11/20/06

0.1

Outline

Frictional Form Closure

Utility of Coulomb Friction

Example where Coulomb is useful - pipe clamp

Example " " " Less useful - Painlevé's Paradox

Next time

Planar Sliding

Limit Surfaces

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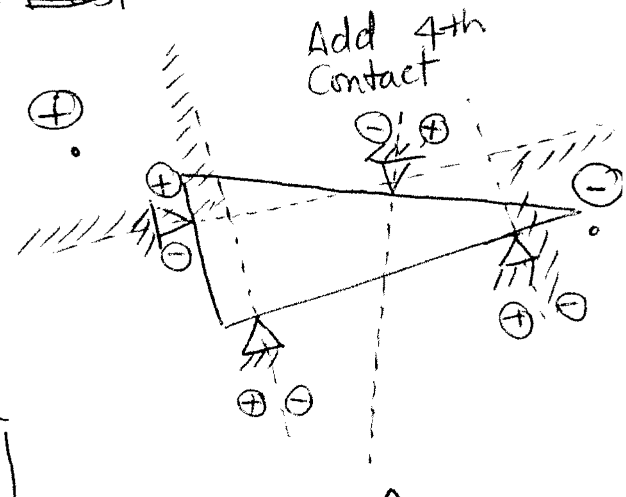
Recall Definitions of Form Closure ~~First~~

(First-Order) $\boxed{W_n^T \nu \geq 0 \Rightarrow \nu = 0}$

$$\Updownarrow$$

$$\boxed{\begin{array}{l} W_n \lambda_n = -g_{ext} \quad \forall\ g_{ext} \in \mathbb{R}^6 \\ \lambda_n \geq 0 \end{array}}$$



4th contact wipes out consistent regions.

W_n is $(6 \times n_c)$

n_c must be greater than:

4 (planar case)

7 (spatial case)

Extend to include friction.

Velocity analysis will not work now.

It can't capture friction force effects

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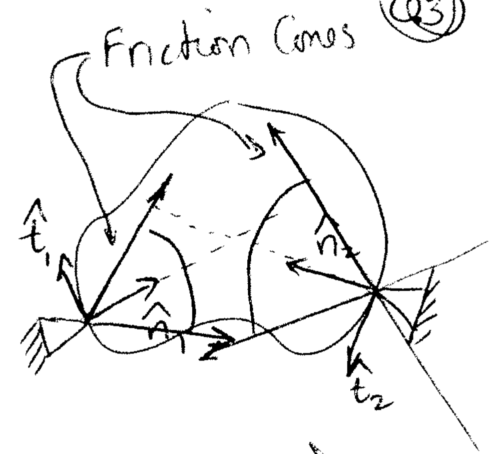
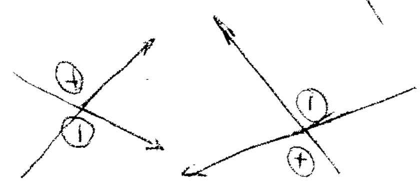
Planar Case

Use Edges of Friction Cones in Definition



$$\left.\begin{aligned} W_n\lambda_n + W_t\lambda_t &= -g_{ext} \\ \lambda_n &\geq 0 \\ -\mu_i\lambda_{in} \leq \lambda_{it} &\leq \mu_i\lambda_{in} \quad \forall\, i \end{aligned}\right\} \forall\, g_{ext}$$



ML yield no consistent regions

Consider one contact

Let $\lambda_{it} = -\mu\lambda_{in} \Rightarrow W_{in}\lambda_{in} + W_{it}\lambda_{it} = (W_{in} - \mu W_{it})\lambda_{in}$

$\lambda_{it} = \mu_i\lambda_{in} \Rightarrow$ " " $= (W_{in} + \mu W_{it})\lambda_{in}$

$\therefore$ Frictional Form Closure is given by:

$$\boxed{\begin{gathered} \begin{bmatrix} W_n + uW_t & W_n - uW_t \end{bmatrix}\begin{bmatrix} \lambda_n \\ \lambda_n \end{bmatrix} = -g_{ext} \quad \forall\, g_{ext} \in \mathbb{R}^3 \\ \lambda_n \geq 0 \end{gathered}}$$

Extension to Spatial Case

(over)

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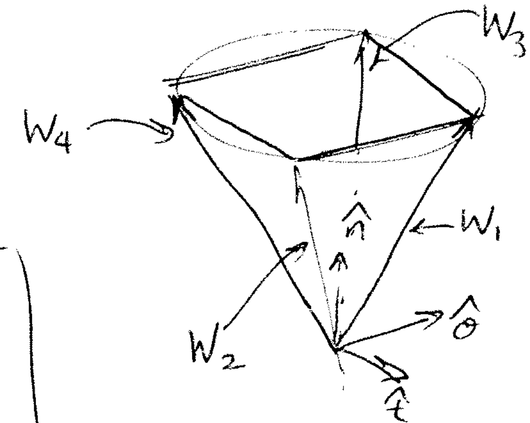
In spatial case, an ∞ # of edges are needed to represent the quadratic friction cone.

Example:

Use four edges



$$\boxed{\begin{aligned} &W\alpha = -g_{ext} \quad \forall\ g_{ext} \in \mathbb{R}^6 \\ &\alpha \geq 0 \end{aligned}}$$

where $W = [W_1\ W_2\ W_3\ W_4]$, $\alpha \in \mathbb{R}^{4n_c}$

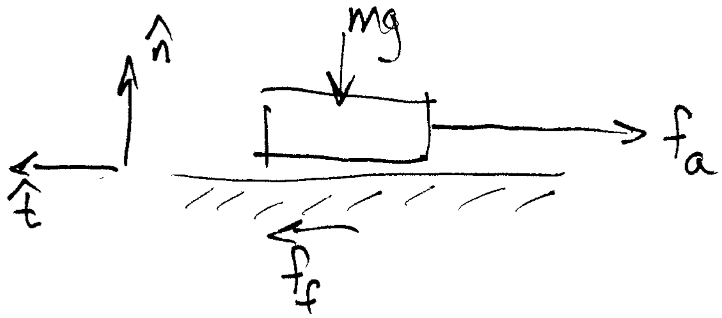
$$W_1 = W_n + \mu(W_t + W_o)$$

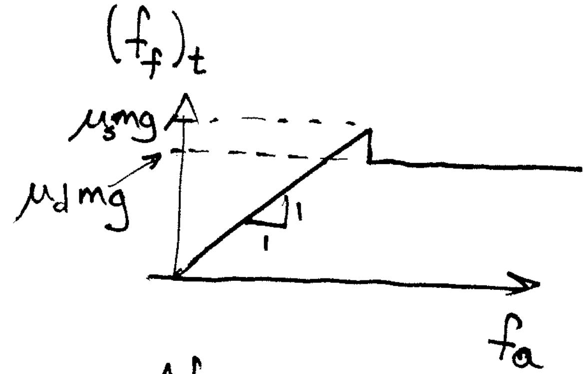
$$W_2 = W_n + \mu(W_t - W_o)$$

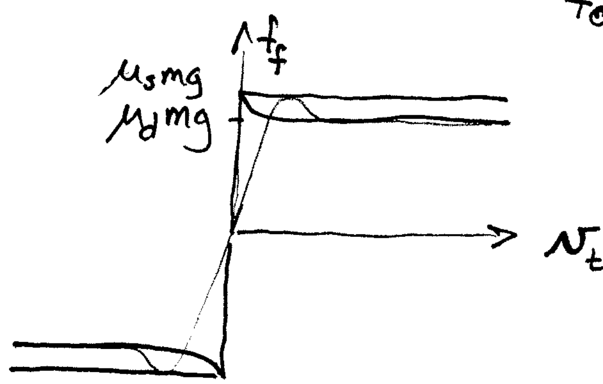
$$W_3 = W_n + \mu(-W_t + W_o)$$

$$W_4 = W_n + \mu(-W_t - W_o)$$

What else should be said about friction?







<u>Other models:</u>

<u>Asperite plastic deformation</u>

Algebraic/Dynamic

In essence, nearly everything but Coulomb friction is too hard/complex to plan with.

Unless doing RRT!

Utility of Coulomb Friction

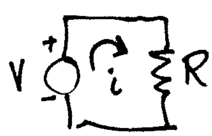
Dynamic Model Does Not Always have Unique Solution

Does that mean its not useful?

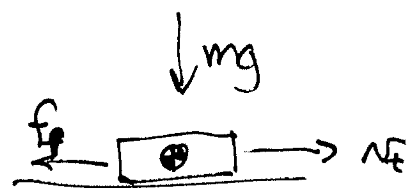
No, but it's harder to use.

Other models are wrong in structure and parameters are uncertain. But they can still be used. H^∞ control

$V = IR$ – this is ideal, but still useful.



~~Book on table~~ Puck Sliding



Don't know location of book cg exactly or angle of table, but doesn't stop us from figuring out when it will stop.

Matt says Mark's Handbook stinks

	μ
metal on metal	0.15 – 0.6
rubber on concrete	0.6 – 0.9
plastic wrap on lettuce	∞
Leonardo's number	0.25 ← not far off

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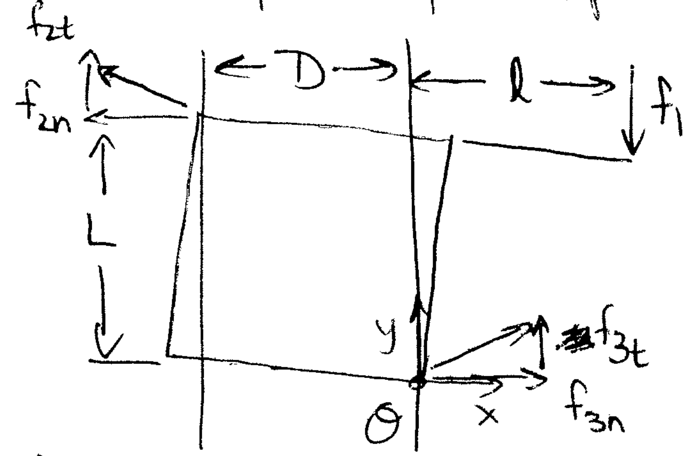
Example of Why Coulomb Frict Model is Useful

Pipe Clamp Example



Equilibrium Possible?

horizontal equilib.

$$f_{3n} - f_{2n} = 0$$

vertical equilib.

$$f_{2t} + f_{3t} - f_1 = 0$$

moment equilib.

$$L f_{2n} - D f_{2t} - \ell f_1 = 0$$

$f_{2n}, f_{3n} \geq 0$

$$\boxed{\begin{aligned} 0 \leq f_{2t} \leq \mu_2 f_{2n} \\ 0 \leq f_{3t} \leq \mu_3 f_{3n} \end{aligned}}$$

Intuitive only

$$\underbrace{\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ -\ell & L & 0 \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -D & 0 \end{bmatrix}}_{A_2} \begin{bmatrix} f_1 \\ f_{2n} \\ f_{3n} \\ f_{2t} \\ f_{3t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} f_1 \\ f_{2n} \\ f_{3n} \end{bmatrix} = -A_1^{-1} A_2 \begin{bmatrix} f_{2t} \\ f_{3t} \end{bmatrix} = \underset{(3\times2)}{\begin{bmatrix} 1 & 1 \\ \frac{\ell + D}{L} & \frac{\ell}{L} \\ \frac{\ell + D}{L} & \frac{\ell}{L} \end{bmatrix}} \begin{bmatrix} f_{2t} \\ f_{3t} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_{2n} \\ f_{3n} \end{bmatrix} \geq 0$$

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2.2

Coulomb Friction Constraints

$$-\mu_i f_{in} \le f_{it} \le \mu_i f_{in}$$

$$\underbrace{\begin{bmatrix} 0 & \mu_2 & 0 & -1 & 0 \\ 0 & 0 & \mu_3 & 0 & -1 \\ 0 & \mu_2 & 0 & 1 & 0 \\ 0 & 0 & \mu_3 & 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} f_1 \\ f_{2n} \\ f_{3n} \\ f_{2t} \\ f_{3t} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Use LP sensitivity to determine l

$$\boxed{\begin{aligned} A(l)\,x &= 0 \\ B(l)\,x &\geq 0 \end{aligned}}$$

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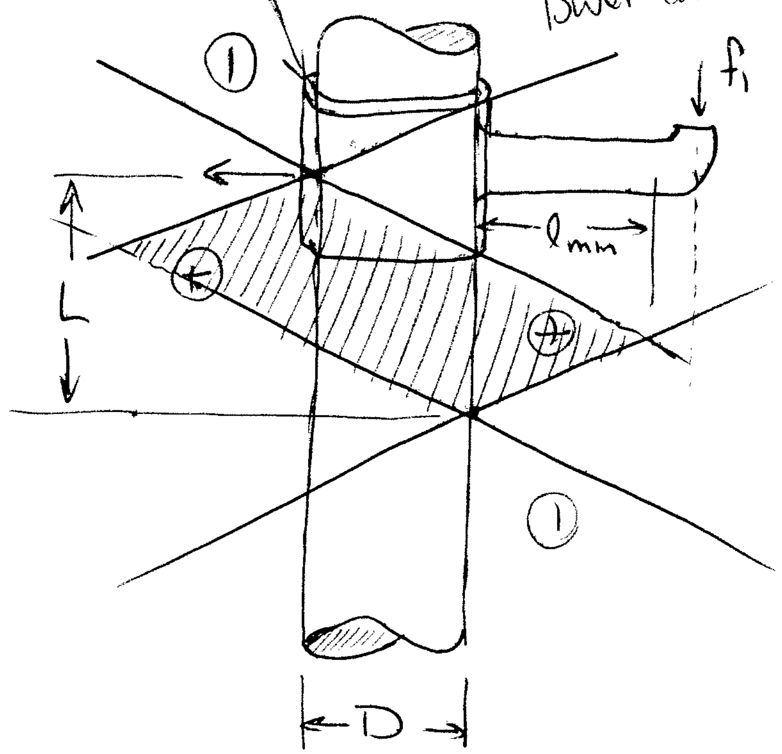
What does moment labeling tell us?

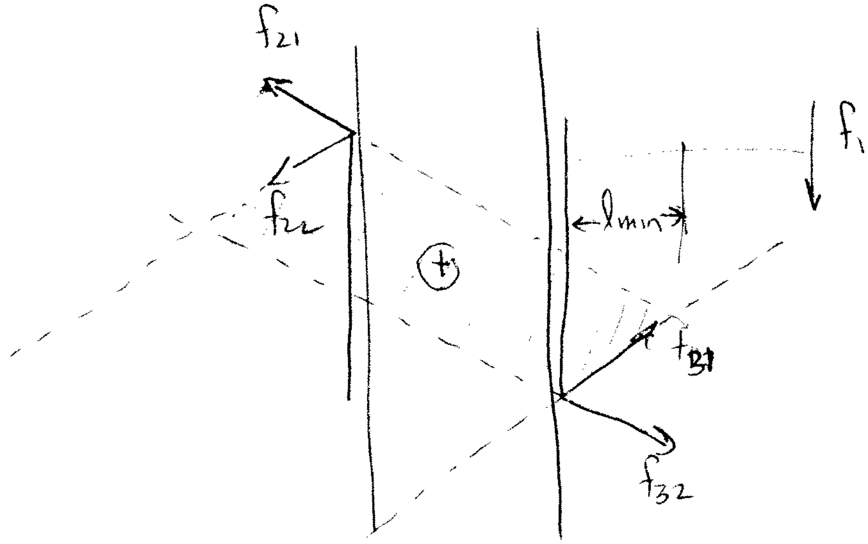
2-3

Picture wrong. Sleeve needs to be lower and longer



f_1 must lie outside the intersection of the two cones.

Larger D, smaller L allows smaller l



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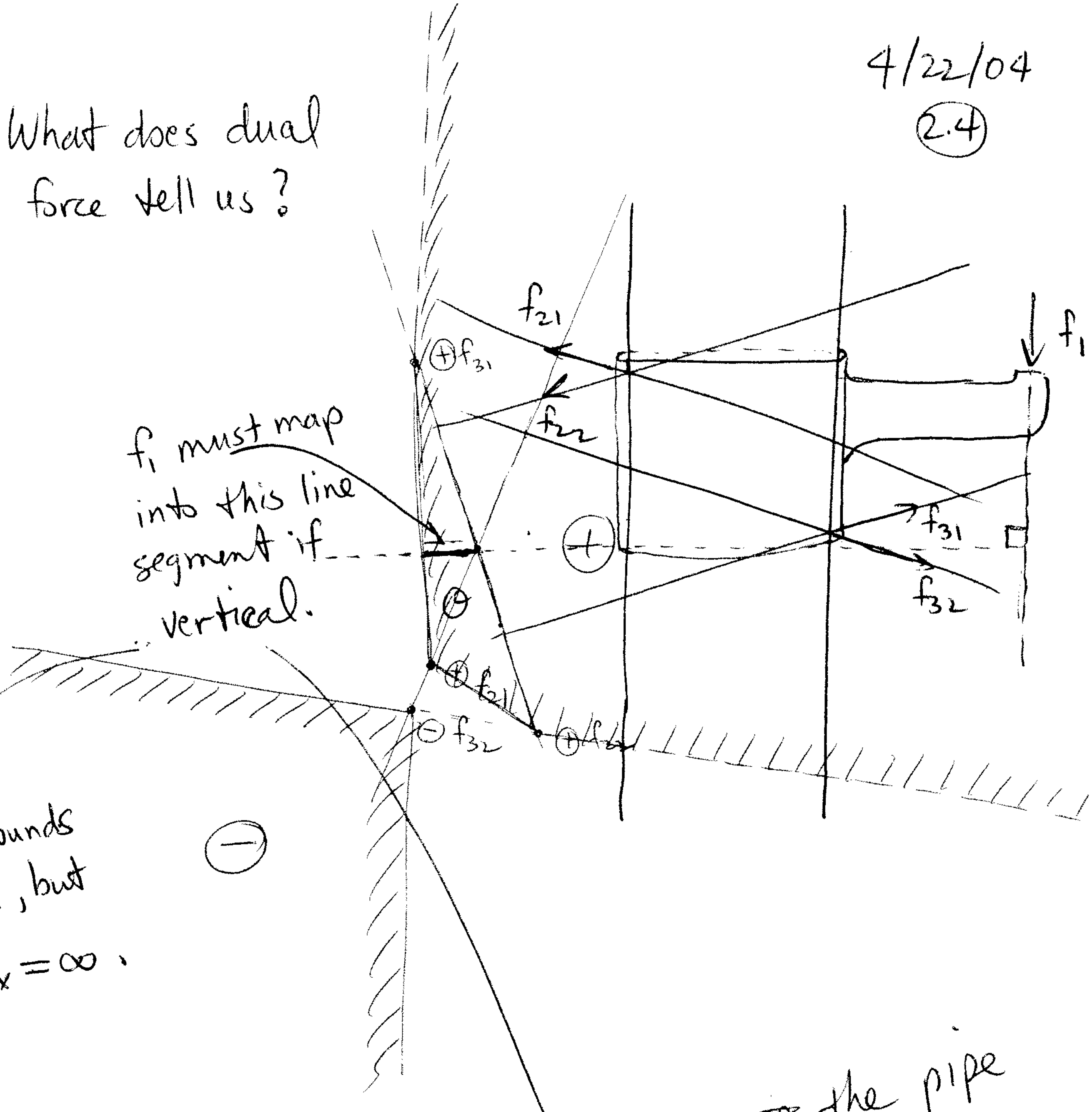
What does dual force tell us?



f_1 must map into this line segment if vertical.

This bounds l_{min}, but $l_{max} = \infty$.

Because ~~we~~ the pipe ~~want a ⊖ pt in~~ ~~a ⊕ region since~~ ~~this gives~~ clamp can resist the negative of what the constraint forces can generate.

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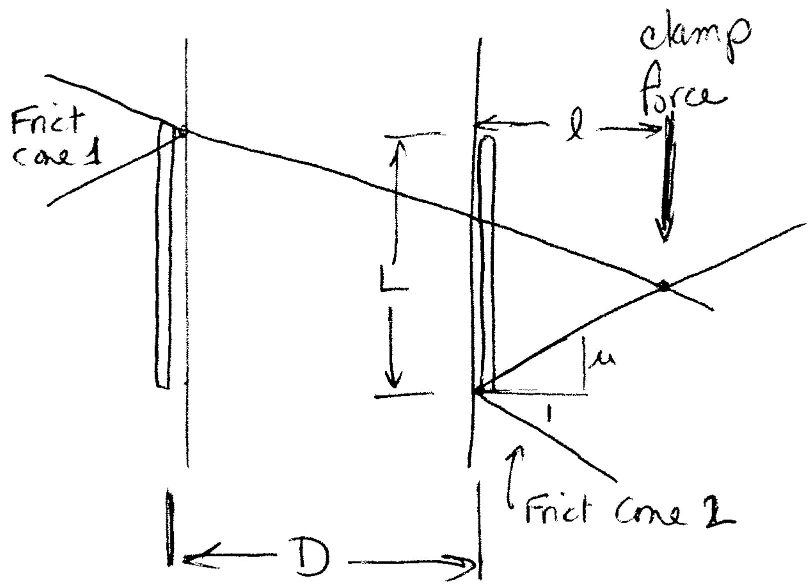
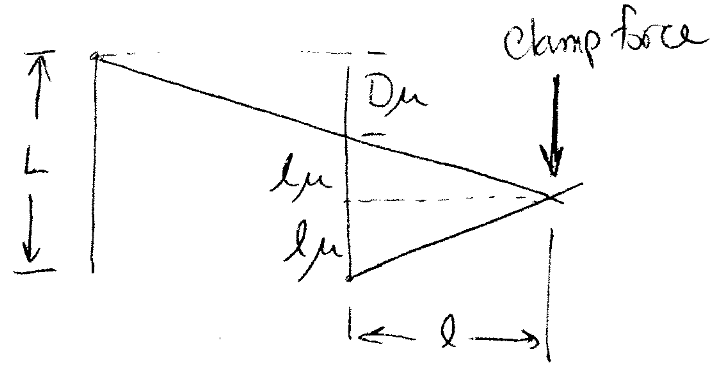
Mason's Analysis in Text (pp. 125-126)



Must have equilibrium.



limiting case

$$L = D\mu + 2l\mu = (D+2l)\mu$$

$$\left(\frac{L}{\mu} - \frac{D\mu}{\mu}\right)\frac{1}{2} = l = \frac{L - D\mu}{2\mu}$$

For safety, l should be greater.

$$l \geq \frac{L - D\mu}{2\mu} = \frac{L}{2\mu} - \frac{D}{2}$$

if μ is uncertain use smallest value!

Pros and Cons for Design

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Where should clamp pad be located to be least sensitive to direction of f_1?

What about sensitivity to other parameters; L, D, ℓ, μ?

Method

LP — Computation solution ~~of~~ LP requires setting direction of f_1

Could find a soln w/ f_1 verticle, then use sensitivity analysis on dir of f_1 & on L, D, μ, ℓ.

Probably requires computation of several LP's & several sensitivity analyses.

ML — Very easy to see how all parameters interact

DF — Difficult to see parameter interactions

Mason's — Fairly easy to see trade-off, but less informative than ML.

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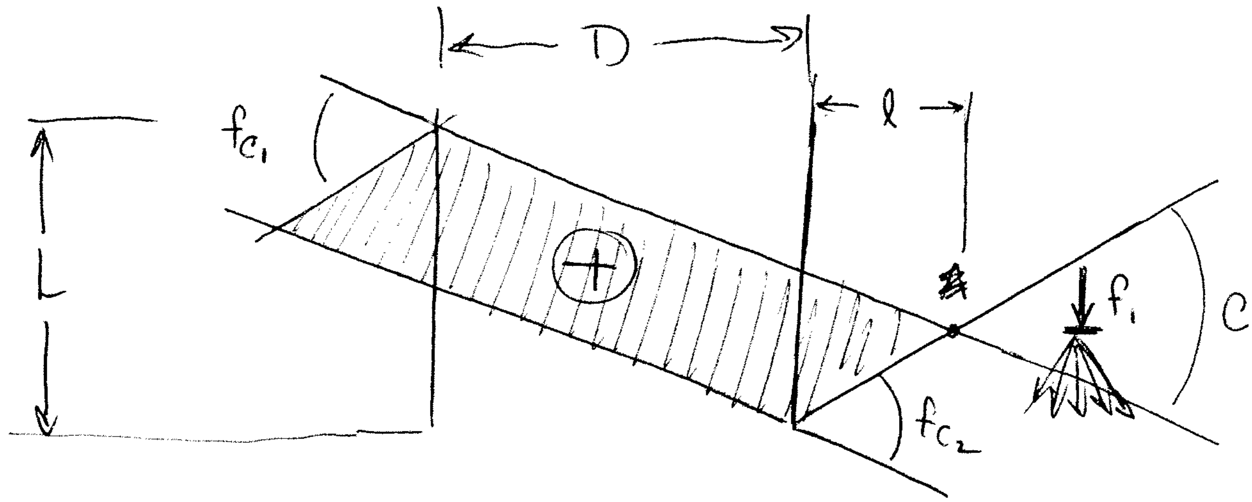
Design w/ ML



as $\mu \downarrow$ $l \uparrow$

as $D \downarrow$ $l \uparrow$

as $L \uparrow$ $l \uparrow$

Assume contact force with clamp pad has no moment (at the contact pt.)

Put clamp pad in cone C to reduce chance that a tangential force component destabilizes clamp.