Chapter 5 - ~~Forces~~ Rigid Body Statics

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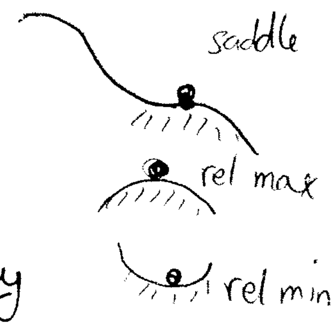
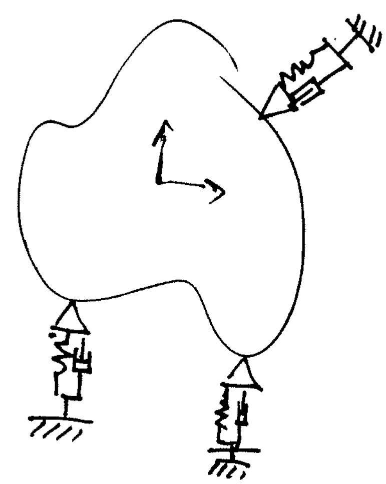
Sum of Ext. forces & Moments ~~is~~ zero	$\sum$ Forces $= \sum$ Moments $= 0$

We care about statics, since a stable grasp must satisfy equilibrium.



Def: A system is stable if when perturbed from an equilibrium config, it eventually returns to the config.



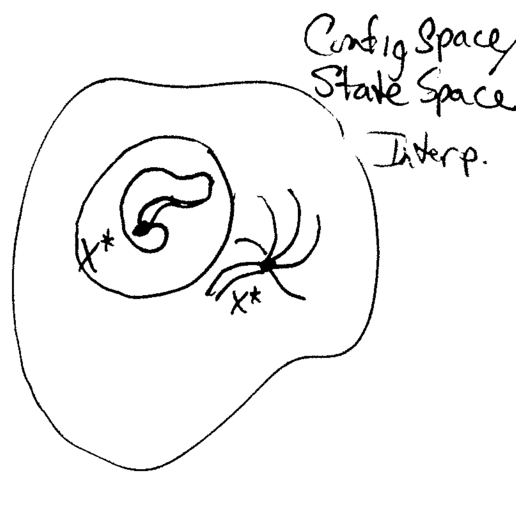
Example: car suspension

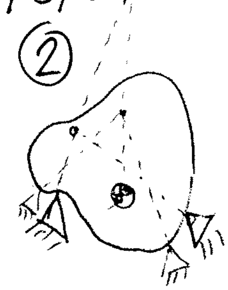
grasp w/ compliant fingers

Big Picture

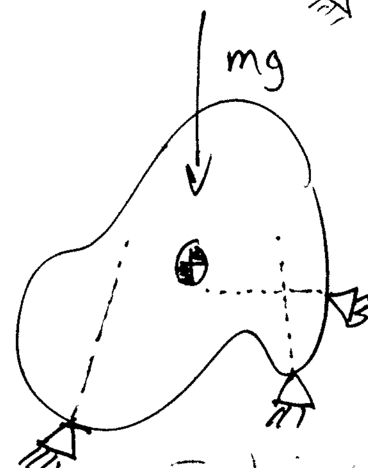
~~what about a rigid~~

We want algorithmic, manual, and analytic methods for reasoning about forces and contacts and predicting motions.





What about a simpler analysis of grasping? Everything rigid?



Is this stable?

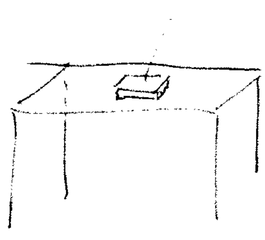
- $\mu = 0$ ⇐ marginally stable

  counting on unmodeled physical effects to damp motion

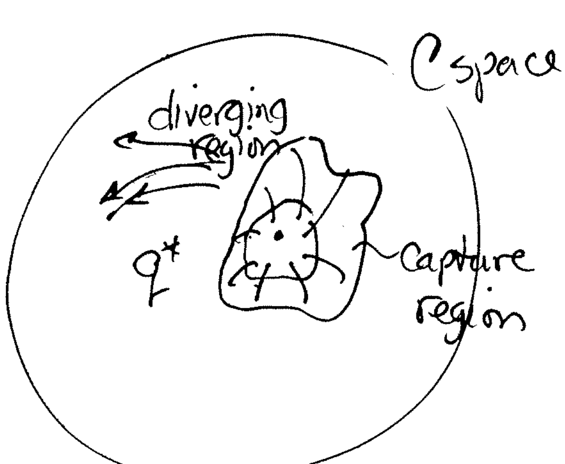
Explain w/ 2 contacts, then 3. Skip to page 2.1

- $\mu > 0$ ⇐ not stable in classical sense, since system will not return from all perturbations, but still operationally ok for many tasks.

Book on table



If not level, perturbations do not recover. Eventually book falls off.



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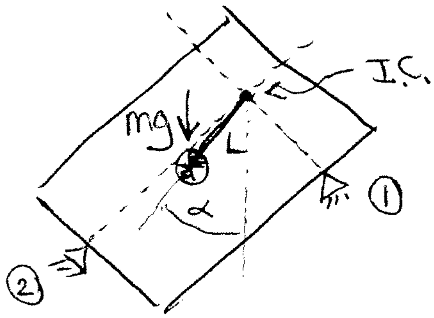
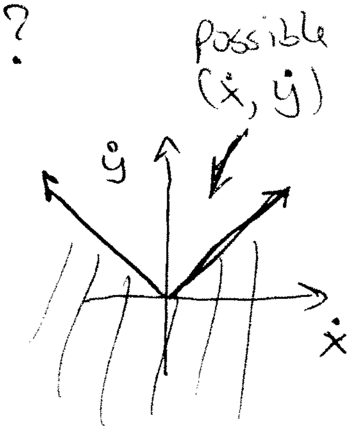
2.1



Is this rectangle in equilibrium with $\mu = 0$?

Contact constraints

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Note: $\dot{\theta}$ is arbitrary

From Physics

Equilibrium w/o friction occurs if c.g. has "locally" smallest y value!

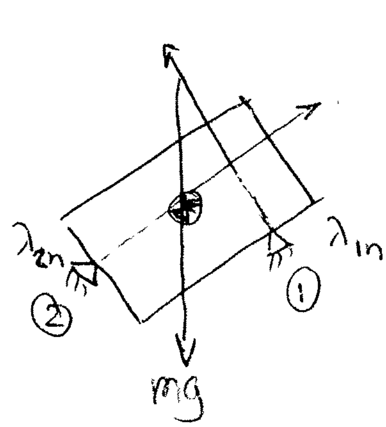
$$\Delta \text{ c.g. height} = -L\sin(\alpha)\,\dot{\theta}$$

$\therefore$ $\dot{\theta} > 0$ causes cg to move down!

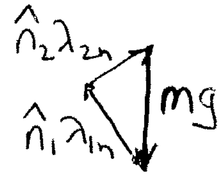
NOT in EQUILIBRIUM!

Geometric Interp

- Forces must intersect at a point, i.e. Σ Mom $= 0$ $\xrightarrow{\Sigma \text{ Moments} \neq 0}$ $\therefore$ not in equilib



- Forces must sum to zero $\xrightarrow{\Sigma \text{ Force} = 0}$ $\therefore$ could be equilib.



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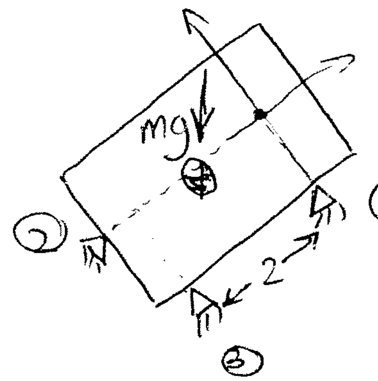
2.2

$$\underbrace{W_n^T}_{}\ \nu \geq \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}-1 & 1 & 0\\ 1 & 1 & 0\\ -1 & 1 & -2\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \geq \begin{bmatrix}0\\0\\0\end{bmatrix}$$



Note: $\dot{\theta}$ is still arbitrary,
but contacts must break.
for any motion, since $(W_n^T)^{-1}$ exists

If forcing equality, $\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = (W_n^T)^{-1}\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

Does there exist $\nu \ni \nu^T f \leq 0$?

where $f = \begin{bmatrix}0\\-1\\1/2\end{bmatrix} mg$

$$\begin{bmatrix}-1 & 1 & 0\\ 1 & 1 & 0\\ -1 & 1 & -2\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \geq \begin{bmatrix}0\\0\\0\end{bmatrix}$$

AND $\begin{bmatrix}0 & -1 & 1/2\end{bmatrix} \leq \begin{bmatrix}0\end{bmatrix}$

This is a Linear Program!

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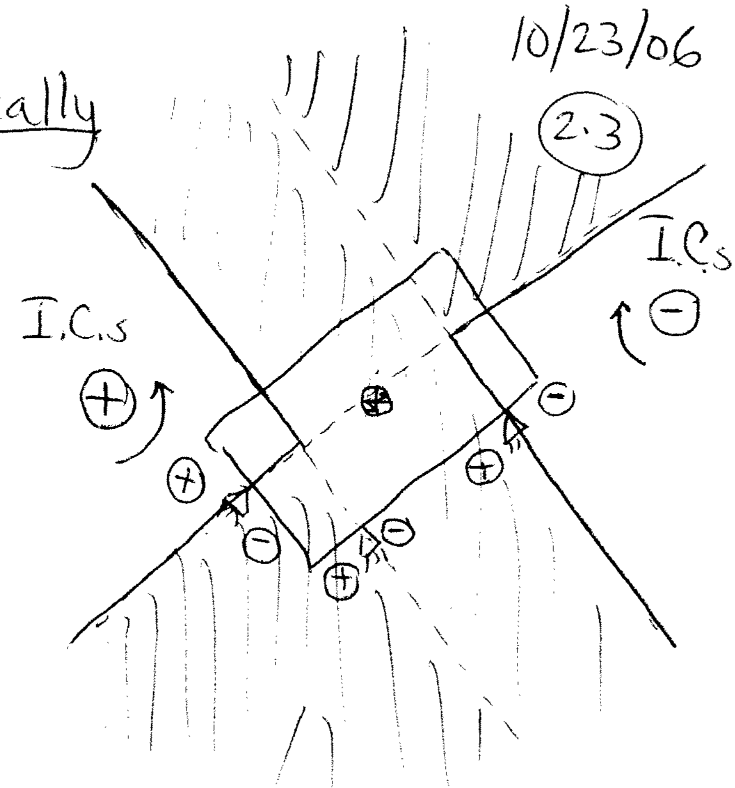
2.3

Solve the LP geometrically (qualitatively)!



All possible I.C's cause cg to move upward.

$\therefore$ $\nu = 0$ is stable!

Solution of LP would give rate of energy gain for each basic motion (breaking one contact)!

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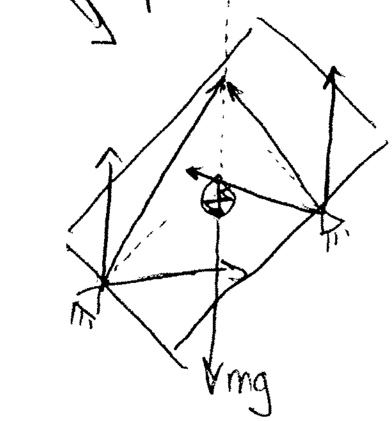
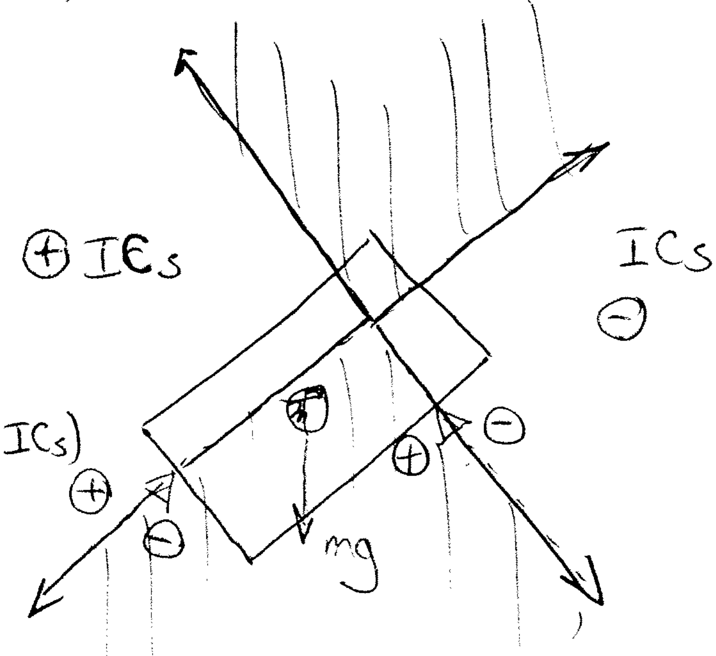
2.4



This is even "more stable" than w/o friction.

ICs = 2 cones



Let $p \in \mathrm{Int}(\oplus ICs) \cup \mathrm{Int}(\ominus ICs)$
then valid $\dot{\theta}$ works against gravity.

If $p \in \partial(\oplus ICs \cup \ominus ICs)$
then valid $\dot{\theta}$ works against gravity & friction!

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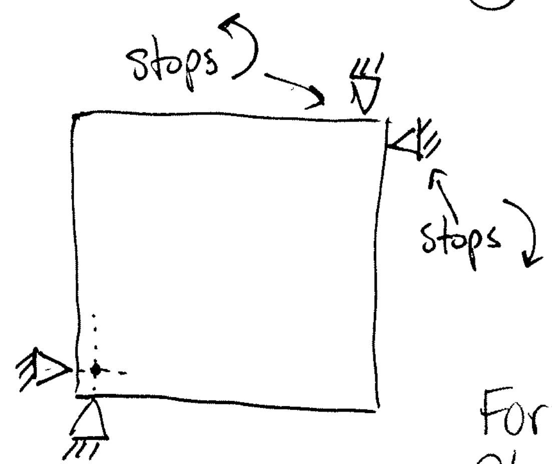
What about this?
Is it stable?



No perturbation is possible, so

Yes.

Form Closure

So forces don't even enter the ~~que~~ picture here.

We will eventually discuss a force/velocity dual

Regardless of ~~how~~ the external force applied, the contacts can alway balance.

$\Updownarrow$

No motion is possible

If ~~the~~ object were flexible, then motion would work againts body stiffness too!

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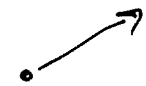
Forces & Moments (Torques)

Forces: cause acceleration (or not)

$$\boxed{f = ma}$$ Newtons eq.



f = force

particle of mass m.

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} m & & \\ & m & \\ & & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}$$

3 equations

Moments cause rotation of rigid bodies

x-y Planar

$$\boxed{n_z = I_{zz}\,\alpha}$$

angular acceleration

moment of inertia wrt z-axis

component of moment in z-direction

$$\boxed{n = I\alpha - \omega \times I\omega}$$ Euler's Eq.

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \Leftarrow$$ Positive Definite

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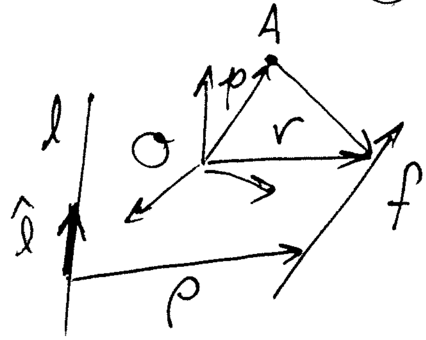
Forces produce moments about lines and points



About origin: $\underline{n_o} = r \times f$ (3x1)

About other point: $\underline{n_A = (r-p) \times f}$ (3x1)

Moment about line:

$$\underline{n_l = \hat{l} \cdot \rho \times f} \quad (1\times1) \text{ scalar}$$

where ρ is diff between two points, one on each line; $\hat{l}$ is ~~an~~ ~~mod~~ unit vector along l.

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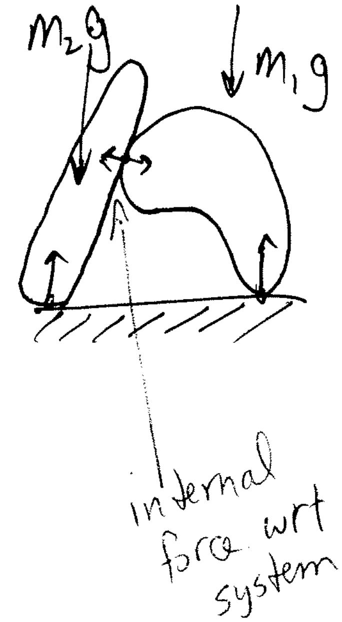
Forces: internal & external



internal – act between particles of {a body} {the system}

external – due to external effects such as gravity, wind

Total Force & Moment ~~(Resultant)~~

$$F \triangleq \Sigma \text{ of all external forces} = \sum_i f_i$$

$$N \triangleq \Sigma \text{ of all external moments} = \sum_i r_i \times f_i$$

Equivalent Systems of forces

If $F_1 = F_2$ & $N_1 = N_2$, then the two systems of forces are said to be <u>equivalent</u>.

Resultant Force ~~Moment~~

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If there exists a single force, f such that $f = F$ and $r \times f = N$

(for a single r), then f is the resultant force of the system.

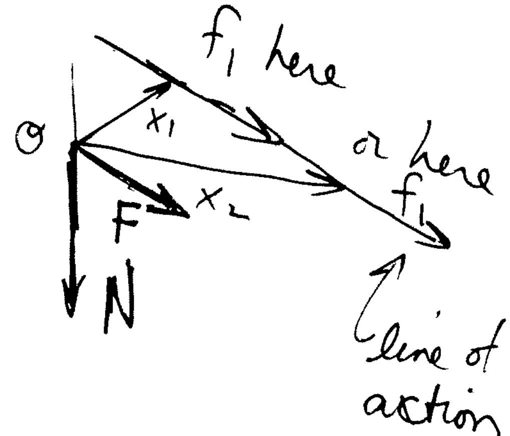
Line of Action – line determines moment.

f applied to a pt is completely characterized by the direction of the force.

i.e. $\underline{f = ma}$

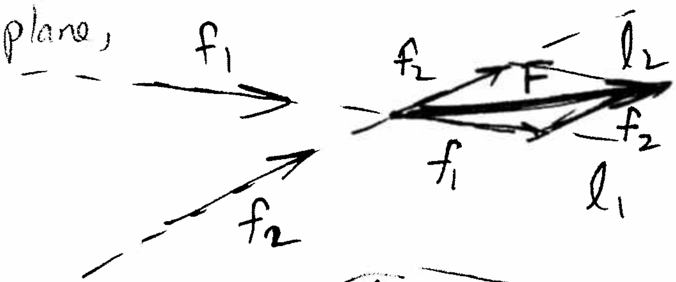
But applied to rigid body,

Point of app. is not important!
Line of action matters!

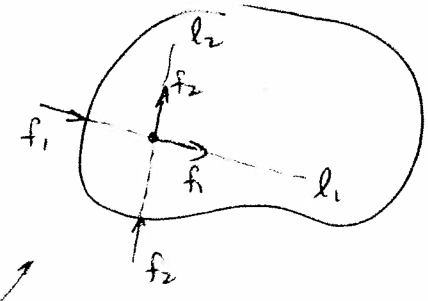


Resultant Force when 2 Lines of Action Intersect

If 2 force lines are in 1 plane,

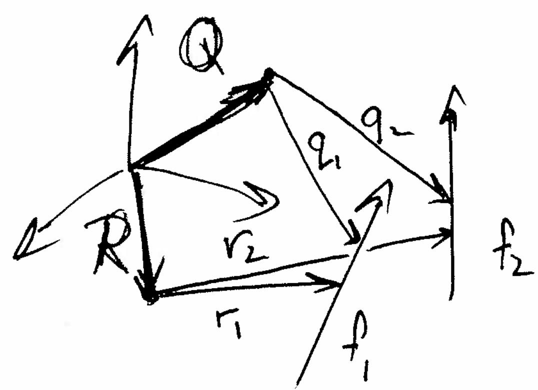


then This is equivalent to two forces acting on a single point!



Change of Reference Point

$$\boxed{\sum_i f_i = F_Q = F_R}$$



$$\sum_i r_i \times f_i = N_R$$

$$\sum_i q_i \times f_i = N_Q$$

Suppose we have N_R and we want N_Q

$$\boxed{N_Q = N_R + (R-Q)\times F}$$

A couple

A system of forces $\ni F = 0$

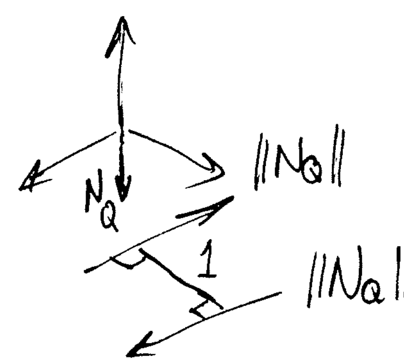
Note N is indep of ref pt.,

Since $N_Q = N_R + (R-Q)\times \cancel{F}^{0}$

A couple is a $\boxed{\text{PURE MOMENT}}$!

One can always construct a system of 2 forces equivalent to a moment!

Note that the couple can be moved rigidly w/o changing moment, N_Q.



Equivalence Theorems

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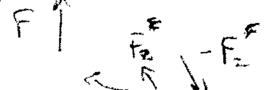
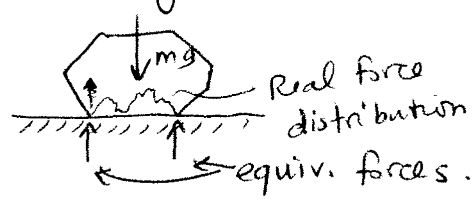
5.1 For any point Q, any system of forces is equiv. to a single force thru Q, plus a couple.

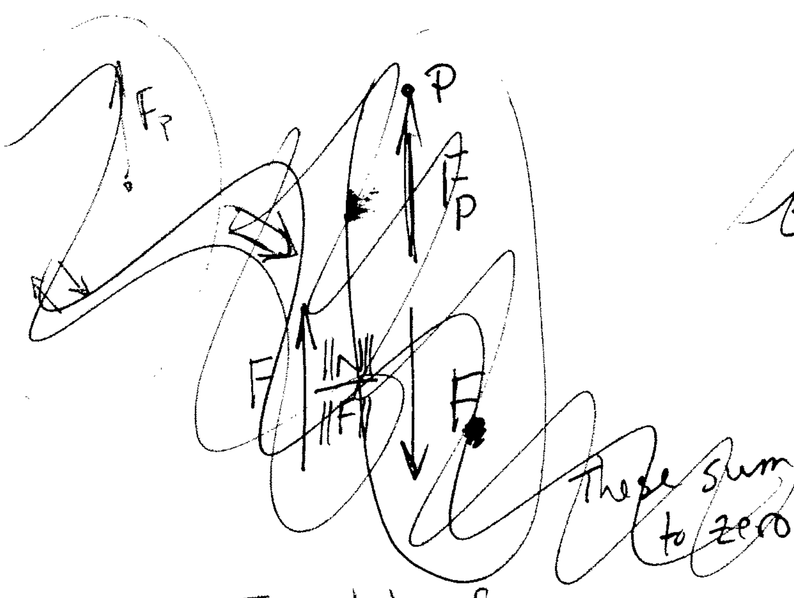
Skip **5.2** Every system of forces is equiv to just 2 forces.



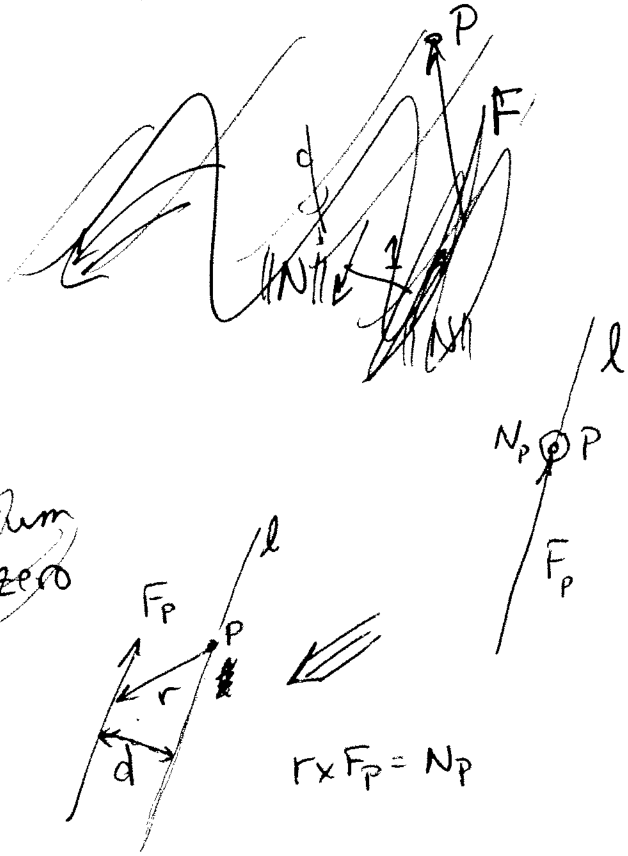
Translate until F_2 intersects F, sum F_2+F



5.3 A system consisting of a single nonzero force plus a couple in the same plane, has a resultant = an equiv. force.



Translate force until $r \times F_p = N_p$!



$r \times F_p = N_p$

Thm. 5.4

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(11)

Poinsot's Theorem

Every system of forces is equivalent to a single force, plus a couple w/ moment parallel to force.

This is the analog to Chasles's theorem.

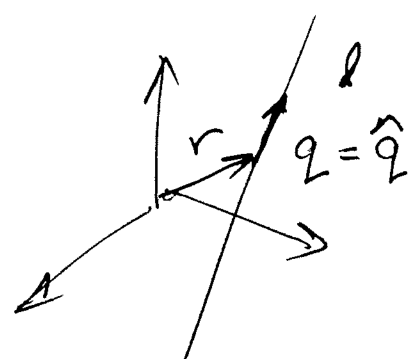
DEFINITION 5.2 Wrench

A wrench is a screw plus a scalar magnitude representing force along the screw axis and moment about the screw axis

pitch is $\cancel{\frac{\|F\|}{\|N\|}}$ $\frac{\|N\|}{\|F\|} = \frac{n}{f}$

force magnitude

$$w = \|f\|\hat{q}$$



$$w_0 = \|f\|q_0 + \|f\|p\hat{q} = \|f\|q_0 + \|n\|\hat{q}$$

$=$

where $q_0 = r \times \hat{q}$

$$= \begin{bmatrix} \hat{q} \\ q_0 + p\hat{q} \end{bmatrix} \|f\| = \text{wrench}$$

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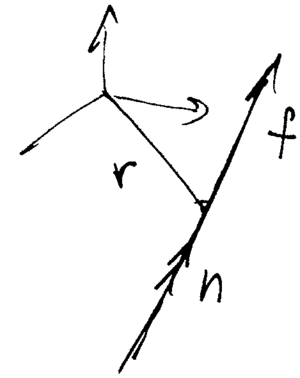
Wrench $\quad w = f \quad (3\times1)$

$\quad w_o = r \times f + n \quad (3\times1)$

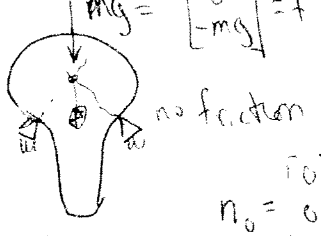


OR $\quad w = f$

$\quad w_o = n_o$

moment of force about origin, plus the moment in the wrench

$mg = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = f$



$n_o = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

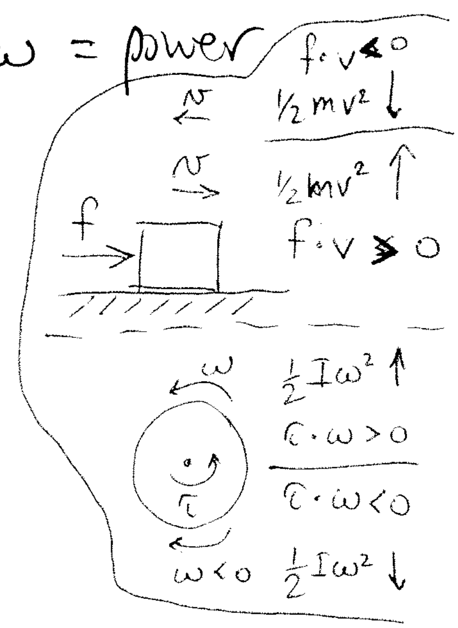
$(f, n_o) + (\omega, v_o) = 0$

Reciprocal Product of wrench & differential twist

$(\omega, v_o) * (f, n_o) = f \cdot v_o + n_o \cdot \omega$ = power

this is instantaneous power

explain w/ example



Put these together to get a wrench & twist

Repelling iff power > 0

Contrary iff power < 0