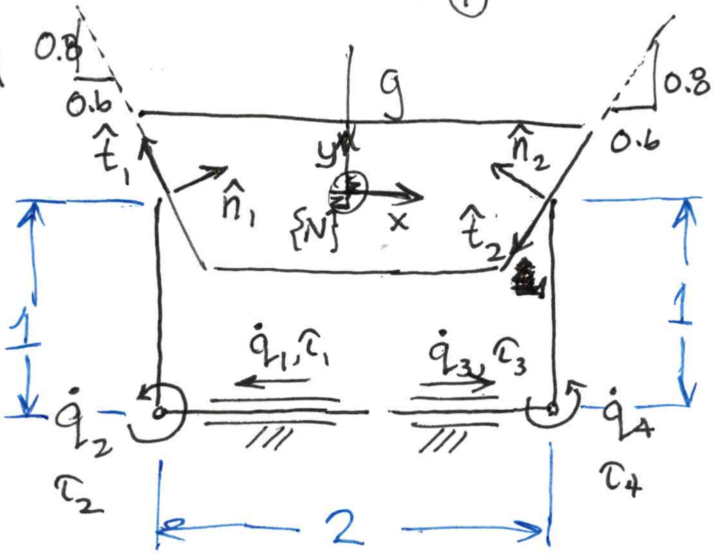
Grasping Homework

4/10/18

All these problems are planar!

1.) A hand with 4 joints grasps a quadrilateral.



SHOW YOUR WORK!

A.) Construct G and J assuming HF contacts

$\mu_1 = \mu_2 = 2/3$

B.) Give a basis for all 4 fundamental subspaces of each of G & J.

C.) Does the grasp have form closure?

D.) Does this grasp have friction form closure?

E.) Does this grasp have force closure?

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F.) Give an example ν of the object that cannot be achieved by choice of $\dot{q}$.

G.) If μ is reduced to 0.5, there ~~is~~ are different values of τ that can balance $g = [0\ -1\ 0]^T$. Give two possible solutions:

i) Maximum internal force.

ii) Minimum internal force.

H.) Keeping contact point 1 fixed on the object, find a range locations for contact point 2 such that if contact 2 is at any point in that range, the grasp will have frictional form closure. Assume $\mu_1 = \mu_2 = 1$.

e.g.

Solution to
Grasping HW from 4/10/18

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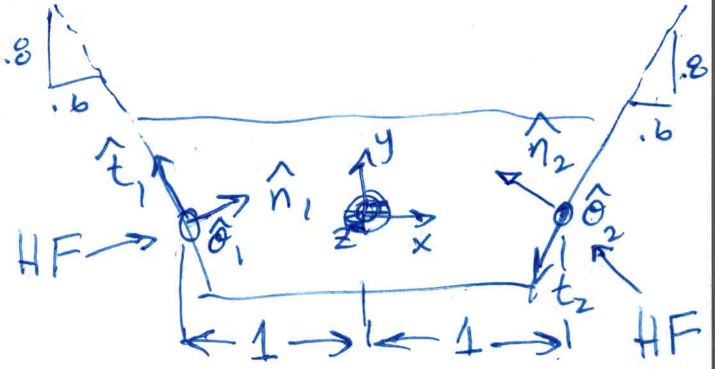
(1)

Part A



$$\tilde{G}_1^T = \begin{bmatrix} \hat{n}_1^T & (r_1 \times \hat{n}_1)^T \\ \hat{t}_1 & (r_1 \times \hat{t}_1)^T \\ \hat{\theta}_1 & (r_1 \times \hat{\theta}_1)^T \\ 0 & \hat{n}_1^T \\ 0 & \hat{t}_1^T \\ 0 & \hat{\theta}_1^T \end{bmatrix}$$

$$H = \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 1 & 0 & O_{3\times 3} \\ 0 & 0 & 1 & \end{array}\right]$$

$$\hat{n}_1^T = \begin{bmatrix} 0.8 & 0.6 & 0 \end{bmatrix} \quad \hat{t}_1^T = \begin{bmatrix} -0.6 & 0.8 & 0 \end{bmatrix} \quad \hat{\theta}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\hat{r}_1^T = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix}$$

$$G_1 = H \tilde{G}_1^T = \begin{bmatrix} \hat{n}_1^T & (r_1 \times \hat{n}_1)^T \\ \hat{t}_1^T & (r_1 \times \hat{t}_1)^T \\ \cancel{\hat{\theta}_1^T} & \cancel{(r_1 \times \hat{\theta}_1)^T} \end{bmatrix}$$

remove this row because we are considering the planar case & forces in $\hat{\theta}$ are out of the plane.

Similarly, for contact 2;

$$\hat{n}_2^T = \begin{bmatrix} -0.8 & 0.6 & 0 \end{bmatrix} \quad \hat{t}_2^T = \begin{bmatrix} -0.6 & -0.8 & 0 \end{bmatrix} \quad \hat{\theta}_2^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$\hat{r}_2^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$G_2^T = H \tilde{G}_2^T = \begin{bmatrix} \hat{n}_2^T & (r_2 \times \hat{n}_2)^T \\ \hat{t}_2^T & (r_2 \times \hat{t}_2)^T \\ \cancel{\hat{\theta}_2} & \cancel{(r_2 \times \hat{\theta}_2)^T} \end{bmatrix}$$

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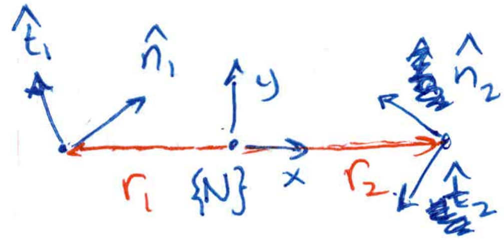
②

$$G^T = \begin{bmatrix} G_1^T \\ G_2^T \end{bmatrix} = \begin{bmatrix} \hat{n}_1^T & (r_1 \times \hat{n}_1)^T \\ \hat{t}_1^T & (r_1 \times \hat{t}_1)^T \\ \hat{\theta}_1^T & (r_1 \times \hat{\theta}_1)^T \\ \hat{n}_2^T & (r_2 \times \hat{n}_2)^T \\ \hat{t}_2^T & (r_2 \times \hat{t}_2)^T \\ \hat{\theta}_2^T & (r_2 \times \hat{\theta}_2)^T \end{bmatrix}$$

This is the 3D version.



Note that inertial $\hat{z}$ and $\hat{\theta}_1$ & $\hat{\theta}_2$ are out of the page.

Convert to planar case.

$$G^T = \begin{bmatrix} n_{1x} & n_{1y} & r_{1x}n_{1y} - r_{1y}n_{1x} \\ t_{1x} & t_{1y} & r_{1x}t_{1y} - r_{1y}t_{1x} \\ n_{2x} & n_{2y} & r_{2x}n_{2y} - r_{2y}n_{2x} \\ t_{2x} & t_{2y} & r_{2x}t_{2y} - r_{2y}t_{2x} \end{bmatrix}_{(4\times 3)}$$

← we removed columns 3, 4, & 5 AND the rows corresponding to $\hat{\theta}_i$

$$\boxed{G^T = \begin{bmatrix} 0.8 & 0.6 & -0.6 \\ -0.6 & 0.8 & -0.8 \\ -0.8 & 0.6 & 0.6 \\ -0.6 & -0.8 & -0.8 \end{bmatrix}_{(4\times 3)}}$$

← Planar case, two HF contacts.

Construct $\tilde{J}$

(see notes from 3/14/18 page (11))

3D $\quad \tilde{J}_i = \bar{R}_i^T \tilde{Z}_i \quad , \quad i = \{1, 2\}$ $\qquad \tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \end{bmatrix}$

Applying the formulas yields:

$$\tilde{J}_1 = \left[\begin{array}{cc|c} -0.8 & -0.8 & \\ 0.6 & 0.6 & \\ 0 & 0 & O_{(6\times 2)} \\ 0 & 0 & \\ 0 & 0 & \\ 0 & 1 & \end{array}\right] \qquad \tilde{J}_2 = \left[\begin{array}{c|cc} & -0.8 & 0.8 \\ & -0.6 & 0.6 \\ O_{(6\times 2)} & 0 & 0 \\ & 0 & 0 \\ & 0 & 0 \\ & 0 & 1 \end{array}\right]$$

Next apply HF selection matrix, $H = [I_{(3\times 3)} \,\vdots\, O_{3\times 3}]$, which eliminates rows 4,5,6 of $\tilde{J}_1$ & $\tilde{J}_2$

$$J_1 = H\tilde{J}_1 = \left[\begin{array}{cc|c} -.8 & -.8 & \\ .6 & .6 & O_{(3\times 2)} \\ 0 & 0 & \end{array}\right] \qquad J_2 = H\tilde{J}_2 = \left[\begin{array}{c|cc} & -.8 & .8 \\ O_{(3\times 2)} & -.6 & .6 \\ & 0 & 0 \end{array}\right]$$

$$J = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \qquad J\dot{q} = \left[\begin{array}{cc|cc} -.8 & -.8 & & \\ .6 & .6 & \multicolumn{2}{c}{O_{(3\times 2)}} \\ 0 & 0 & & \\ \hline & & -.8 & .8 \\ \multicolumn{2}{c|}{O_{(3\times 2)}} & -.6 & .6 \\ & & 0 & 0 \end{array}\right] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} N_{1n} \\ N_{1t} \\ N_{1o} \\ N_{2n} \\ N_{2t} \\ N_{2o} \end{bmatrix}$$

Convert to planar case.

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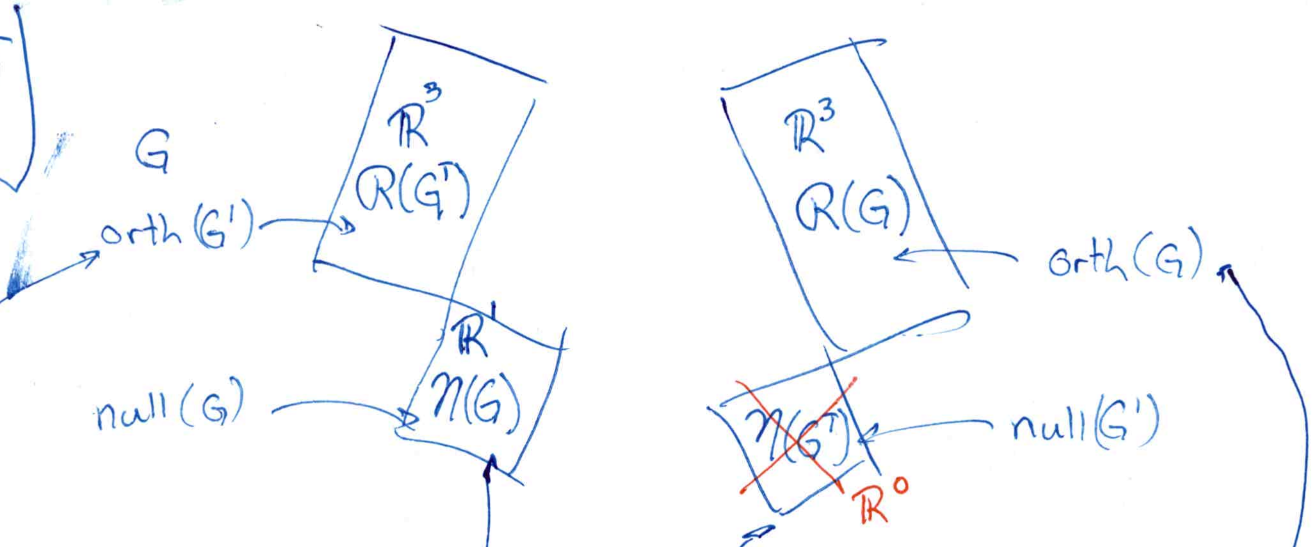
velocities in $\hat{\theta}_i$, $i \in \{1,2\}$ are irrelevant.

$\therefore$ remove rows 3 & 6 from J

$$\boxed{J = \begin{bmatrix} -.8 & -.8 & \multicolumn{2}{c}{0_{(2\times2)}} \\ .6 & .6 & & \\ \multicolumn{2}{c}{0_{(2\times2)}} & -.8 & .8 \\ & & -.6 & .6 \end{bmatrix}}$$

PART B



Row space basis

$$= \begin{bmatrix} 0 & -.57 & .6 \\ 0 & .42 & .8 \\ -.6 & .57 & 0 \\ .8 & .42 & 0 \end{bmatrix}$$

Column space basis

$$\begin{bmatrix} 0 & 0 & -1 \\ .707 & -.707 & 0 \\ .707 & .707 & 0 \end{bmatrix}$$

Null space basis

$$\begin{bmatrix} .57 & -.42 & .57 & .42 \end{bmatrix}^T$$

Left null space basis

0

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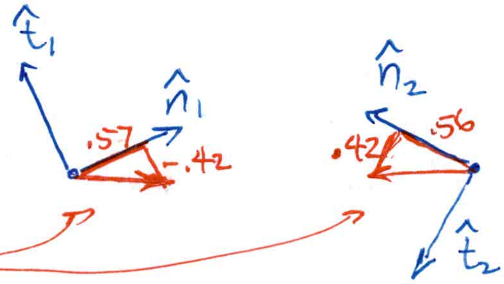
Physical interpretation of basis of $\mathcal{N}(G)$.

$$N(G) = \begin{bmatrix} 0.57 & -0.42 & 0.57 & 0.42 \end{bmatrix}^T$$

N(G) corresponds to internal grasp force or squeezing force.

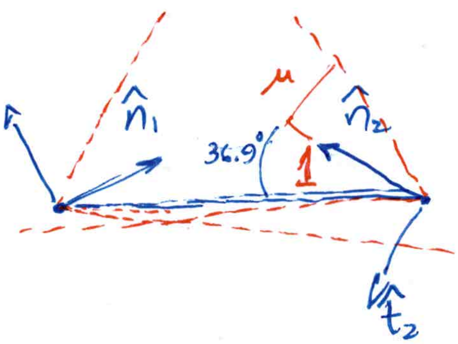


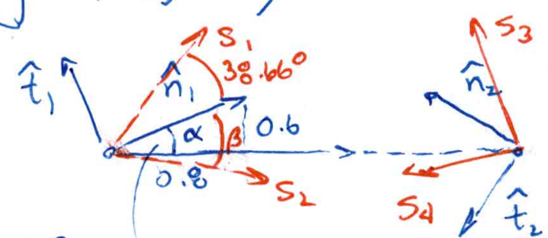
Part C

No form closure. Fails # contacts test. Must have at least 4 contacts in plane.

Part D

Yes, if $\mu > 0.75$





Let $\mu = 0.8$

$\alpha = \text{atan}(3/4)\, 180/\pi = 36.87°$

$\beta = \text{atan}(\mu/1)\, 180/\pi = 38.67°$

$\therefore$ cones can "see" each other $\therefore$ frictional form closure exists (if ~~[illegible]~~ $\mu = 0.8$)

Part E

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Force Closure exists if

$$\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$$

$$N(J) = \begin{bmatrix} .6 & 0 \\ .8 & 0 \\ 0 & -.6 \\ 0 & .8 \end{bmatrix} \qquad N(G) = \begin{bmatrix} .8 \\ -.6 \\ .8 \\ .6 \end{bmatrix}$$

Is it possible that $N(J)\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + N(G)\,\beta = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

even if $\alpha_1, \alpha_2, \beta \neq 0$?

first row

$$.6\alpha_1 + .8\beta = 0 \rightarrow \qquad \alpha_1 = -\frac{.8}{.6}\beta = -\frac{4}{3}\beta$$

$$.8\alpha_1 - .6\beta = 0 \qquad \leftarrow \text{substitute}$$

$$.8\left(-\frac{4}{3}\beta\right) - .6\beta = 0$$

$$\left(-\frac{3.2}{3} - .6\right)\beta = 0 \Rightarrow \beta = 0 \Rightarrow \alpha_1 = 0$$

And $\beta = 0 \Rightarrow \alpha_2 = 0$.

Since $\alpha_1 = \alpha_2 = \beta = 0$ is required,

$$\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$$

$\therefore$ The grasp has force closure.

Part F

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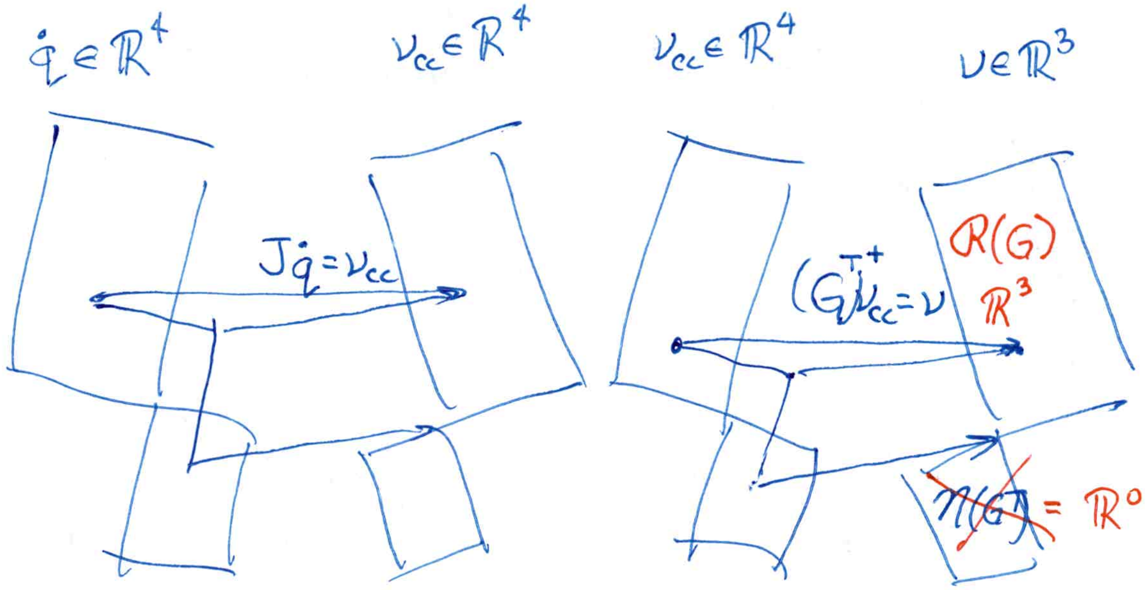


$$J\dot{q} = \nu_{cc} = G^T \nu$$

General solution is:

$$\nu = (G^T)^+ J \dot{q} + \underbrace{\cancel{N(G^T)}}_{0} \alpha$$

$$\nu = \begin{bmatrix} -.5 & -.5 & .5 & -.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} \nu_x \\ \nu_y \\ \omega_z \end{bmatrix}$$

$\therefore$ No choice of $\dot{q}$ can cause object to move upward or rotate!

$\boxed{\text{Part G}}$

Use $G\lambda = g$

where $\mu = 0.5$, $g = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$. $\lambda = \begin{bmatrix} \lambda_{1n} \\ \lambda_{1t} \\ \lambda_{2n} \\ \lambda_{2t} \end{bmatrix}$

general soln.

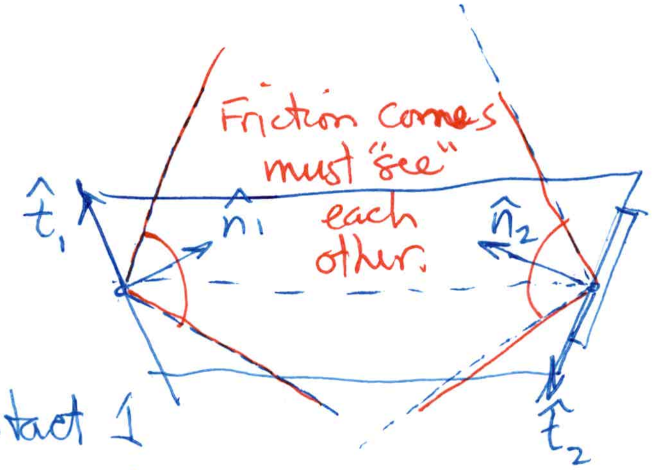
$$\lambda = G^{+} g + N(G)\alpha \qquad \cancel{\&} \; -\tfrac{1}{2}\lambda_{in} \le \lambda_{it} \le \tfrac{1}{2}\lambda_{in}$$

$$\begin{bmatrix} \lambda_{1n} \\ \lambda_{1t} \\ \lambda_{2n} \\ \lambda_{2t} \end{bmatrix} = \begin{bmatrix} -.3 \\ -.4 \\ -.3 \\ .4 \end{bmatrix} + \begin{bmatrix} .8 \\ -.6 \\ .8 \\ .6 \end{bmatrix} \alpha$$

Solve the 4 inequalities to find limits on α.

These correspond to the minimum and maximum squeezing forces on the object to prevent slipping.

$\boxed{\text{Part H}}$



Keeping contact 1 fixed, determine how far contact 2 can be slid up edge and keep contact 1 in its friction cone.