H.W. 2/24/13

(1)

Time stepping subproblem

$$\begin{bmatrix}0\\0\\\rho_n^{l+1}\\\rho_f^{l+1}\\\sigma^{l+1}\end{bmatrix} = \begin{bmatrix} M^l & -G_B^l & -G_n^l & -G_f^l D^l & 0\\ (G_B^T)^l & 0 & 0 & 0 & 0\\ (G_n^T)^l & 0 & 0 & 0 & 0\\ (D^T G_f^T)^l & 0 & 0 & 0 & E^l\\ 0 & 0 & U^l & -(E^T)^l & 0\end{bmatrix}\begin{bmatrix}\nu^{l+1}\\ p_B^{l+1}\\ p_n^{l+1}\\ \alpha^{l+1}\\ \beta^{l+1}\end{bmatrix} + \begin{bmatrix}-M\nu^l - p_{app}^l + F^l\\ \Psi_B^l/h + \partial\Psi_B/\partial t\\ \Psi_n^l/h + \partial\Psi_n/\partial t\\ \partial\Psi_f/\partial t\\ 0\end{bmatrix}$$

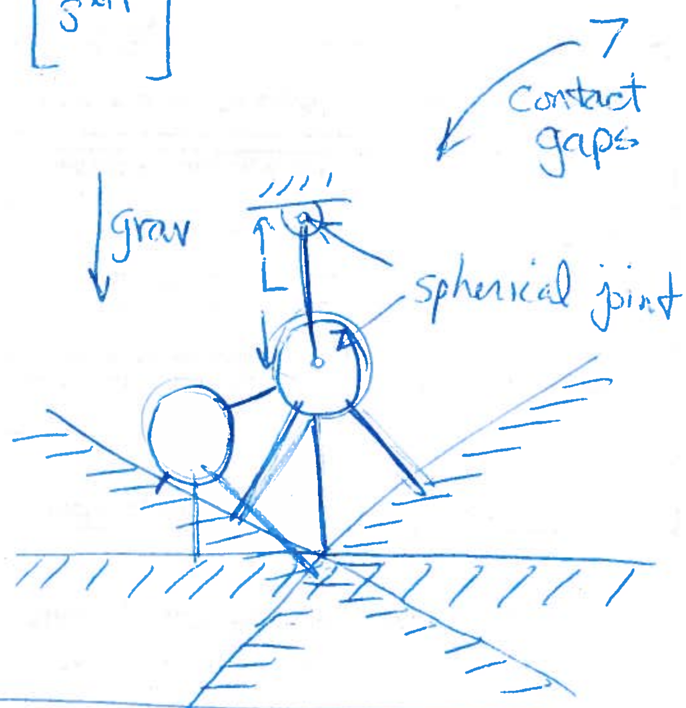
$$0 \le \begin{bmatrix}\rho_n^{l+1}\\ \rho_f^{l+1}\\ \sigma^{l+1}\end{bmatrix} \perp \begin{bmatrix}p_n^{l+1}\\ \alpha^{l+1}\\ \beta^{l+1}\end{bmatrix} \ge 0$$

System

Two spheres

Two spherical joints

Three or more
plane to create
environment



After solving the LCP, update ~~state~~ positions

$$u^{l+1} = u^l + V\nu^{l+1} h$$

AND NORMALIZE QUATERNIONS !

2/4/13

2

<u>Steps & partial credit</u>

- 36 Two spheres interacting on 3 plane support. w/o friction. Animate simulation
- 20 Add friction (get rotations right)
- 20 Add bilateral constraints
- 2 Add joint torques
- 2 Add collision restitution (bounce)
- 15 ~~Implemen~~ Solve with prox PGS
- 20 Plot state & impulse trajectories

~~Plot~~ ~~Animate~~

Total 115

I will provide animation code & other graphics functions.

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{(14\times 1)} \qquad u_i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ e_{i0} \\ e_{i1} \\ e_{i2} \\ e_{i3} \end{bmatrix}_{(7\times 1)}, \quad i=\{1,2\}$$

$$\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}_{(12\times 1)} \qquad \nu_i = \begin{bmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \\ \omega_{ix} \\ \omega_{iy} \\ \omega_{iz} \end{bmatrix} \quad i=\{1,2\}$$

$$p_B = \begin{bmatrix} p_{1B} \\ p_{2B} \end{bmatrix}_{(6\times 1)} \qquad p_{iB} = \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{bmatrix} \quad i=\{1,2\}$$

$$p_n = \begin{bmatrix} p_{1n} \\ p_{2n} \\ \vdots \\ p_{7n} \end{bmatrix}_{(7\times 1)} \qquad \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_7 \end{bmatrix}_{(7n_d\times 1)} \qquad s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_7 \end{bmatrix}_{(7\times 1)}$$

Now we know the size of the big mLCP matrix

$18+7(2+n_d)$ PATH can solve this, but not Lemke's algorithm.

mLCP

$$Cx + Dw + c = 0$$

$$0 \le (Fx + Bw + a) \perp x \ge 0$$

Convert to pure LCP.

$$Dw = -Cx - c$$

$$w = -D^{-1}(Cx + c)$$

$$0 \le Fx + B(-D^{-1}(Cx + c)) + a \perp x \ge 0$$

$$\boxed{0 \le (F - BD^{-1}C)x - BD^{-1}c + a \perp x \ge 0}$$

Compare to big mLCP formulation to determine the definitions of B, C, D, F, a, c, x, w.

Calling Lemke solver

$$[z, err] = \text{lemke}(M, q, z_0)$$

here $M = F - BD^{-1}C$ $\qquad$ z_0 = initial guess

$q = a - BD^{-1}c$ $\qquad$ z_0 = zero is usually fine.

2/24/13

4

$M = \mathrm{diag}(M_1, M_2)$

$$M_i = \begin{bmatrix} m_i I_{(3\times3)} & 0 \\ 0 & J_{(3\times3)} \end{bmatrix} \qquad i = \{1, 2\}$$

not Jacobian

m_i = mass of body i

J_i = inertia matrix of body i =

$$J_i = \cancel{\mathrm{diag}}\ \frac{2}{5} m R^2 I_{(3\times3)}$$

where R = radius, not rotation matrix

$$G_B = \Bigg[$$