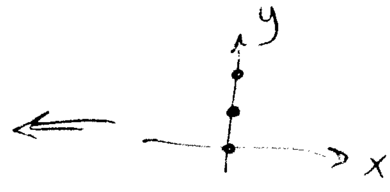
Definition - Minkowski Sum (or Addition)

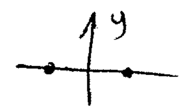
Let A & B be two sets.

$$A + B = \{ a + b \mid a \in A,\ b \in B \}$$

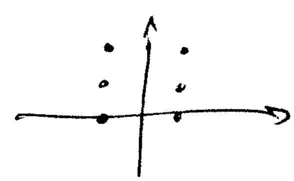
$A = \{(0,0), (0,1), (0,2)\}$ ⟵



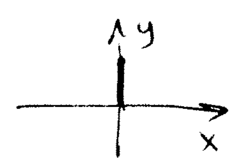
$B = \{(1,0), (-1,0)\}$



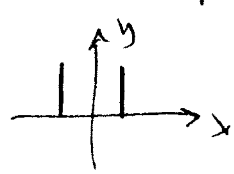
$A+B = \{(1,0), (-1,0), (1,1), (-1,1), (1,2), (-1,2)\}$



Suppose $A = \{ a \mid a_x = 0,\ 0 \le a_y \le 2 \}$



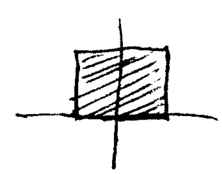
$A + B =$



Suppose $B = \{ b \mid -1 \le b_x \le 1,\ b_y = 0 \}$

$$A + B = \{ a + b \mid -1 \le a_x + b_x \le 1,\ 0 \le b_y + a_y \le 2 \}$$

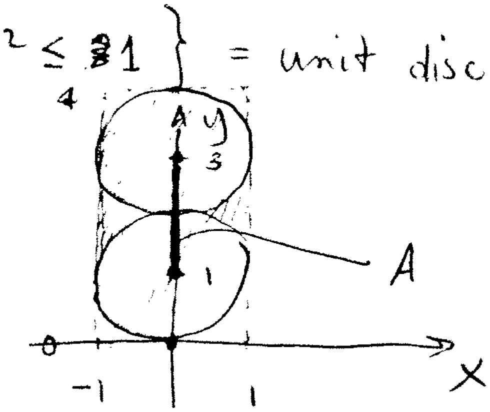
= square.



Suppose $A = \{ (x,y) \mid x = 0, \; 1 \le y \le 3 \}$

$B = \{ (u,v) \mid u^2 + v^2 \le 1 \}$ = unit disc

Minkowski Sum $A \oplus B$

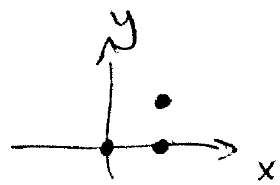


Minkowski Difference of A ~~−~~ B $\triangleq$ $A \ominus B$

$=$ $\qquad\qquad = A \oplus (-B)$

$A = \{(0,0), (1,1), (1,0)\}$

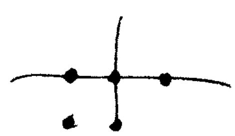
$B = \{(0,0), (1,0)\}$



$A \ominus B \Rightarrow$

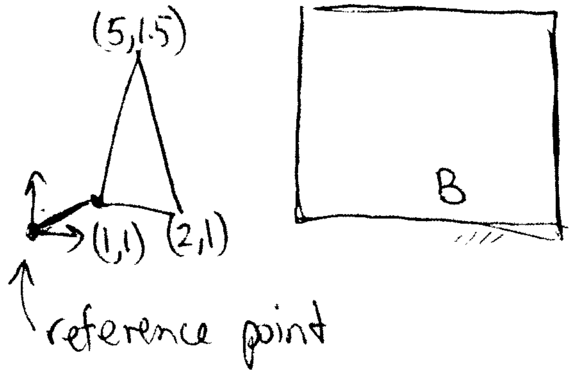
$\neq$

$B \ominus A \Rightarrow$



Not commutative in general.

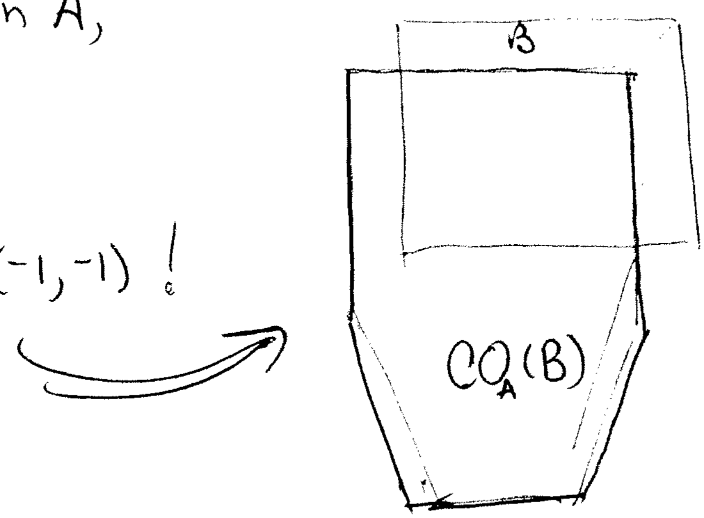
What about change of CO due to ref. pt. choice?



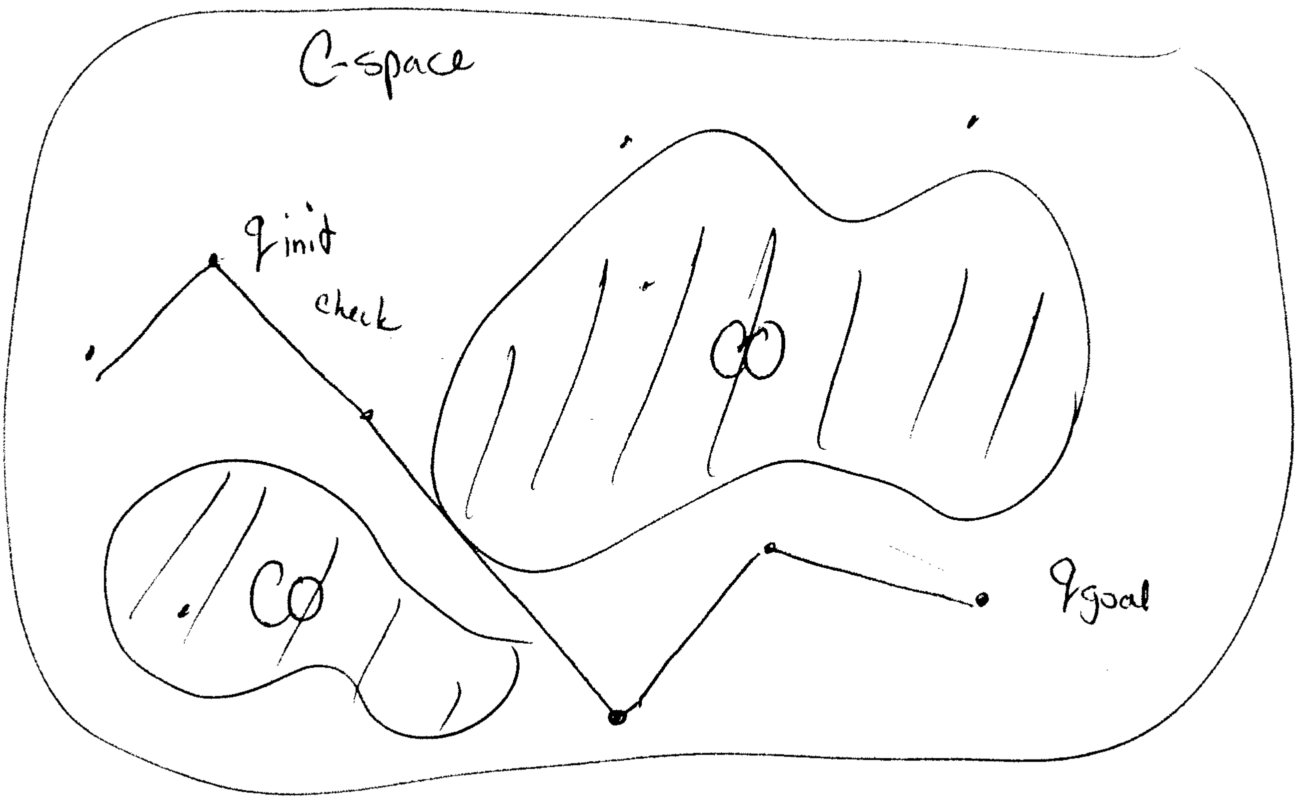
If we add (1,1) to every point in A,

then

b−a shifts (−1,−1)!



PRM - Probabilistic Road Map Approach



Plaster C-space w/ points. Hope to construct C-space structure.

attempt ~~to~~ connect w/ "local planner". Chose points "close enough."

Use C-space hints to refine sampling

Not complete?

How many points to sample?

Exponential complexity in dimension to cover C-space well & return C-space structure

Metric Space – a set for which a notion of distance is defined between set elements!

10/2/06 (15.2)

Possible potential functions – See Koditschek's paper from 90's

$C_1 \|q - q_{goal}\|^2 \Leftarrow$ quadratic surface with q_{goal} the lowest point.

$C_1 \in \mathbb{R}^+$

Let $d_i(q)$ = distance of ~~object~~ robot to obstacle i

$\frac{C_{2i}}{d_i(q)} \Leftarrow$ hyperbolic function that grows as distance

$C_{2i} \in \mathbb{R}^+$

Potential Function, F

$$F(q) = C_1 \|q - q_{goal}\|^2 + \sum_{i=1}^{N_{obst}} \frac{C_{2i}}{d_i(q)}$$

Ideally F(q) has a unique global minimum.

Then just follow the gradient

Barraquand & Latombe
Best First Search

2/26/04

(18.1)

Main data structure as a priority queue.

A priority queue is a container for which you can access only the highest priority item

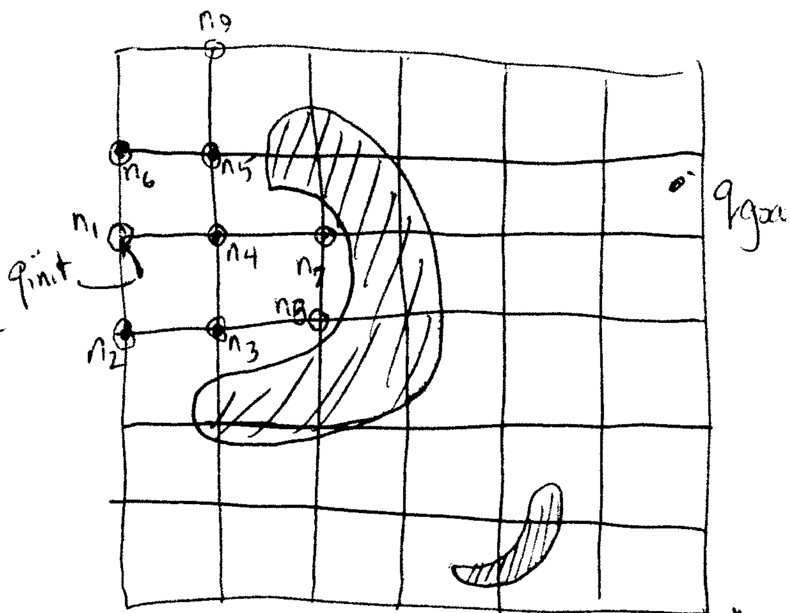
Need an objective fcn the defines "best"

e.g. Potential field plus distance.

HOW DO YOU CHOOSE GRID SIZE?

Depends on whether you use collision check or swept volumes.

C space



must be removed ↓

Iterations ~~actions~~	insertions queue	best	visited
1	n_1	n_1	n_1
2	n_2, n_4, n_6	n_4 remove n_1	all nodes
3	n_3, n_5, n_7	n_7 remove n_4	added one
4	n_8	n_8 remove n_7	visited according to alg.
5	none	n_5 remove n_8	
6	n_9	n_9 remove n_5	
⋮	⋮	⋮	⋮

2/23/04

18.2

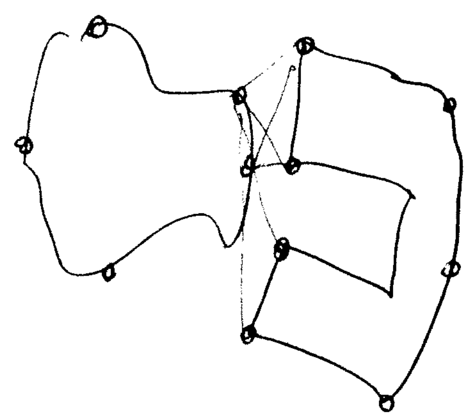
Feature of BFP.

- No need to compute C-space obstacle, which is exponential in dimension of C-space.
- Just need to do collision check when visiting a node.

Distance computation

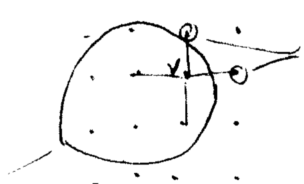
- You get this as a by-product of collision checking
- Could also use

  total distance between pairs of points



Behavior:

- If lucky, alg walks down slope to goal
- If unlucky, alg reaches potential well and visits many points in the well before escaping



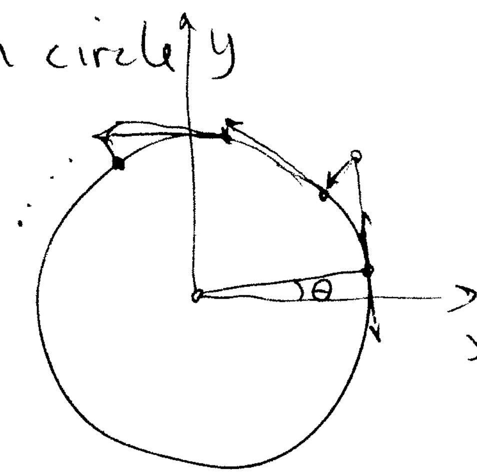
List of visited nodes becomes very large.

Applicability

- Holonomic systems?
  - Yes, in principle, but ~~must~~ should be able to eliminate constrained variables, i.e. Need a lowest dimensional representation of C-space.

Suppose we have a ~~system~~ point robot constrained to lie on a circle

$$x^2+y^2-r^2=0$$

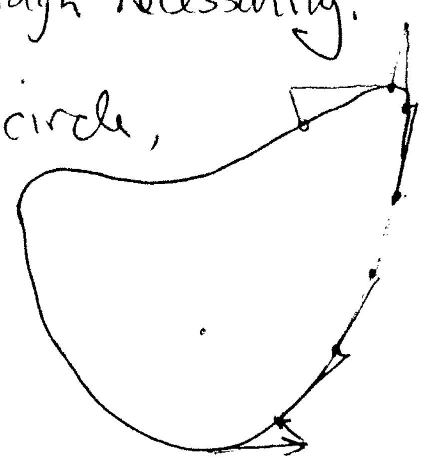


We need to grid on the variable θ, not x & y.

Tangent space is not enough necessarily.

It could work for the circle,

Could lead to non-uniform coverage of C-space.

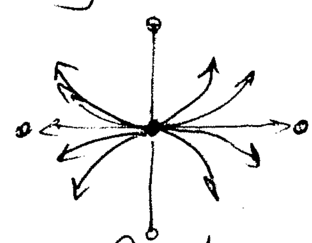


Want points not "too close". Use geodesics in n-dimensional space

What about nonholonomic systems?

~~Mason says "no!"~~

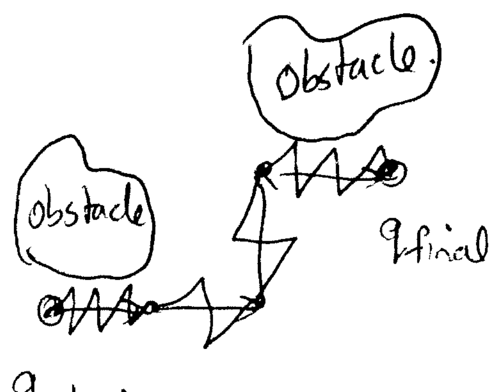
Why? Can't get to arbitrary nearest neighbor easily"



Car type robots

We **can**!

If constraints are Pfaffian, then we can plan a "free-flying" path and then do Lie bracket maneuvers to reach various sub goals along the way



How could we modify the alg to produce ~~motions~~ plans with fewer Lie bracket motions.

Integrate system forward over time δt with input a.

node $= n =$ int$(q, a, \delta t)$

Must discretize the space of actions.

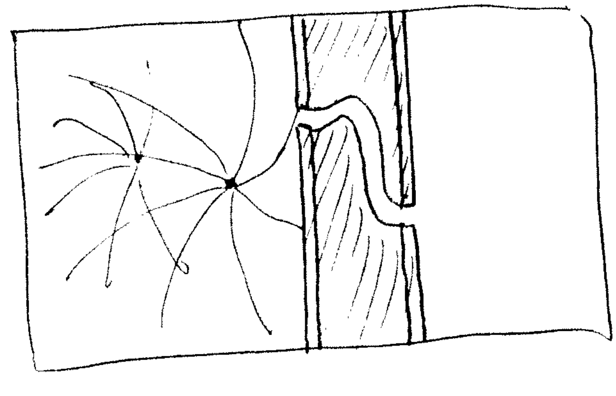
What's a suitable δt?

Running time is exponential in # of actions

Need function to determine closest node to a config, q.

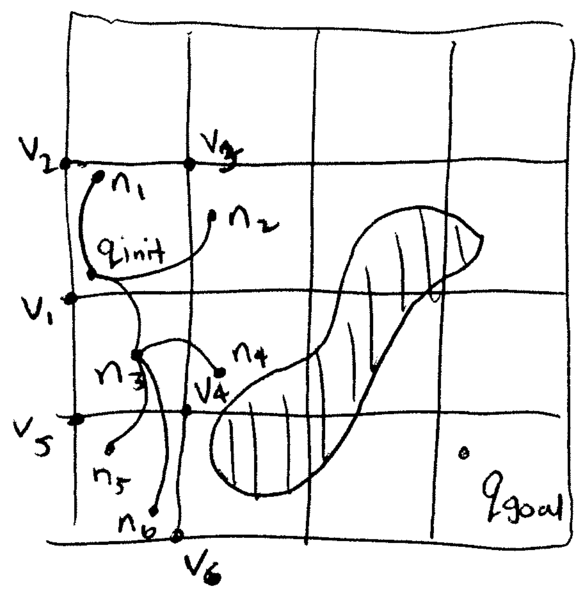
What is the cost function for "best" node?

How do we choose discretization to ensure coverage of reachable set.



Nonholonomic Planner, NHP

	open	best	visited
	q_{init}	q_{init}	v_1
actions	n_1, n_2, n_3	n_3	v_2, v_3, v_4
actions	n_5, n_6	n_6	

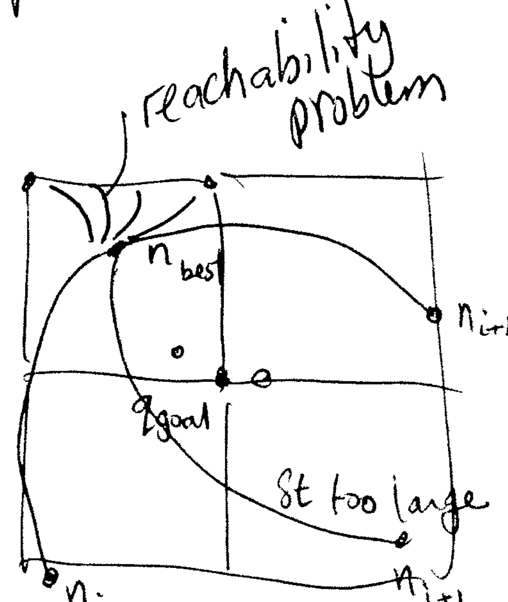


PROBLEMS !?

Planner is at best Resolution Complete.

Can get stuck.

What if $a, \delta t, n$ have been chosen such that you can't make progress toward goal?



What if δt is too small for some portion of grid.

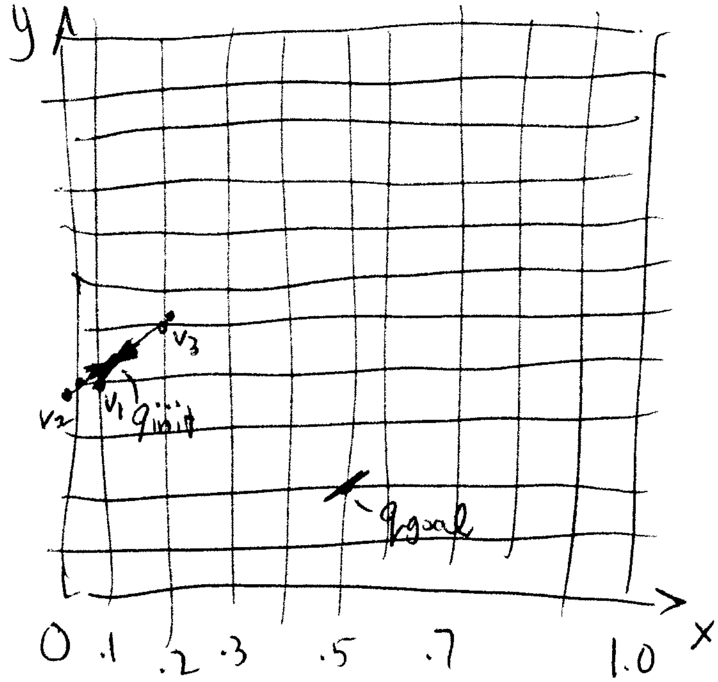
Planner requires space & time exponential in { dimension of C space, # of actions

10/02/06

(23.1)

4 actions.

open	best	visited
q_{init}	q_{init}	v_1
4 actions n_1, n_2, n_3, n_4		v_2, v_3, v_4, v_5



Note that v_4 & v_5 are in same (x, y) position as v_1, but above & below on x, y, θ grid