Grasping: Chapter 38 : Springer Handbook of Robotics

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(1)

Our goals:

- Build models of grasping systems (e.g., object & hand) sufficient for designing, planning, & executing grasping and dexterous manipulation
- Grasp security analysis – "form closure" and "force closure"
- Fwd and inv. velocity kinematics
- Contact models

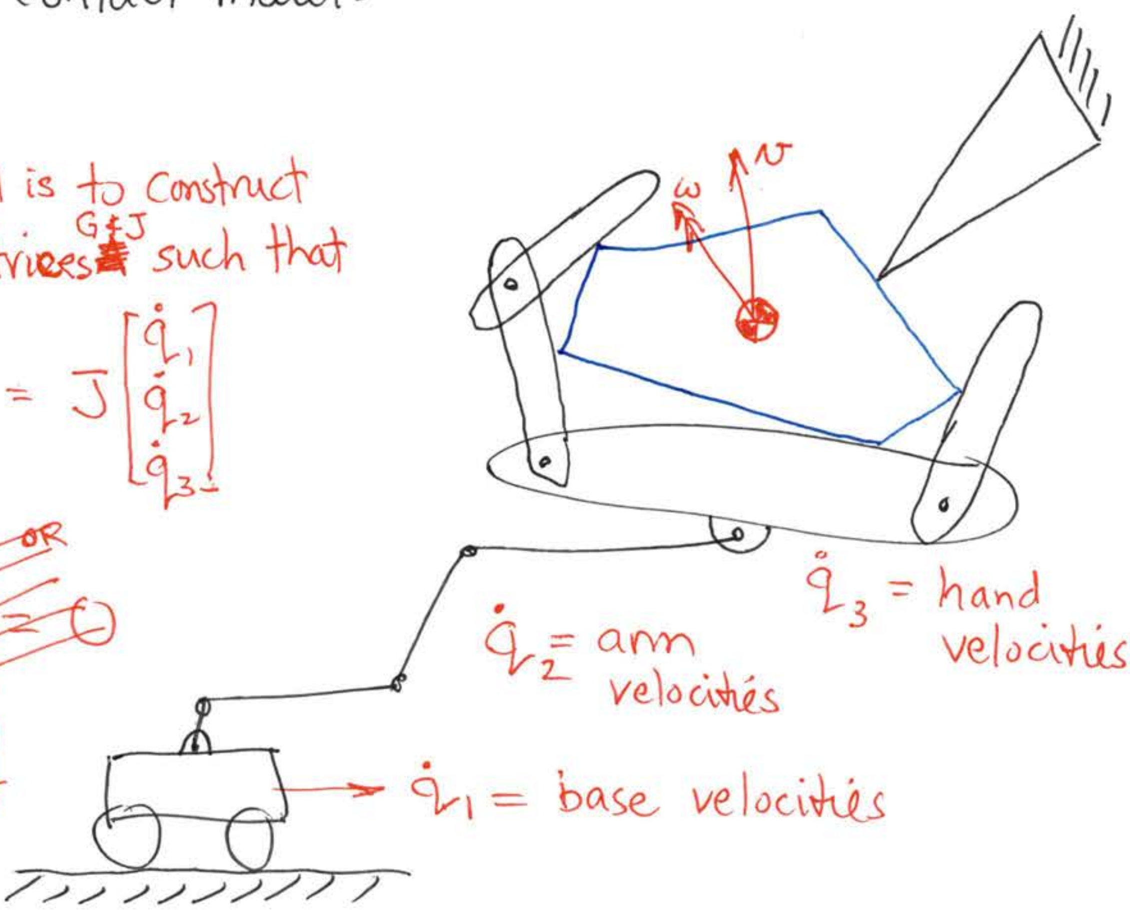
Goal is to construct matrices G&J such that

$$G^T \begin{bmatrix} \nu \\ \omega \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

~~OR~~

$$\cancel{A \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0}$$

$$\cancel{A = \begin{bmatrix} J & -G^T \end{bmatrix}}$$



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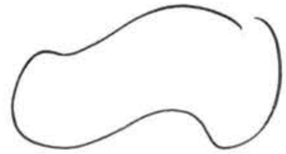
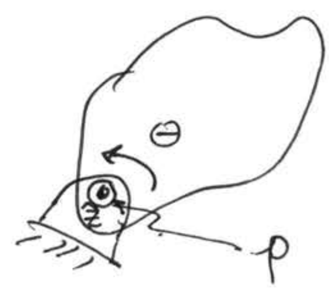
Position Kinematics of Grasps

Object to grasp has 6 dof. (3 d.of. in plane)



Connect it to ground thru revolute joint.

How many d.o.f.s now? $\underline{\underline{1}}$.



Count constraints:

planar case: p on body is fixed in the plane

$$\left.\begin{array}{l} p_x = \text{const} = A \\ p_y = \text{const} = B \end{array}\right\}$$

3 d.o.f − 2 constr equations = 1 d.o.f.

spatial case: p on body is fixed in space

point on jnt axis → $p_x = A$, $p_y = B$, $p_z = C$

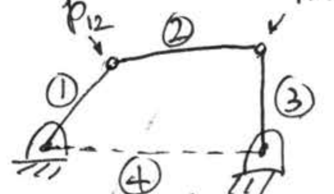
$$\begin{array}{cc} & \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \end{array} \\ {}^N R_B & \left[\begin{array}{ccc} & & r_{13}(\alpha,\beta,\gamma) \\ & & r_{23}(\alpha,\beta,\gamma) \\ & & r_{33}(\alpha,\beta,\gamma) \end{array}\right] \end{array} \begin{array}{c} = \\ = \\ \end{array} \left[\begin{array}{c} {}^N\hat{z}_x \\ {}^N\hat{z}_y \\ {}^N\hat{z}_z \end{array}\right]$$

two more constraint equations

(not 3, because if 2 values are known, the other can be computed.)

known numbers

Consider a four-bar linkage:



planar case:

How many d.o.f.s w/o joints?

4 bodies × 3 dofs/body = 12 dofs

Now consider joints:

ground can't move ← 3 constraints

pt on link 1 is fixed ← 2 constrs

pt of link 3 is fixed ← 2 constr

pt of link 1 is fixed to pt of link 2 ← 2 constr

pt of link 3 is fixed to pt of link 2 ← 2 constr

+ ______

Add to find # of d.o.fs lost due to constrs. 11

Remaining dofs $\geq$ 12 − 11 = 1

actually, this is not an equality because sometimes constraints a dependent.

spatial case:

4 bodies × 6 d.o.fs / body = 24 dofs

ground imposes 6 constr

each joint imposes 5 constr

dofs $\geq$ 24 − 6 − 4·5 $\geq$ −2 ← Does this make sense?

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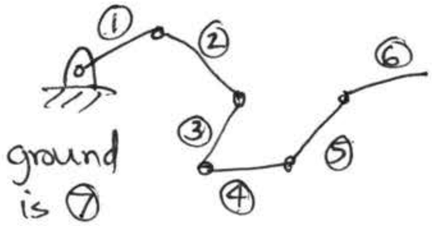
Really there is 1 dof.

- This means of the 20 constraint equations associated with the 4 joints, 3 are dependent.
- If a body or system of bodies only has n dofs, it's impossible to remove $N > n$ dofs by adding constraints

Consider the std IK problem for a 6-jointed robot.



- 7 bodies $\Rightarrow$ 42 dof

1 is fixed $\Rightarrow$ 6 constrs

6 jnts remove 5 dof's each $\Rightarrow$ 30 constrs

$\doteq$ 6 dofs

this is tight, ~~since no body~~ ~~but if all jnts~~ ~~were~~ ~~are revolute~~ ~~prismatic~~

Now fix the end effector.

That is what we do when we specify an IK problem.

Link 6 becomes part of ground.

6 bodies $\Rightarrow$ 36 dof

1 fixed $\Rightarrow$ 6 constr

5 jnts $\Rightarrow$ 30 constr

= 0 dof.

correct! IK solns are isolated.

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Consider WAM IK

8 bodies → 48 dof

(base) 1 fixed → 6 constr

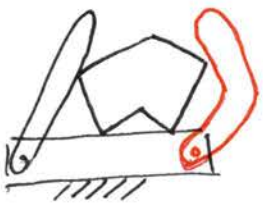
7 jnts → 35 constr

(end eff) 1 fixed → 6 constr ← caused by IK problem

= 1 dof

Apply freedoms & constraints to grasp security

planar example



but contacts can break! How to stop that

Dofs: finger → 1 dof

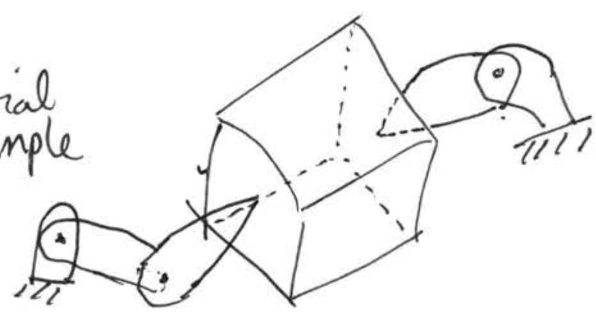
Object → 3 dof

Constrs: fix finger → 1 constr

maintain contact at 3 points → 3 constr

= 0 dof → secure

spatial example



Dofs w/ fingers locked = 6

point of object stays at point on finger → 3 constr

2nd point contact → 3 constr

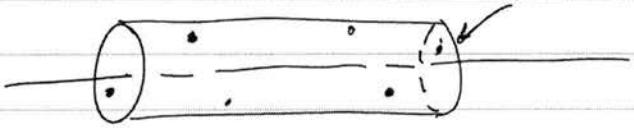
= 1 dof (not zero)

(b) Assuming no obstacles, what does a curve in C-space represent in the world or physical space of the robot.

A continuous motion of the system

② Someone claims to have a form closure grasp of a cylinder using 7 contacts. Is there a way to think about this problem such that the claim makes sense? <u>Explain</u>.

Yes. You can form close 5 dof (excluding rotation about the cylinder's axis. It is reasonable



to ignore rotation since the geom. is independent of it anyway.

Alternatively, if the person meant "frictional form closure," the statement could be correct.

Second alternative: the cylinder is not exactly a cylinder.

From Page 1, 3/14/18, $\boxed{G^T \nu = J\dot{q}}$

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Forward velocity kinematics

Given robot velocity $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$, compute

object "twist" (or velocity) $\nu = \begin{bmatrix} v \\ \omega \end{bmatrix}$

$$\boxed{\nu = G^{-T} J \dot{q}}$$ ← true iff G^{-1} exist

Inverse velocity kinematics

Given desired object twist ν_*, compute the corresponding robot velocity $\dot{q}$.

$$\boxed{\dot{q} = J^{-1} G^T \nu}$$ ← true iff J^{-1} exists

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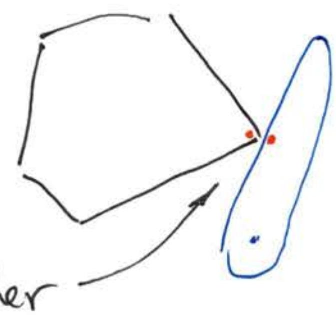
$J\dot{q}$ = twists of contact points on hand

$G^T\nu$ = twists of contact points on object

$G^T\nu = J\dot{q}$ means the contacts are maintained. i.e., contact points on object and hand move together



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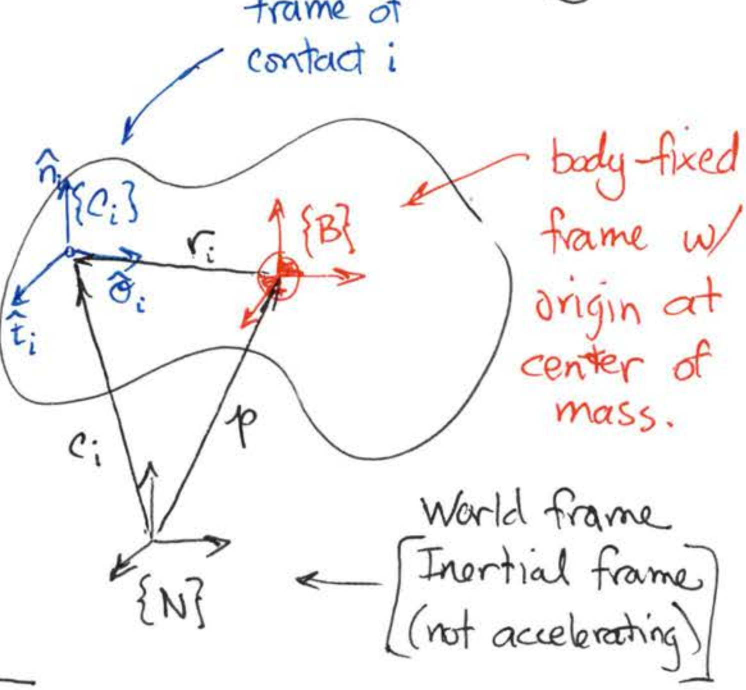
Deriving G^T

p = position of c.g.

$\dot{p} = v$

Given: $(v^N, \omega^N) = \nu^N$

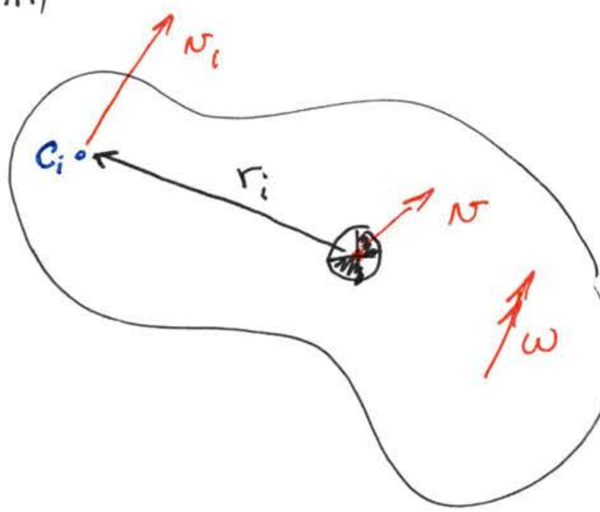


Determine:

$(v^{c_i}, \omega^{c_i}) = \nu^{c_i}$

(Text drops c_i, so ν^{c_i} becomes ν_i)

Transform v to contact point



$$v_{c_i}^N = v^N + \omega^N \times r_i^N$$

translational velocity of contact pt i.

where $r_i^N = c_i^N - p^N$

$$v_{c_i}^N = v^N - S(r_i)\,\omega^N$$

cross product matrix

$$S(r_i) = \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix}$$

$\cancel{v_{c_i}^N = v^N - S(r_i}$

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write in matrix form:

$$v_{c_i}^N = \left[I_{(3\times3)} \; -S(r_i) \right]_{(3\times6)} \begin{bmatrix} v^N \\ \omega^N \end{bmatrix}_{(6\times1)}$$

Now t'form ω to contact point.

No change! (Body is rigid)

$$\omega_{c_i}^N = \omega^N$$

$$\omega_{c_i}^N = \left[O_{(3\times3)} \quad I_{(3\times3)} \right] \begin{bmatrix} v^N \\ \omega^N \end{bmatrix}$$

Put them together to tranform the body twist to the contact point.

$$\underbrace{\begin{bmatrix} v_{c_i}^N \\ \omega_{c_i}^N \end{bmatrix}}_{\nu_{c_i}^N} = \begin{bmatrix} I_{(3\times3)} & -S(r_i^N) \\ O_{(3\times3)} & I_{(3\times3)} \end{bmatrix} \underbrace{\begin{bmatrix} v^N \\ \omega^N \end{bmatrix}}_{\nu^N}$$

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Now express the twist at contact i (on the object) in frame $\{C_i\}$, because it makes it easier to apply the contact models.

Recall $R^A_B v^B = v^A$

Let $R_i = R^N_{c_i}$ represent the orientation of $\{C_i\}$ w.r.t. $\{N\}$

$$R_i = \left[\hat{n}_i^N \; \hat{t}_i^N \; \hat{o}_i^N \right]$$

then $R_i^T v^N_{c_i} = v^{c_i}_{c_i} \triangleq v_{i,obj}$

$R_i^T \omega^N_{c_i} = \omega^{c_i}_{c_i} \triangleq \omega_{i,obj}$

$$\nu_{i,obj} = \underbrace{\begin{bmatrix} v_{i,obj} \\ \omega_{i,obj} \end{bmatrix}}_{(6\times1)} = \underbrace{\underbrace{\begin{bmatrix} R_i^T & 0 \\ 0 & R_i^T \end{bmatrix}}_{(6\times6)}}_{\triangleq \bar{R}_i^T} \underbrace{\underbrace{\begin{bmatrix} I & S(r_i^N)^T \\ 0 & I \end{bmatrix}}_{(6\times6)}}_{\triangleq P_i^T} \underbrace{\begin{bmatrix} v^N \\ \omega^N \end{bmatrix}}_{(6\times1)} = \nu^N \triangleq \nu$$

$$\boxed{\nu_{i,obj} = \tilde{G}_i^T \nu}, \text{ where } \tilde{G}_i^T = \bar{R}_i^T P_i^T$$

$\leftarrow$ equation (28.5) in "Grasping"

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$$\tilde{G}_i^T = \begin{bmatrix} R_i^T & R_i^T \left(S(r_i^N)\right)^T \\ 0 & R_i^N \end{bmatrix}$$

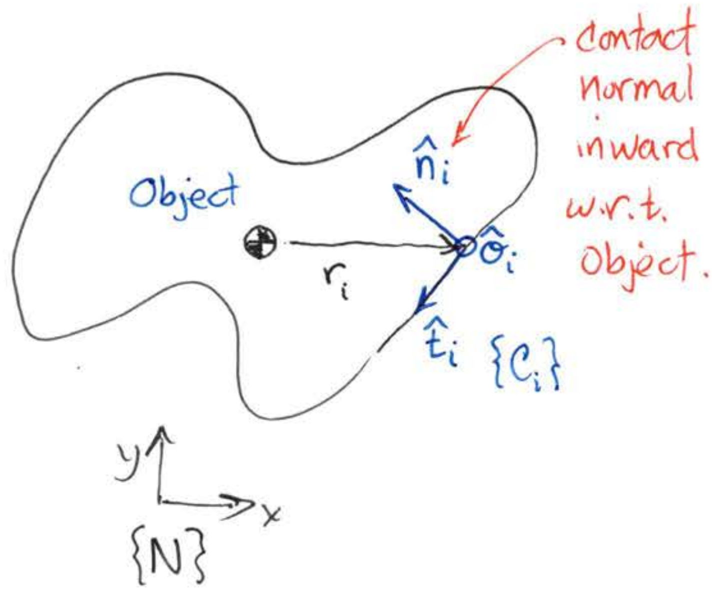
$$= \left[\begin{array}{c|c} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{\theta}_i^T & (r_i \times \hat{\theta}_i)^T \\ 0_{(1\times3)} & \hat{n}_i^T \\ 0_{(1\times3)} & \hat{t}_i^T \\ 0_{(1\times3)} & \hat{\theta}_i^T \end{array}\right]_{(6\times6)} \longleftarrow$$

All quantities are expressed in {N}

Example

$$R_i = \begin{bmatrix} -0.8 & -0.6 & 0 \\ 0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r_i^N = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix}^T$$



$$S^T(r_i^N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$$

Expand $\nu_{i,obj} = \tilde{G}_i^T \nu$

$$\nu_{i,obj} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \\ \hline \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix} = \left[\begin{array}{ccc|ccc} -0.8 & +0.6 & 0 & 0 & 0 & +1.2 \\ -0.6 & -0.8 & 0 & 0 & 0 & -1.6 \\ 0 & 0 & 1 & 0 & -2 & 0 \\ \hline & & & -0.8 & +0.6 & 0 \\ & 0 & & -0.6 & -0.8 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} v_x^N \\ v_y^N \\ v_z^N \\ \hline \omega_x^N \\ \omega_y^N \\ \omega_z^N \end{bmatrix}$$

$$\begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \\ \hline \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix} = \begin{bmatrix} -0.8\, v_x^N + 0.6\, v_y^N + 1.2\, \omega_z^N \\ -0.6\, v_x^N - 0.8\, v_y^N + 1.6\, \omega_z^N \\ v_z^N - 2\omega_y^N \\ \hline -0.8\, \omega_x^N + 0.6\, \omega_y^N \\ -0.6\, \omega_x^N - 0.8\, \omega_y^N \\ \omega_z^N \end{bmatrix}$$

Consider special cases to build intuition

$v_x^N = 1$, all other components of ν are zero.

$\omega_z^N = 1$, " " " " " " "

⋮

etc

⋮

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Extend to multiple contact points on object

$$\nu_{c,obj} \triangleq \begin{bmatrix} \nu_{1,obj} \\ \nu_{2,obj} \\ \vdots \\ \nu_{n_c,obj} \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix}}_{\tilde{G}^T} \underset{(6\times 1)}{\nu}$$

$(6n_c \times 1)$ $\quad$ $(6n_c \times 6)$

where n_c is # of contact points

$$\boxed{\nu_{c,obj} = \tilde{G}^T \nu} \quad \leftarrow \text{eq. (38.12)}$$

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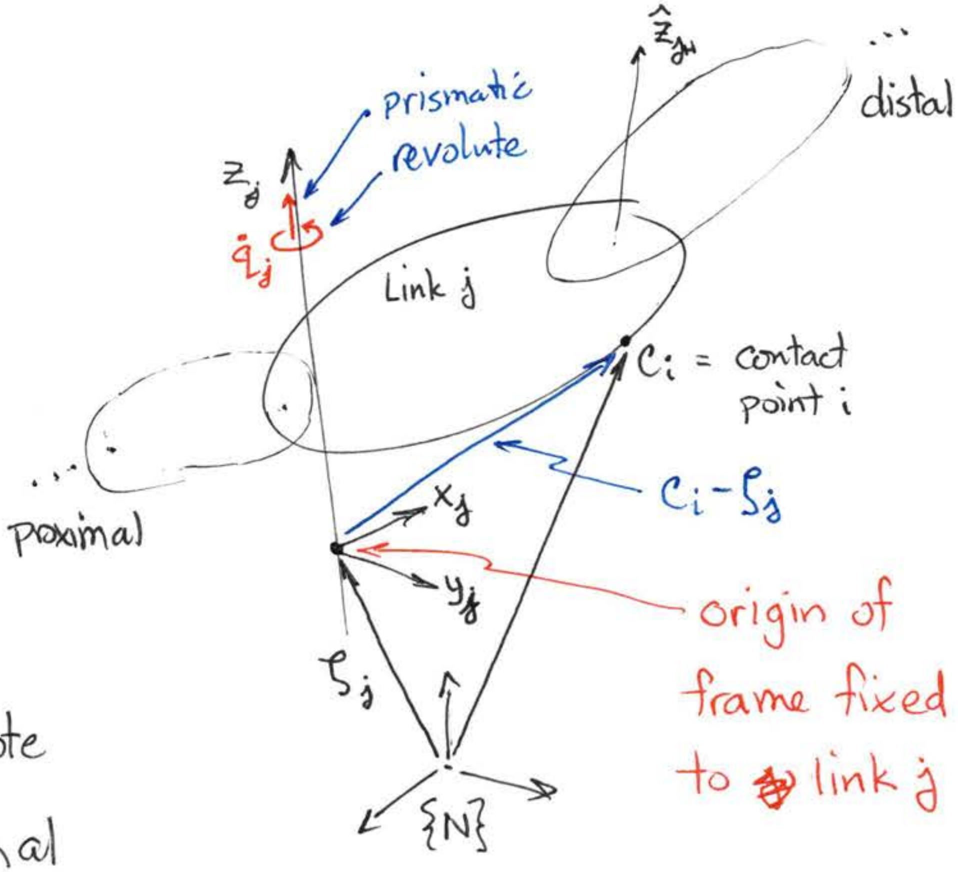
Deriving J

$$\underset{(6\times n_q)}{\tilde{J}_i}\ \underset{(n_q\times 1)}{\dot{q}} = \underset{(6\times 1)}{\nu^{c_i}_{i,hnd}} \triangleq \nu_{i,hnd} \quad \leftarrow \text{Grasping eq. (38.10)}$$

Follow approach used for object; first map joint velocities to hand contact points, expressed in $\{N\}$, then t'form to $\{C_i\}$.



Let ν_{ij} denote the translational velocity of pt. c_i induced by motion of joint j.

$$v_{ij}^N = \begin{cases} 0_{(3\times1)} & \leftarrow \text{joint } j \text{ is distal to contact } i \\ \hat{z}_j^N \dot{q}_j & \leftarrow \text{joint } j \text{ is prismatic} \\ \hat{z}_j^N \dot{q}_j \times (c_i^N - \rho_j^N) & \leftarrow \text{joint } j \text{ is revolute} \end{cases}$$

Rearrange into form that exposes $\dot{q}_j$ nicely

$$-(c_i^N - \rho_j^N) \times \hat{z}_j^N \dot{q}_j$$

$$= -S(c_i - \rho_j)\hat{z}_j^N \dot{q}_j = S^T(c_i - \rho_j)\hat{z}_j^N \dot{q}_j$$

Now $\boxed{v_{ij}^N = d_{ij}^N \dot{q}_j}$, where d_{ij}^N is ...

$$d_{ij}^N = \begin{cases} 0_{(3\times1)} & \leftarrow j \text{ distal to } i \\ \hat{z}_j^N & \leftarrow j \text{ prismatic} \\ S^T(c_i^N - \rho_j^N)\,\hat{z}_j^N & \leftarrow j \text{ revolute} \end{cases}$$

Map joint velocities to angular vel. of contact points

$\boxed{\omega_{ij}^N = l_{ij}^N \dot{q}_j}$ where $l_{ij}^N = \begin{cases} 0_{(3\times1)} & \leftarrow j \text{ distal to } i \\ 0_{(3\times1)} & \leftarrow j \text{ prismatic} \\ \hat{z}_j^N & \leftarrow j \text{ revolute} \end{cases}$

Add the effects of all joints at contact i:

$$\nu^N_{i,hnd} = \underbrace{\begin{bmatrix} d^N_{i,1} & d^N_{i,2} & \cdots & d^N_{i,n_q} \\ l^N_{i,1} & l^N_{i,2} & \cdots & l^N_{i,n_q} \end{bmatrix}}_{Z_i} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_{n_q} \end{bmatrix} = Z_i \dot{q}$$

Now t'form $\nu^N_{i,hnd}$ to $\{C_i\}$ as before

$$\underbrace{\bar{R}_i^T Z_i}_{\triangleq \tilde{J}_i} \dot{q} = \nu^{C_i}_{i,hnd} \triangleq \nu_{i,hnd}$$

$$\boxed{\nu_{i,hnd} = \tilde{J}_i \dot{q}} \qquad \text{eq. (38.10)}$$

Extend to case of multiple contacts.

$$\boxed{\nu_{c,hnd} = \tilde{J} \dot{q}} \qquad \text{eq. (38.13)}$$

where $\nu_{c,hnd} \triangleq \begin{bmatrix} \nu_{1,hnd} \\ \nu_{2,hnd} \\ \vdots \\ \nu_{n_c,hnd} \end{bmatrix}$ $(6n_c \times 1)$ $\qquad \tilde{J} \triangleq \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}$ $(6n_c \times n_q)$

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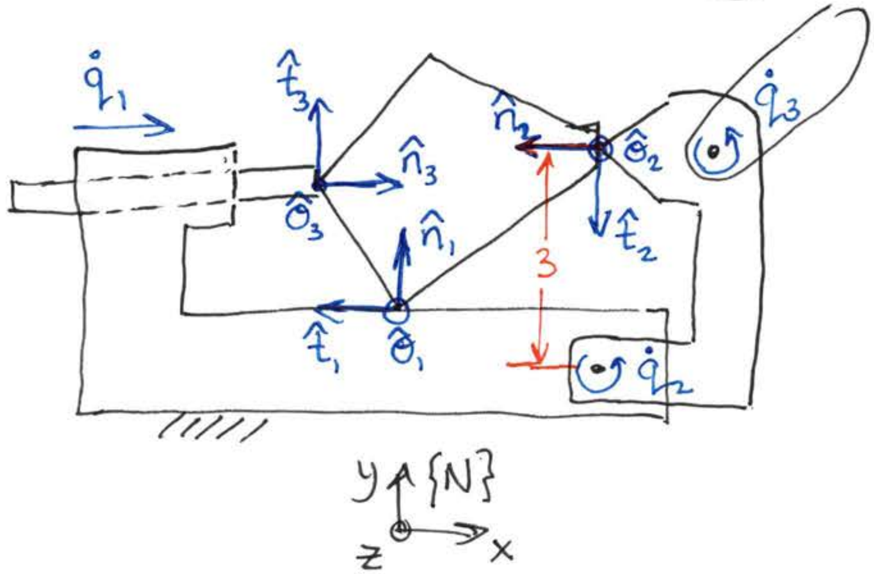
Example: Hand Jacobian

$$R_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{z}_1^N = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad z_2^N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \hat{z}_3^n$$

joint axis directions

Contact 1

No joint affects contact 1, $\therefore$ $Z_1 = 0_{(6\times3)}$

$\therefore$ $\underbrace{\bar{R}_1^T Z_1}$ $= 0_{(6\times3)}$

$\therefore$ $\tilde{J}_1 = 0_{(6\times3)}$

$$\nu_{1,hnd} = \tilde{J}_1 \dot{q}$$

$$\begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \nu_{1\theta} \\ \omega_{1n} \\ \omega_{1t} \\ \omega_{1\theta} \end{bmatrix} = \begin{bmatrix} \\ & 0_{(6\times3)} & \\ \\ \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Contact 2

$$Z_2 = \begin{bmatrix} d_{2,1}^N & d_{2,2}^N & d_{2,3}^N \\ \ell_{2,1}^N & \ell_{2,2}^N & \ell_{2,3}^N \end{bmatrix} \qquad R_2 = \begin{bmatrix} I_{(3\times3)} \end{bmatrix}$$

all zeros, $0_{(6\times1)}$, because motion of $\dot{q}_1$, $\dot{q}_3$ does not cause motion of contact point 2 on the hand.

$$d_{2,2}^N = S^T(c_2^N - \xi_2^N)\,\hat{z}_2^N \qquad c_2^N - \xi_2^N = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$d_{2,2}^N = \begin{bmatrix} 0 & 0 & +3 \\ 0 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} +3 \\ 0 \\ 0 \end{bmatrix}$$

$$\ell_{2,2}^N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{J}_2 = \bar{R}_2^T Z_2 = I_{(6\times6)} \left[\begin{array}{c|c|c} 0 & \begin{matrix} +3 \\ 0 \\ 0 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} & 0 \end{array}\right] = \tilde{J}_2$$

$$\nu_{2,hnd} = \tilde{J}_2 \, \dot{q}$$

$$\begin{bmatrix} \nu_{2x} \\ \nu_{2y} \\ \nu_{2z} \\ \omega_{2x} \\ \omega_{2y} \\ \omega_{2z} \end{bmatrix} = \left[\begin{array}{c|c|c} & +3 & \\ & 0 & \\ 0 & 0 & 0 \\ & 0 & \\ & 0 & \\ & 1 & \end{array} \right] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \dot{q}_2 \begin{bmatrix} +3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Check this result against figure. Does it make sense?

Contact 3

$$Z_3 = \begin{bmatrix} d_{3,1}^N & \underbrace{d_{3,2}^N \quad d_{3,3}^N} \\ l_{3,1}^N & l_{3,2}^N \quad l_{3,3}^N \end{bmatrix} = \left[\begin{array}{c|c} 1 & \\ 0 & \\ 0 & 0_{(6\times2)} \\ 0 & \\ 0 & \\ 0 & \end{array} \right]$$

no effect of c_3 on hand.

$$R_3 = \begin{bmatrix} +1 & & 0 \\ & +1 & \\ 0 & & +1 \end{bmatrix}$$

$$\therefore \quad \tilde{J}_3 = \left[\begin{array}{c|c} +1 & \\ 0 & \\ 0 & 0_{(6\times2)} \\ 0 & \\ 0 & \\ 0 & \end{array} \right] \qquad \nu_{3,hnd} = \begin{bmatrix} +1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{q}_1$$