrenches.

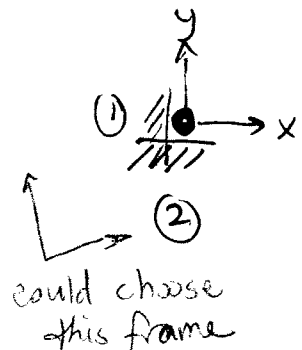
(5)

Contact constraint is:

$$(c, c_0) * (\omega, \nu_0) \geq 0$$

$$(\hat{u}, p \times \hat{u}) * (\omega, \cancel{p \times \omega} + \frac{\|N\|}{\|\omega\|} \omega) \geq 0$$

Choice of frame
is free, so ... →



$$\hat{u}^T \nu_0 + (p \times \hat{u})^T \omega \geq 0$$

$$\begin{bmatrix} \hat{u}^T & (p \times \hat{u})^T \end{bmatrix} \begin{bmatrix} \nu_0 \\ \omega \end{bmatrix} \geq 0$$

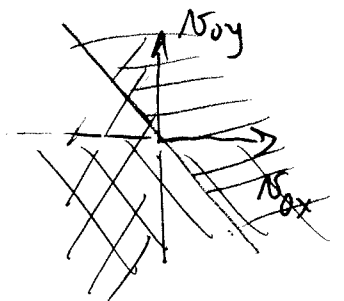
$$\overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n_{3x} & n_{3y} & 0 \end{bmatrix}}^{W_n^T} \begin{bmatrix} \nu_0 \\ \omega \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ n_{3x} & n_{3y} \end{bmatrix} \begin{bmatrix} \nu_{0x} \\ \nu_{0y} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Note:

$$p_i = 0$$

$$i = 1, 2, 3, \dots$$



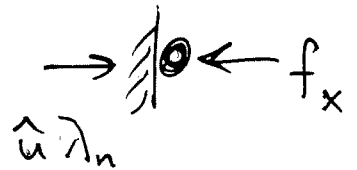
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Dual Analysis in Force Space (6)

What f_{ext} can be balanced? \uparrow
 \hat{n}

1 contact: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_n = \begin{bmatrix} -f_x \\ -f_y \end{bmatrix}$

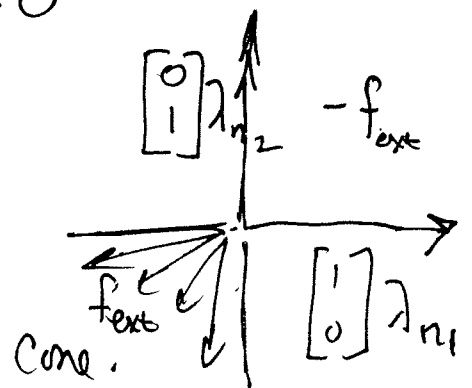
Can balance only forces
opposing contact!



2 contacts: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -f_x \\ -f_y \end{bmatrix}$

$\lambda_1, \lambda_2 \geq 0$

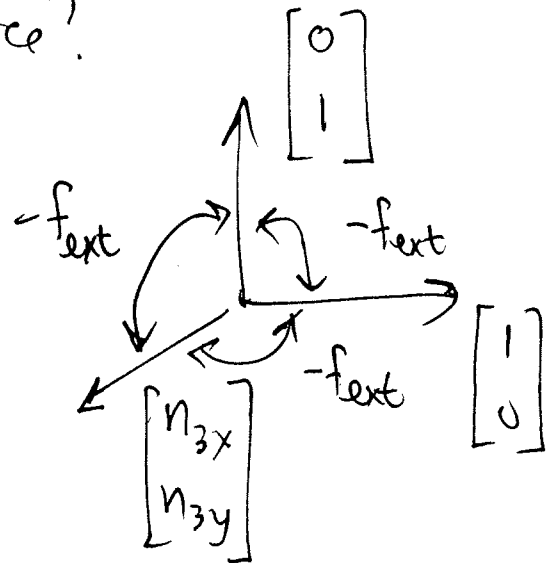
Can balance
everything in cone



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(7)

How can we balance every possible external force?



Geometric Test in Wrench Space

Interior of
Convex hull
of unit

wrenches

(contact screws) ~~strictly~~ contains

the origin. ~~in its~~



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