

Grasping: Chapter 38: Springer
Handbook of Robotics

3/14/18

①

Our goals:

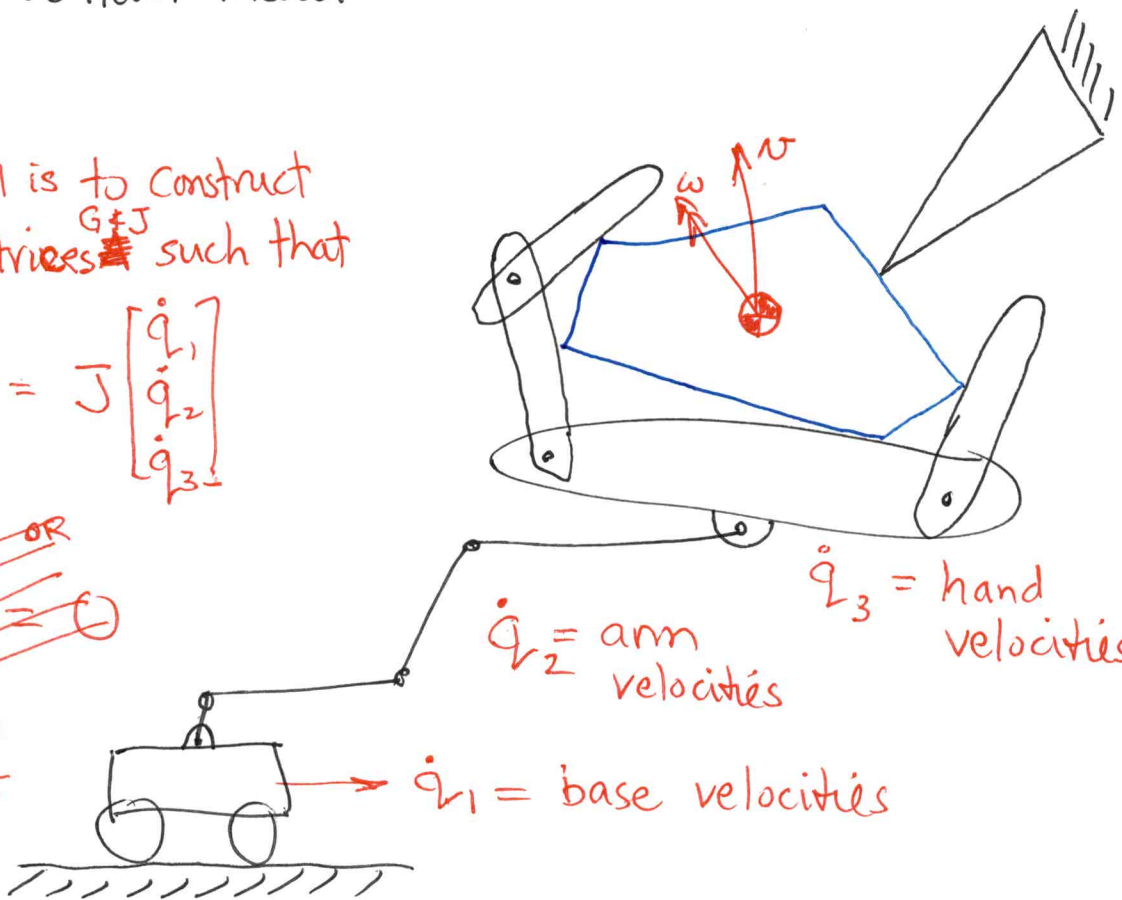
- Build models of grasping systems (e.g., object & hand) sufficient for designing, planning, & executing grasping and dexterous manipulation
- Grasp security analysis – “form closure” and “force closure”
- Fwd and inv. velocity kinematics
- Contact models

Goal is to construct matrices ~~G~~^{G & J} such that

$$G^T \begin{bmatrix} v \\ \omega \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

~~$A \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = 0$~~

~~$A = [J \quad -G^T]$~~

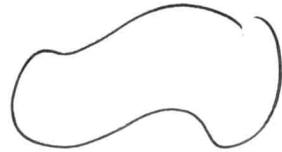


3/18/18

Position Kinematics of Grasps

(1)

Object to grasp has 6 dof. (3 d.o.f. in plane)



Connect it to ground thru revolute joint.

How many dof.s
now? 1.



Count constraints:

planar case: p on body is fixed in the plane

$$\left. \begin{aligned} p_x = \text{const} = A \\ p_y = \text{const} = B \end{aligned} \right\} \begin{array}{l} 3 \text{ d.o.f} - 2 \text{ constr} \\ \text{equations} = 1 \text{ d.o.f.} \end{array}$$

spatial case: p on body is fixed in space

point on }
jnt axis } $\rightarrow p_x = A, p_y = B, p_z = C$

$${}^N R_B \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} r_{13}(\alpha, \beta, \gamma) \\ r_{23}(\alpha, \beta, \gamma) \\ r_{33}(\alpha, \beta, \gamma) \end{bmatrix} = \begin{bmatrix} {}^N \hat{z}_x \\ {}^N \hat{z}_y \\ {}^N \hat{z}_z \end{bmatrix}$$

$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \text{two more constraint equations}$
 $\leftarrow \text{ (not 3, because if 2 values are known, the other can be computed.)}$
 $\leftarrow \text{ known numbers}$

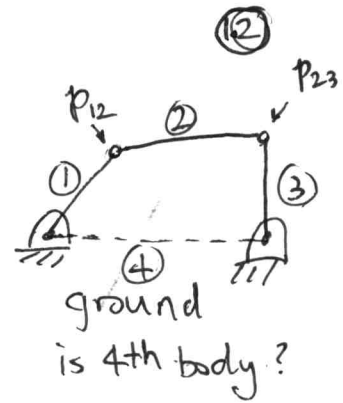
3/18/18

Consider a four-bar linkage:

planar case:

How many d.o.f.s w/o joints?

$$4 \text{ bodies} \times 3 \text{ d.o.f.s/body} = 12 \text{ d.o.f.s}$$



Now consider joints:

ground can't move \leftarrow 3 constraints

pt on link 1 is fixed \leftarrow 2 constrs

pt of link 3 is fixed \leftarrow 2 constr

pt of link 1 is fixed to pt of link 2 \leftarrow 2 constr

pt of link 3 is fixed to pt of link 2 \leftarrow 2 constr

+

Add to find # of d.o.f.s
lost due to constrs.

11

$$\text{Remaining d.o.f.s} \geq 12 - 11 = 1$$

actually, this is not an equality because sometimes constraints are dependent.

spatial case:

$$4 \text{ bodies} \times 6 \text{ d.o.f.s/body} = 24 \text{ d.o.f.s}$$

ground imposes 6 constr

each joint imposes 5 constr

$$\text{d.o.f.s} \geq 24 - 6 - 4 \cdot 5 \geq -2 \quad \leftarrow \text{Does this make sense?}$$

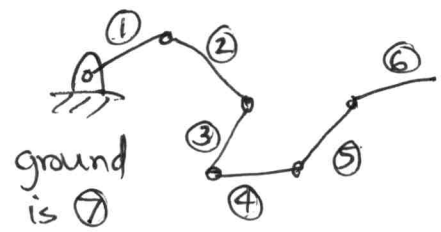
3/18/18

Really there is 1 dof.

(13)

- This means of the 20 constraint equations associated with the 4 joints, 3 are dependent.
- If a body or system of bodies only has n dofs, it's impossible to remove $N > n$ dofs by adding constraints

Consider the std IK problem for a 6-jointed robot.



• 7 bodies \Rightarrow 42 dof
 1 is fixed \Rightarrow 6 constrs
 6 jnts remove
 5 dofs each \Rightarrow 30 constrs

 \Rightarrow 6 dofs

Now fix the end effector.
 That is what we do when we specify an IK problem.
 Link 6 becomes part of ground.

6 bodies \Rightarrow 36 dof
 1 fixed \Rightarrow 6 constr
 6 jnts \Rightarrow 30 constr

 $=$ 0 dof.

↑ this is tight,
~~since no body~~
~~but if all joints~~
~~have been~~
~~are revolute~~
~~prismatic~~

← correct! IK solns are isolated.

3/18/18

(14)

Consider WAM IK

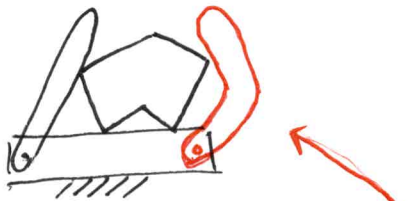
8 bodies \rightarrow 48 dof
 (base) 1 fixed \rightarrow 6 constr
 7 jnts \rightarrow 35 constr
 (end eff) 1 fixed \rightarrow 6 constr

$= 1$ dof

\leftarrow caused by IK problem

Apply freedoms & constraints to grasp security

planar example



but contacts can break! How to stop that

Dofs: finger \rightarrow 1 dof

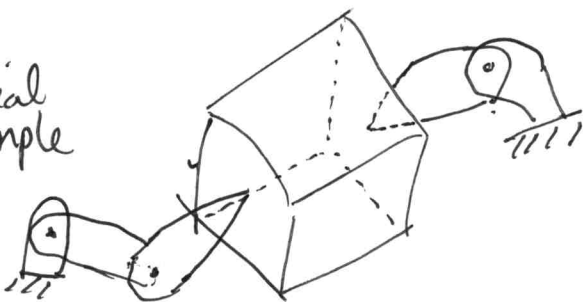
Object \rightarrow 3 dof

Constrs: Fix finger \rightarrow 1 constr

maintain contact at \rightarrow 3 constr
3 points

$= 0$ dof \Rightarrow secure

spatial example



Dofs w/ fingers locked = 6

point of object stays at point on finger \rightarrow 3 constr

2nd point contact \rightarrow 3 constr

$= 1$ dof (not zero)

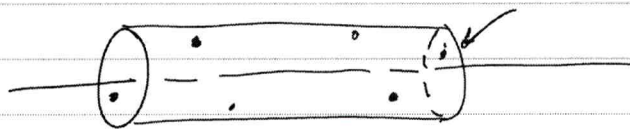
(b) Assuming no obstacles, what does a curve in C -space represent in the world or physical space of the robot,

A continuous motion of the system

② Someone claims to have a form closure grasp of a cylinder using 7 contacts.

Is there a way to think about this problem such that the claim makes sense? Explain.

Yes. You can form close 5 dof (excluding rotation about the cylinder's axis. It is reasonable



to ignore rotation since the geom. is independent of it anyway.

Alternatively, if the person meant "frictional form closure," the statement could be correct.

Second alternative: the cylinder is not exactly a cylinder.

From Page 1, 3/14/18,

$$\boxed{G^T v = J \dot{q}}$$

3/14/18

Forward velocity kinematics

(2)

Given robot velocity $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$, compute

object "twist" (or velocity) $v = \begin{bmatrix} v \\ \omega \end{bmatrix}$

$$\boxed{v = G^{-T} J \dot{q}} \quad \leftarrow \text{true iff } G^{-1} \text{ exist}$$

Inverse velocity kinematics

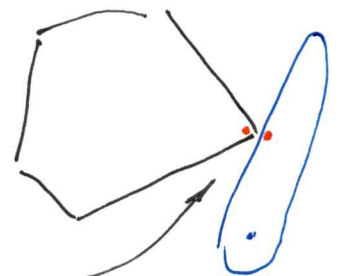
Given desired object twist v , compute the corresponding robot velocity \dot{q} .

$$\boxed{\dot{q} = J^{-1} G^T v} \quad \leftarrow \text{true iff } J^{-1} \text{ exists}$$

$J \dot{q}$ = twists of contact points on hand

$G^T v$ = twists of contact points on object

~~Hand~~
 $G^T v = J \dot{q}$ means the contacts are maintained. i.e., contact points on object and hand move together



3/14/18

(3)

Deriving G^T

p = position of c.g.

$\dot{p} = N$

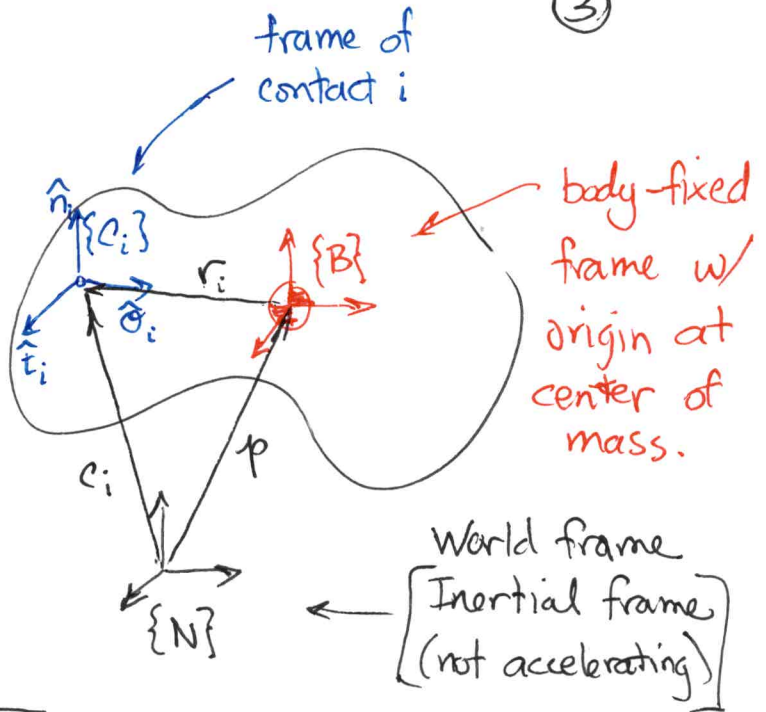
Given: $(N^N, \omega^N) = v^N$

Determine:

$$(N^{c_i}, \omega^{c_i}) = v^{c_i}$$

(Text drops c_i , so

v^{c_i} becomes v_i)

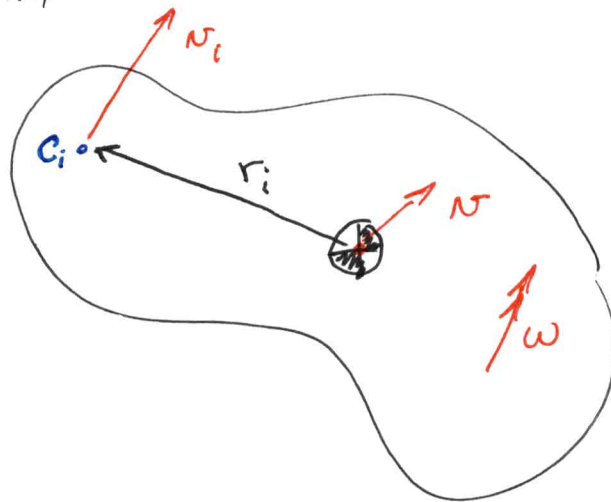


Transform N to ~~the~~ contact point

$$N_{c_i}^N = N^N + \omega^N \times r_i^N$$

↳ translational velocity of contact pt i .

where $r_i^N = c_i^N - p^N$



$$N_{c_i}^N = N^N - S(r_i) \omega^N$$

↳ cross product matrix

$$S(r_i) = \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix}$$

~~$N_{c_i}^N = N^N - S(r_i)$~~

write in matrix form:

3/14/18

(4)

$$N_{c_i}^N = \begin{bmatrix} I_{(3 \times 3)} & -S(r_i) \\ 0_{(3 \times 3)} & I_{(3 \times 3)} \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix}$$

Now t'form ω to contact point.

No change! (Body is rigid)

$$\omega_{c_i}^N = \omega^N$$

$$\omega_{c_i}^N = \begin{bmatrix} 0_{(3 \times 3)} & I_{(3 \times 3)} \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix}$$

Put them together to transform the body twist to the contact point.

$$\underbrace{\begin{bmatrix} N_{c_i}^N \\ \omega_{c_i}^N \end{bmatrix}}_{V_{c_i}^N} = \begin{bmatrix} I_{(3 \times 3)} & -S(r_i) \\ 0_{(3 \times 3)} & I_{(3 \times 3)} \end{bmatrix} \underbrace{\begin{bmatrix} N^N \\ \omega^N \end{bmatrix}}_{V^N}$$

3/14/18

⑤

Now express the twist at contact i

(on the object) in frame $\{C_i\}$, because

it makes it easier to apply the contact models.

Recall $R_B^A v^B = v^A$

Let $R_i = R_{C_i}^N$ represent the orientation of $\{C_i\}$ w.r.t. $\{N\}$

$$R_i = \begin{bmatrix} \hat{n}_i^N & \hat{t}_i^N & \hat{o}_i^N \end{bmatrix}$$

then $R_i^T N_{C_i}^N = N_{C_i}^{C_i} \triangleq N_{i,obj}$

$R_i^T \omega_{C_i}^N = \omega_{C_i}^{C_i} \triangleq \omega_{i,obj}$

$$v_{i,obj} = \begin{bmatrix} N_{i,obj} \\ \omega_{i,obj} \end{bmatrix} = \underbrace{\begin{bmatrix} R_i^T & 0 \\ 0 & R_i^T \end{bmatrix}}_{\triangleq \bar{R}_i^T} \underbrace{\begin{bmatrix} I & S(r_i^N)^T \\ 0 & I \end{bmatrix}}_{\triangleq P_i^T} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix} = v^N \triangleq v$$

$v_{i,obj} = \tilde{G}_i^T v$

where $\tilde{G}_i^T = \bar{R}_i^T P_i^T$

← equation (28.5) in "Grasping"

3/14/18

⑥

$$\tilde{G}_i^T = \begin{bmatrix} R_i^T & R_i^T (S(r_i^N))^T \\ 0 & R_i^N \end{bmatrix}$$

$$= \begin{bmatrix} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{\sigma}_i^T & (r_i \times \hat{\sigma}_i)^T \\ 0_{(1 \times 3)} & \hat{n}_i^T \\ 0_{(1 \times 3)} & \hat{t}_i^T \\ 0_{(1 \times 3)} & \hat{\sigma}_i^T \end{bmatrix}_{(6 \times 6)}$$

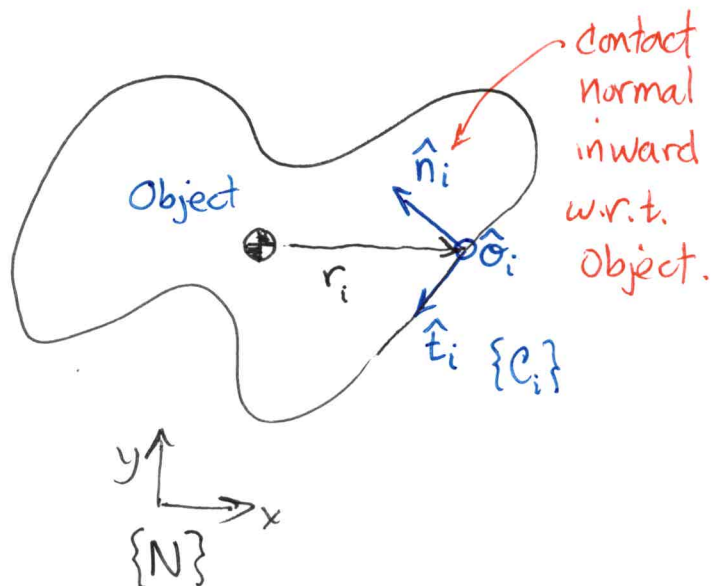
← All quantities are expressed in $\{N\}$

Example

$$R_i = \begin{bmatrix} -0.8 & -0.6 & 0 \\ 0.6 & -0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r_i^N = [2 \ 0 \ 0]^T$$

$$S^T(r_i^N) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$$



3/14/18

Expand $v_{i,obj} = \tilde{G}_i^T v$

(7)

$$v_{i,obj} = \begin{bmatrix} N_{in} \\ N_{it} \\ N_{io} \\ \hline \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 & 0 & 0 & +1.2 \\ 0.6 & -0.8 & 0 & 0 & 0 & -1.6 \\ 0 & 0 & 1 & 0 & -2 & 0 \\ \hline & & & 0.8 & 0.6 & 0 \\ & & & -0.6 & -0.8 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_x^N \\ N_y^N \\ N_z^N \\ \hline \omega_x^N \\ \omega_y^N \\ \omega_z^N \end{bmatrix}$$

$$\begin{bmatrix} N_{in} \\ N_{it} \\ N_{io} \\ \hline \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix} = \begin{bmatrix} -0.8 N_x^N + 0.6 N_y^N + 1.2 \omega_z^N \\ -0.6 N_x^N - 0.8 N_y^N + 1.6 \omega_z^N \\ N_z^N - 2\omega_y^N \\ \hline -0.8 \omega_x^N + 0.6 \omega_y^N \\ -0.6 \omega_x^N - 0.8 \omega_y^N \\ \omega_z^N \end{bmatrix}$$

special

Consider cases to build intuition

 $N_x^N = 1$, all other components of v are zero. $\omega_z^N = 1$, " " " " " " " "

⋮

etc

⋮

3/14/18

(7.1)

Extend to multiple contact points on object

$$\underset{(6 \times 1)}{v_{c,obj}} \stackrel{\Delta}{=} \begin{bmatrix} v_{1,obj} \\ v_{2,obj} \\ \vdots \\ v_{n_c,obj} \end{bmatrix} = \begin{bmatrix} \tilde{G}_1^T \\ \tilde{G}_2^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix} \underset{(6 \times 1)}{v} \leftarrow \tilde{G}^T \underset{(6 \times 6)}{v}$$

where n_c is # of contact points

$$\boxed{v_{c,obj} = \tilde{G}^T v} \leftarrow \text{eq. (38.12)}$$

3/14/18

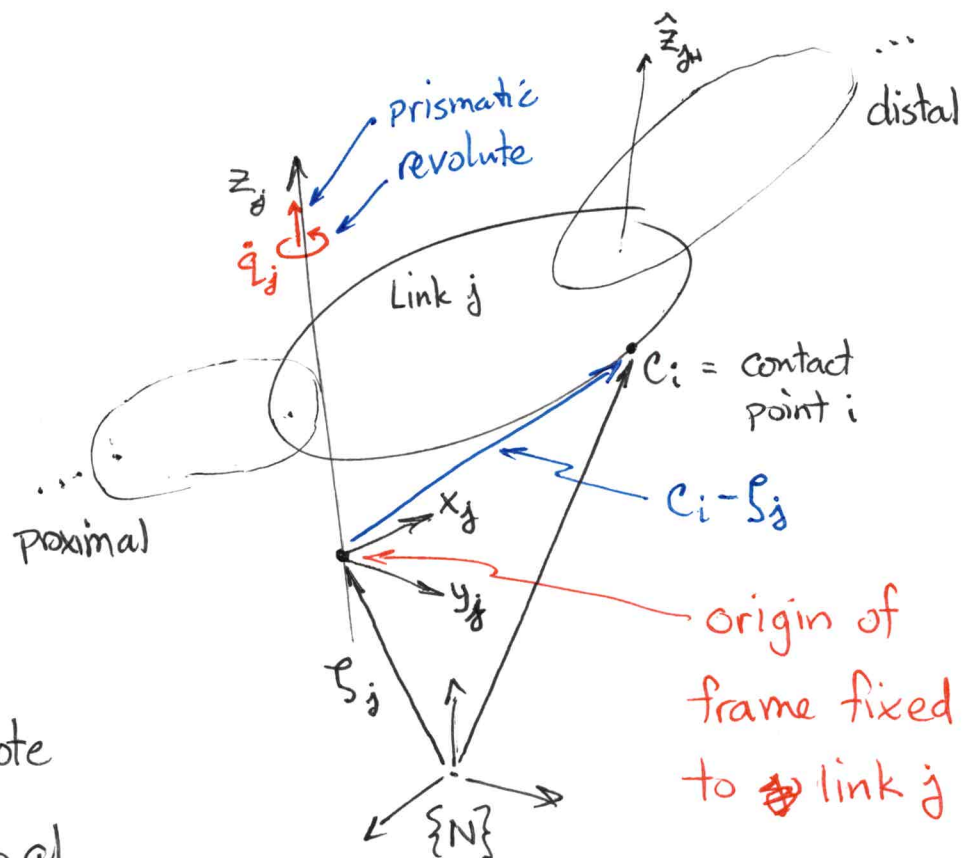
(8)

Deriving J

$$\tilde{J}_i \dot{q}_j = v_{i,hnd}^e \triangleq v_{i,hnd} \quad \leftarrow \text{Grasping eq. (38.10)}$$

$$\begin{array}{ccc} \swarrow & \swarrow & \swarrow \\ (6 \times n_q) & (n_q \times 1) & (6 \times 1) \end{array}$$

Follow approach used for object; first map joint velocities to hand contact points, expressed in $\{N\}$, then transform to $\{C_i\}$.



Let N_{ij} denote the translational velocity of pt. e_i induced by motion of joint j .

3/14/18

$$N_{ij}^N = \begin{cases} O_{(3 \times 1)} & \leftarrow \text{joint } j \text{ is distal to contact } i \quad \textcircled{9} \\ \hat{z}_j^N \dot{q}_j & \leftarrow \text{joint } j \text{ is prismatic} \\ \hat{z}_j^N \dot{q}_j \times (c_i^N - p_j^N) & \leftarrow \text{joint } j \text{ is revolute} \end{cases}$$

Rearrange into form that exposes \dot{q}_j nicely

$$\begin{aligned} & -(c_i^N - p_j^N) \times \hat{z}_j^N \dot{q}_j \\ & = -S(c_i^N - p_j^N) \hat{z}_j^N \dot{q}_j = S^T(c_i^N - p_j^N) \hat{z}_j^N \dot{q}_j \end{aligned}$$

Now $N_{ij}^N = d_{ij}^N \dot{q}_j$, where d_{ij}^N is ...

$$d_{ij}^N = \begin{cases} O_{(3 \times 1)} & \leftarrow j \text{ distal to } i \\ \hat{z}_j^N & \leftarrow j \text{ prismatic} \\ S^T(c_i^N - p_j^N) \hat{z}_j^N & \leftarrow j \text{ revolute} \end{cases}$$

Map joint velocities to angular vel. of contact points

$$\omega_{ij}^N = l_{ij}^N \dot{q}_j \quad \text{where } l_{ij}^N = \begin{cases} O_{(3 \times 1)} & \leftarrow j \text{ distal to } i \\ O_{(3 \times 1)} & \leftarrow j \text{ prismatic} \\ \hat{z}_j^N & \leftarrow j \text{ revolute} \end{cases}$$

3/14/18

Add the effects of all joints at contact:

(10)

$$v_{i,hnd}^N = \underbrace{\begin{bmatrix} d_{i,1}^N & d_{i,2}^N & \dots & d_{i,n_q}^N \\ l_{i,1}^N & l_{i,2}^N & \dots & l_{i,n_q}^N \end{bmatrix}}_{Z_i} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_{n_q} \end{bmatrix} = Z_i \dot{q}$$

Now transform $v_{i,hnd}^N$ to $\{C_i\}$ as before

$$\underbrace{\bar{R}_i^T}_{\approx \tilde{J}_i} Z_i \dot{q} = v_{i,hnd}^{C_i} \triangleq v_{i,hnd}$$

$$\triangleq \tilde{J}_i$$

$$\boxed{v_{i,hnd} = \tilde{J}_i \dot{q}}$$

eq. (38.10)

Extend to case of multiple contacts.

$$\boxed{v_{c,hnd} = \tilde{J} \dot{q}} \quad \text{eq. (38.13)}$$

where

$$v_{c,hnd} \triangleq \begin{bmatrix} v_{1,hnd} \\ v_{2,hnd} \\ \vdots \\ v_{n_c,hnd} \end{bmatrix} \quad \tilde{J} \triangleq \begin{bmatrix} \tilde{J}_1 \\ \tilde{J}_2 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}$$

(6n_c x 1) (6n_c x n_q)

3/14/18

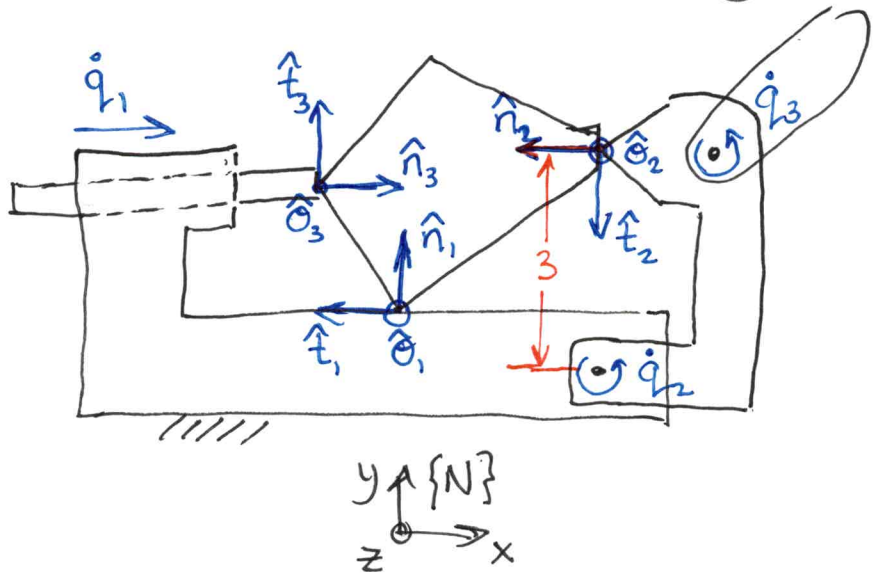
(11)

Example: Hand Jacobian

$$R_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\hat{z}_1^N = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad z_2^N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \hat{z}_3^N$$

joint axis directions

Contact 1

No joint affects

contact 1, $\therefore Z_1 = O_{(6 \times 3)}$

$$\therefore \bar{R}_1^T Z_1 = O_{(6 \times 3)}$$

$$\therefore \tilde{J}_1 = O_{(6 \times 3)}$$

$$v_{i, \text{hnd}} = \tilde{J}_i \dot{q}$$

$$\begin{bmatrix} v_{1n} \\ v_{1t} \\ v_{1\theta} \\ \omega_{1n} \\ \omega_{1t} \\ \omega_{1\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{(6 \times 3)} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3/14/18

(12)

Contact 2

$$Z_2 = \begin{bmatrix} d_{2,1}^N & d_{2,2}^N & d_{2,3}^N \\ l_{2,1}^N & l_{2,2}^N & l_{2,3}^N \end{bmatrix} \quad R_2 = \begin{bmatrix} I_{(3 \times 3)} \end{bmatrix}$$

all zeros, $O_{(6 \times 1)}$, because motion of \dot{q}_1, \dot{q}_3 does not cause motion of contact point 2 on the hand.

$$d_{2,2}^N = S^T (c_2^N - \xi_2^N) \hat{z}_2^N \quad c_2^N - \xi_2^N = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$d_{2,2}^N = \begin{bmatrix} 0 & 0 & +3 \\ 0 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} +3 \\ 0 \\ 0 \end{bmatrix}$$

$$l_{2,2}^N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{J}_2 = \bar{R}_2^T Z_2 = I_{(6 \times 6)} \begin{bmatrix} \text{O} & +3 & \text{O} \\ \text{O} & 0 & \text{O} \\ \text{O} & 0 & \text{O} \\ \text{O} & 0 & \text{O} \\ \text{O} & 0 & \text{O} \\ \text{O} & 1 & \text{O} \end{bmatrix} = \tilde{J}_2$$

3/14/18

(13)

$$V_{2,hnd} = \tilde{J}_2 \dot{q}$$

$$\begin{bmatrix} N_{2x} \\ N_{2y} \\ N_{2z} \\ \omega_{2x} \\ \omega_{2y} \\ \omega_{2z} \end{bmatrix} = \begin{bmatrix} +3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \dot{q}_2 \begin{bmatrix} +3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Check this result against figure.
Does it make sense?

Contact 3

$$Z_3 = \begin{bmatrix} d_{3,1}^N & d_{3,2}^N & d_{3,3}^N \\ l_{3,1}^N & l_{3,2}^N & l_{3,3}^N \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \circlearrowleft_{(6 \times 2)} \end{bmatrix}$$

no effect of c_3 on hand.

$$R_3 = \begin{bmatrix} +1 & & \\ & +1 & \\ & & +1 \end{bmatrix}$$

$$\therefore \tilde{J}_3 = \begin{bmatrix} +1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \circlearrowleft_{(6 \times 2)} \end{bmatrix}$$

$$V_{3,hnd} = \begin{bmatrix} +1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \dot{q}_1$$