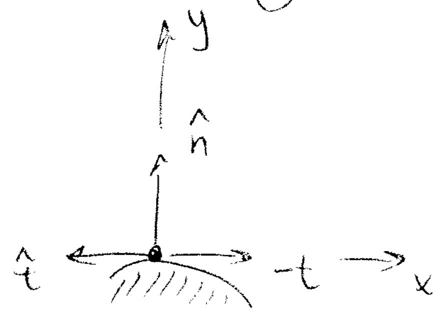
LCP HW Solution

2/25/08

(1)



$$0 \le \lambda_f \perp G_f \nu + \mathbb{1} s \ge 0$$

$$0 \le s \perp \mu \lambda_n - \mathbb{1}^T \lambda_f \ge 0$$

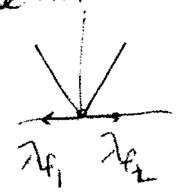
$$0 \le \lambda_{f_1} \perp -\dot{x} + s \ge 0$$

$$0 \le \lambda_{f_2} \perp \dot{x} + s \ge 0$$

$$0 \le s \perp \mu \lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0$$

$$G_f = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \qquad \lambda_f = \begin{bmatrix} \lambda_{f_1} \\ \lambda_{f_2} \end{bmatrix}$$

Case 1: $\boxed{\text{STICKING}}$ This case is inconsistent if equalities are not retained



3 eqs.

$$\left.\begin{matrix} +\dot{x} = s \\ \dot{x} = -s \end{matrix}\right\} \Rightarrow s = \dot{x} = 0 \Rightarrow \text{sticking}$$

$$\left\{\begin{matrix} \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} = 0 \\ \lambda_{f_1}, \lambda_{f_2} \ge 0 \end{matrix}\right\} \Rightarrow \text{in friction cone}$$

Case 2: $\boxed{\text{STICKING}}$

3 eqs

$$\left.\begin{matrix} \dot{x} = s \\ \dot{x} = -s \end{matrix}\right\} \Rightarrow s = \dot{x} = 0 \Rightarrow \text{sticking}$$

$s = 0$ is consistent with

different way to write same thing

$$\left\{\begin{matrix} \lambda_{f_1} \ge 0 \\ \lambda_{f_2} \ge 0 \\ \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} \ge 0 \end{matrix}\right\} \Rightarrow \text{contact force in friction cone.}$$

Case 3: $\boxed{\text{RIGHT SLIDING}}$

$\dot{x} = s$

$\lambda_{f_2} = 0$

$$\left.\mu\lambda_n - \lambda_{f_1} - \underset{0}{\lambda_{f_2}} = 0\right\} \Rightarrow \mu\lambda_n = \lambda_{f_1}$$

$\lambda_{f_1} \ge 0$ valid

$\dot{x} \ge -s \Rightarrow \dot{x} \ge -\dot{x}$

$s \ge 0$ $\quad\Downarrow$

$\dot{x} \ge 0$

$\Rightarrow$ includes degenerate case when $\dot{x} = 0 = s$

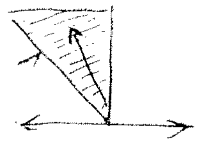
8 cases

Case #	Left	Right
1	+ + +	0 0 0
2	+ + 0	0 0 +
3	+ 0 +	0 + 0
4	+ 0 0	0 + +
5	0 + +	+ 0 0
6	0 + 0	+ 0 +
7	0 0 +	+ + 0
8	0 0 0	+ + +

this is forced

LCP HW Soln

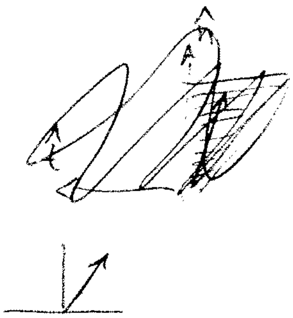
Case 4: $\boxed{\text{STICKING}}$



3 eqs
$$\left.\begin{array}{l} \dot{x} = s \\ \lambda_{f_2} = 0 \\ s = 0 \end{array}\right\} \Rightarrow \text{sticking with } \lambda_{f_2} = 0$$

consistent inequalities
$$\left[\begin{array}{l} \lambda_{f_1} \geq 0 \\ \dot{x} \geq -s \quad \cancel{\geq 0} \\ \mu\lambda_n - \lambda_{f_1} \geq 0 \end{array}\right.$$

Case 5: $\boxed{\text{LEFT SLIDING}}$



3 eqs.
$$\left[\begin{array}{l} \lambda_{f_1} = 0 \\ \dot{x} = -s \\ \mu\lambda_n - \underset{0}{\cancel{\lambda_{f_1}}} - \lambda_{f_2} = 0 \Rightarrow \lambda_{f_2} = \mu\lambda_n \end{array}\right.$$

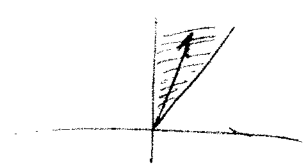
third $-\dot{x} \geq -s \Rightarrow \dot{x} \leq s$

second — $\lambda_{f_2} \geq 0$

first — $s \geq 0$ with $\dot{x} = -s \Rightarrow \dot{x} \leq 0$. This makes $\dot{x} \leq s$ consistent

---

Case 6! $\boxed{\text{STICKING}}$



3 eqs
$$\left[\begin{array}{l} \lambda_{f_1} = 0 \\ \dot{x} = -s \\ s = 0 \end{array}\right. \left.\begin{array}{l} \\ \\ \end{array}\right\} \Rightarrow \dot{x} = s = 0$$

$-\dot{x} \geq -s \; \cancel{\Rightarrow} \; -\dot{x} \geq 0 \Rightarrow \dot{x} \leq 0$

$\lambda_{f_2} \geq 0$

$\mu\lambda_n - \lambda_{f_2} \geq 0 \Rightarrow \mu\lambda_n \geq \lambda_{f_2}$

Case 7: $\boxed{\text{Degenerate Sliding}}$

2/25/08

3

3 eqs
$$\begin{cases} \lambda_{f_1} = 0 \\ \lambda_{f_2} = 0 \\ \mu\lambda_n - \lambda_{f_1} - \lambda_{f_2} = 0 \end{cases} \Rightarrow \lambda_n = 0 \Rightarrow \text{no contact force}$$

$$\left.\begin{array}{ll} -\dot{x} \geq -s \Rightarrow & \dot{x} \leq s \Rightarrow \\ \dot{x} \geq -s & \dot{x} \geq -s \Rightarrow \\ s \geq 0 & s \geq 0 \end{array}\right\} \Rightarrow -s \leq \dot{x} \leq s$$

always true for any $\dot{x}$, ~~s~~ S can be made large enough

Case 8: $\boxed{\text{Degenerate Sticking}}$

3 eqs
$$\begin{cases} \left.\begin{array}{l}\lambda_{f_1} = 0 \\ \lambda_{f_2} = 0\end{array}\right\} = \text{no friction} \\ s = 0 \Rightarrow \text{sticking} \end{cases}$$

$$\left.\begin{array}{ll} -\dot{x} \geq -s \Rightarrow & \dot{x} \leq s \\ \dot{x} \geq -s \longrightarrow & \end{array}\right\} -s \leq \dot{x} \leq s$$

$$\mu\lambda_n - \underbrace{\lambda_{f_1}}_{0} - \underbrace{\lambda_{f_2}}_{0} \geq 0 \Rightarrow \lambda_n \geq 0$$

2/27/08

④

$\mathbb{1}$ is empty
E is empty

2. $M = \begin{bmatrix} m & & 0 \\ & m & \\ 0 & & J \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$ $\qquad p_{ext} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$$\cancel{M} \nu^{l+1} = G_n p_n^{l+1} + \cancelto{0}{M \nu^l} + p_{ext}$$

$$\boxed{\nu^{l+1} = G_n p_n^{l+1} + p_{ext}}$$

$$G_n = \begin{bmatrix} -\sqrt{2}/2 & 0 & 1 & 1 & 0 & 0 \\ \sqrt{2}/2 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -a & 1 & -1 & 0 \end{bmatrix}$$

$$\rho_n^{l+1} = G_n^T \nu^{l+1} + \frac{\psi_n^l}{\cancelto{1}{h}} + \frac{\partial \psi_n^l}{\partial t}$$

$$\psi_n^l = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0 \\ 0 \end{bmatrix}$$

$$\rho_n^{l+1} = G_n^T G_n p_n^{l+1} + G_n^T p_{ext} + \psi_n^l + \frac{\partial \psi_n^l}{\partial t}$$

$$\rho_n^{l+1} = A p_n^{l+1} + b$$

where $A = G_n^T G_n \qquad b = G_n^T p_{ext} + \psi_n^l + \frac{\partial \psi_n^l}{\partial t}$

$$0 \le \rho_n^{l+1} \perp p_n^{l+1} \ge 0$$

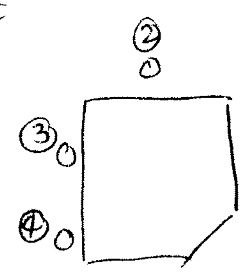
Standard Approach

Set first 4 elements of ρ_n^{l+1} to zero (only 3 are indep.)

last 2 elements of p_n^{l+1} to zero

Easiest to start with $\rho_n^{l+1} = G_n^T \nu^{l+1} + \psi_n^l + \frac{\partial \psi_n^l}{\partial t}$

Note that ν^{l+1} is the velocity to be maintained over the time step that will cause contacts at ②, ③, ④.



Only 3 of the ρ_n^{l+1} eqs. are linearly indep.

$\therefore$ Take 3 and solve them. I choose the eqs. corresponding to ~~eqs~~ contacts 2, 3, 4, because we want to close these gaps.

$$\begin{bmatrix} \rho_{1n}^{l+1} \\ \rho_{2n}^{l+1} \\ \rho_{3n}^{l+1} \\ \rho_{4n}^{l+1} \\ \rho_{5n}^{l+1} \\ \rho_{6n}^{l+1} \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -0.1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}}_{\nu^{l+1}} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} d \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d \text{ unknown}$$

Solution yields $\nu_{l+1} = \begin{bmatrix} -0.1 \\ 0.1 \\ 0 \end{bmatrix}$

Then solve for d (the movement of finger 1).

$$d = -0.2414$$

Substitute back to check $\Longrightarrow$ $\rho_n^{l+1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.1 \\ 0.1 \end{bmatrix}$

Now we have b & A in

$$\rho_n^{\ell+1} = A p_n^{\ell+1} + b$$

First 4 ~~eqs~~ elements of $\rho_n^{\ell+1}$ are zero. Solve for $p_n^{\ell+1}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = A_4 p_n^{\ell+1} + b_4$$

where A_4 is first 4 rows of A

b_4 is first 4 rows of b

$$p_n^{\ell+1} = \underbrace{-A_4^+ b_4}_{\begin{bmatrix} 0.44 \\ -0.79 \\ 0.19 \\ 0.02 \end{bmatrix}} + \underbrace{\text{null}(A_4)}_{\begin{bmatrix} 0.72 \\ 0.51 \\ 0.46 \\ 0.05 \end{bmatrix}} \alpha$$

$$\begin{bmatrix} 0.44 \\ -0.79 \\ 0.19 \\ 0.02 \end{bmatrix} + \begin{bmatrix} 0.72 \\ 0.51 \\ 0.46 \\ 0.05 \end{bmatrix} \alpha$$

Part C result

I picked $\alpha = 1.7$

$$\Rightarrow \quad p_n^{\ell+1} = \begin{bmatrix} 1.66 \\ 0.08 \\ 0.98 \\ 0.10 \\ 0 \\ 0 \end{bmatrix}$$

Part C result

For part D, a changes from 0.1 to -0.1.

$$p_n^{l+1} = -A_4^+ b_4 + \mathrm{null}(A_4)\alpha$$

$$\begin{bmatrix} 0.41 \\ -0.81 \\ 0.21 \\ -0.02 \end{bmatrix} + \begin{bmatrix} 0.67 \\ 0.49 \\ 0.54 \\ -0.05 \end{bmatrix} \alpha$$

impossible to "fix" both.

∴ the initial assumptions made (i.e., p_n^{l+1} does not have it's first 4 elements equal to zero.)