MP HW 1 Solution
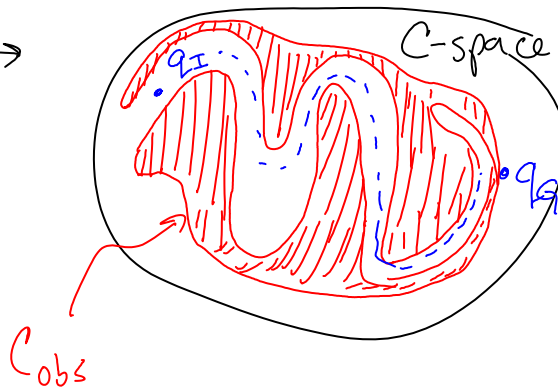
LaValle text. Chapter 2, problems 2 & 6a
Chapter 3, problems 1 & 6

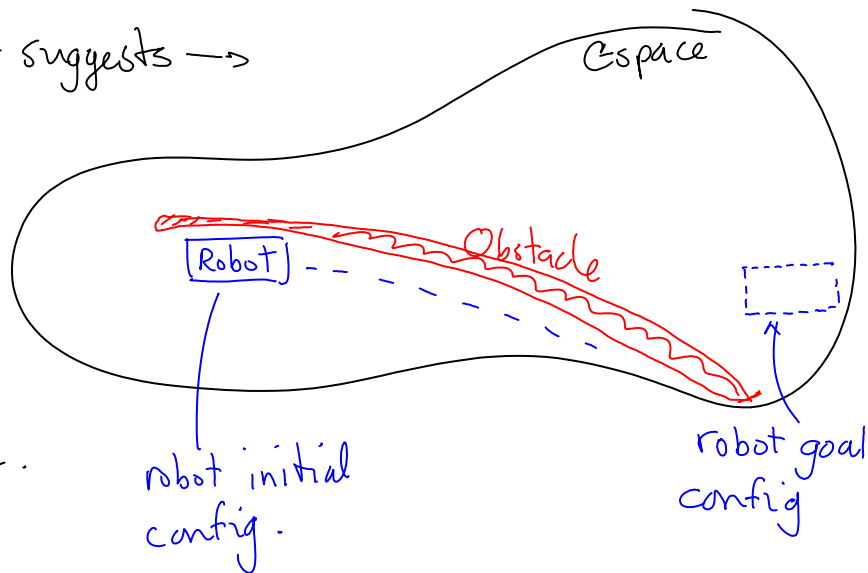(2.2) There are two interpretations: C-space is 2D, or the robot moves in a 2-D world.

The former suggests →

The latter suggests →

robot will move to dead end then backtrack.

(2.6a) See email from me and LaValle sent to entire class about this problem.

(3.1) Make a nose with three linear inequalities. Assume origin is at the center of base of nose.

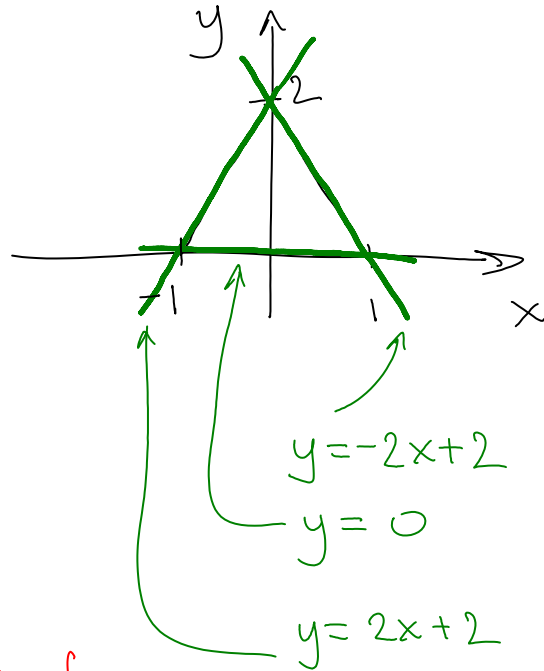We want to exclude the interior of the nose.

Interior is given by:

$$y \leq -2x + 2 \implies$$

So we get :

$$y + 2x - 2 \leq 0$$
$$-y \leq 0$$
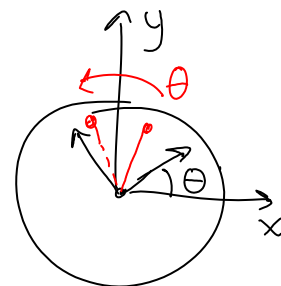$$y - 2x - 2 \leq 0$$

Interior of nose

$$y = -2x + 2$$
$$y = 0$$
$$y = 2x + 2$$

To complete this problem, put results in form of primitives that remove nose interior.

(3.6) Consider rotation of a disc primitive about its center.

$$x^2 + y^2 - 1 \leq 0$$

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x^2 + y^2 - 1 \leq 0$$

$$\begin{bmatrix} x_{new} \\ y_{new} \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} c_\theta x - s_\theta y \\ s_\theta x + c_\theta y \end{bmatrix}$$

Substitute into primitive.

$$(c_\theta x - s_\theta y)^2 + (s_\theta x + c_\theta y)^2$$

$$= c_\theta^2 x^2 - 2 c_\theta s_\theta xy + s_\theta^2 y^2$$

$$\frac{+ \left(s_\theta^2 x^2 + 2 s_\theta c_\theta xy + c_\theta^2 y^2 \right)}{x^2 + 0 + y^2}$$

∴ if the original (x,y) satisfies the primitive, so will the rotated (x,y).