Example-graspVelKin

Wednesday, February 06, 2013
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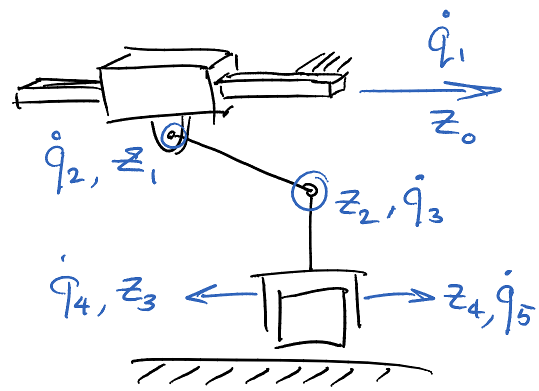


① Construct $\tilde{J}$, $\tilde{G}$, J, and G for the system shown.

ⓐ Do this first in 3D with hard finger contacts and

ⓑ then with soft finger contacts

② Then reduce the result to the planar case.

③ Then specialize to the case where $\dot q_4 = \dot q_5$

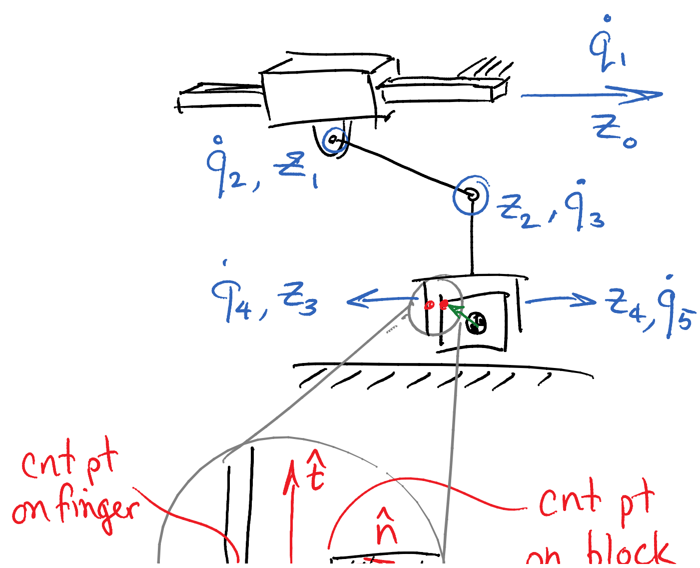
Solution Sketch

Consider one hard contact.

First consider the block.

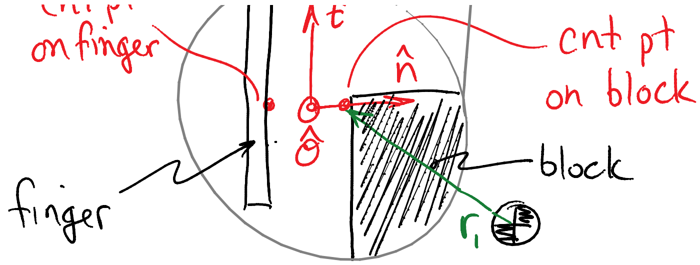
$$\tilde{G}_1^T = \bar{R}_1^T P^T(r_1)$$

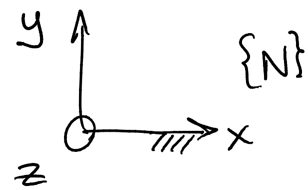
$$r_1^G = (-1, 1, 1)$$



$r_1^{C_1} = (-1, 1, 1)$

$$R_1 = \begin{bmatrix} \hat{n}^N & \hat{t}^N & \hat{o}^N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





$$\tilde{G}_1^T = P^T(r_1) = \begin{bmatrix} I & S^T(r_1) \\ 0 & I \end{bmatrix}$$

Moment arm effect

$$\tilde{G}_1^T = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ \hline & & & 1 & 0 & 0 \\ & 0 & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \underbrace{\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}}_{\text{body twist in }\{N\}} = \underbrace{\begin{bmatrix} v_n \\ v_t \\ v_o \\ \omega_n \\ \omega_t \\ \omega_o \end{bmatrix}}_{\text{body twist in }\{C_1\}} = \begin{bmatrix} v_x + \omega_y - \omega_z \\ v_y - \omega_x - \omega_z \\ v_z + \omega_x + \omega_y \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Check

$$\tilde{G}_1^T \nu = \nu_1^c$$

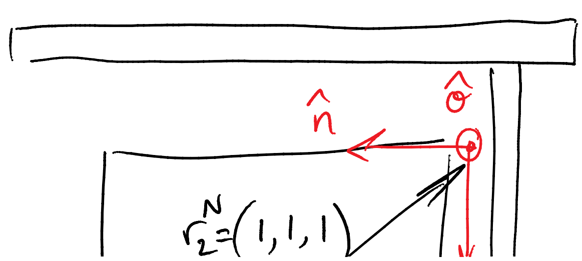
What is G_1^T for S.F. contact?

The first four rows of $\tilde{G}_1^T$.

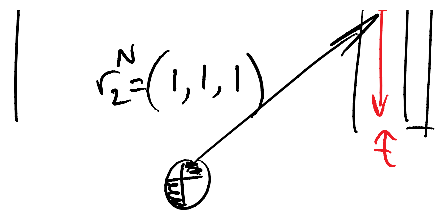
What is G_1^T for H.F. contact?

The first three rows.

What changes for contact 2



What changes for contact 2



$r_2=(1,1,1)$

$$R_2=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Dimensional analysis

$\tilde{G}^T\nu = \nu_{cc}$ — If we model each rectangular contact patch with 4 H.F. contacts, then we have 8 contacts.

$\therefore$ $\tilde{G}^T$ (48×6) $\quad \nu\ (6\times 1)$ $\quad \nu_{cc}\ (48\times 1)$

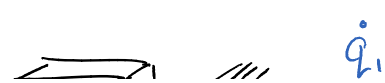
8 H.F. $\quad G^T\ (24\times 6)$ $\quad (24\times 1)$

2 S.F. $\quad G^T\ (8\times 6)$ $\quad (8\times 1)$

1 SF, 1 HF $\quad (7\times 6)$ $\quad (7\times 1)$

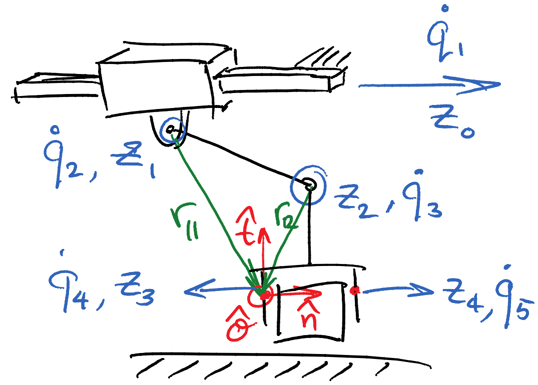
1 SF and 1 HF gives just enough constraint to grasp the object and prevent motion relative to the palm.

Now for the Jacobians: $\tilde{J}\dot{q} = \nu_{cc}$



Now for the Jacobians: $J\dot{q} = \nu_{cc}$

$\tilde{J}$ and J



Consider just contact 1 first.

$$\begin{array}{c|c}
 & \begin{array}{ccccc} z_0^{c_1} & z_1^{c_1} & z_2^{c_1} & z_3^{c_1} & z_4^{c_1} \end{array} \\
\begin{array}{c} \tilde{J}_p \\ \\ \tilde{J}_\phi \end{array} &
\underbrace{\begin{bmatrix} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}}_{\tilde{J}_1}
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} \nu_n \\ \nu_t \\ \nu_o \\ \hline \omega_n \\ \omega_t \\ \omega_o \end{bmatrix}^{c_1}
\end{array}$$

use $S^T(c-\xi)z_1$

$r_{11}^N = (1, -2, 1)$

$r_{12}^N = (-\frac{1}{2}, -1, 1)$

What changes if we consider contact 2?

$r_{21}^N = (3, 2, 1) \qquad r_{22}^N = (3/2, -1, 1)$

Everything else stays the same.

Lets get rid of the ~ .

HF contact $\qquad J_1\dot{q} = \begin{bmatrix} \nu_n \\ \nu_t \\ \nu_o \end{bmatrix}^{c_1}$

$J = \tilde{J}_\phi$

Dimensional analysis

Dimensional analysis

$\tilde{J}\dot{q} = \nu_{cc}$ $\qquad \tilde{J}$ (48×5)

8HF contacts $\Rightarrow$ J (24×5)

2SF contacts $\Rightarrow$ J (8×5)

SF, HF $\Rightarrow$ J (7×5)

Specialize to planar case.

What is not a planar motion? ω_n & ν_o

Look at case of 2 SF contacts

$$G^T \nu = J\dot{q} = \begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \cancel{\nu_{1o}} \\ \cancel{\omega_{1n}} \\ \nu_{2n} \\ \nu_{2t} \\ \cancel{\nu_{2o}} \\ \cancel{\omega_{2n}} \end{bmatrix}$$

Eliminate 4 rows of J

and

$$G^T \begin{bmatrix} \nu_x \\ \nu_y \\ \cancel{\nu_z} \\ \cancel{\omega_x} \\ \cancel{\omega_y} \\ \omega_z \end{bmatrix} = \begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \nu_{2n} \\ \nu_{2t} \end{bmatrix}$$

Also eliminate 3 columns of G^T !

$$\boxed{\begin{array}{c} G^T \nu = J\dot{q} \\ (4\times 3)\,(3\times 1) = (4\times 5)\,(5\times 1) \end{array}}$$