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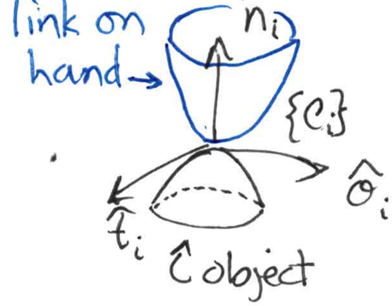
Contact Modeling for Grasping

Most easily thought about & expressed in contact frames.

Relative velocity at contact i.



$$\nu_{cc,i} = \nu_{i,hand} - \nu_{i,obj}$$

$$\nu_{cc,i} = \tilde{J}_i \dot{q} - \tilde{G}_i^T \nu$$

$$\nu_{cc,i} = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \\ \omega_{it} \\ \omega_{io} \end{bmatrix}$$

ν_{in} ← setting = to zero means contact cannot separate

ν_{it}, ν_{io} ← setting = to zero means contact cannot slide

ω_{in} ← setting = to zero means bodies can't twist relative to each other about the contact normal.

$$\nu_{in} = 0 \implies \underbrace{[1\ 0\ 0 \vdots 0\ 0\ 0]}_{\substack{\text{constraint} \\ \text{selection} \\ \text{matrix}}} \nu_{cc,i} = 0$$

$(\kappa_i \times 6)$

κ_i = # constraints

κ_i here is 1

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②

3 Standard Models : PwoF, HF, SF

PwoF - point w/o friction

- set $v_{in} = 0$ ← recall v_{in} is relative velocity of object & hand at the contact point.
- let other five comp. of relative twist be free
- remove one dof from hand/object system by enforcing 1 equation

$$H_i \nu_{cc,i} = v_{in} = 0$$

$$\text{where } H_i = [1\ 0\ 0\ 0\ 0\ 0] \quad \leftarrow l_i = 1$$

HF - hard finger (point with friction)

- set $v_{in} = v_{it} = v_{io} = 0$
- let other 3 rel. twists be free
- remove 3 dofs from system by enforcing

$$H_i \nu_{cc,i} = \begin{bmatrix} v_{in} \\ v_{it} \\ v_{io} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{where } H_i = \left[I_{3\times3} \,\vdots\, 0_{3\times3} \right] \quad \leftarrow l_i = 3$$

SF - Soft Finger

- set $\nu_{in} = \nu_{it} = \nu_{io} = \omega_{in} = 0$
- let other 2 rel. twist components be free
- remove 4 dofs from system by

$$H_i \nu_{cc,i} = \begin{bmatrix} \nu_{in} \\ \nu_{it} \\ \nu_{io} \\ \omega_{in} \end{bmatrix} = 0_{4\times1}$$

where $H_i = \left[I_{4\times4} \;\vdots\; 0_{4\times2} \right]$ $\leftarrow$ $k_i = 4$

$(k_i \times 6)$

Kinematic grasp modeling process

1.) Choose contact types that are reasonable for your situation

2.) Enforce the contact types by

$$H_i \nu_{cc,i} = 0 \qquad \forall i$$

$(k_i \times 6)$ where k_i is the number of dofs removed

$$H_i(\nu_{i,hnd} - \nu_{i,obj}) = 0$$

$$H_i \begin{bmatrix} \tilde{J}_i & -\tilde{G}_i^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

$$\begin{bmatrix} J_i & -G_i^T \end{bmatrix} \begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$

k_i equations in $n_q + n_\nu$ unknowns

where $J_i = H_i \tilde{J}_i$ & $G_i^T = H_i \tilde{G}_i^T$

Put all contacts together

$$H\left(\nu_{c,hnd} - \nu_{c,obj}\right) = 0$$

where $H = \mathrm{diag}(H_1, H_2, \dots, H_{n_c})$

$$H\begin{bmatrix}\tilde{J} & -\tilde{G}^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0$$

$$\begin{bmatrix}J & -G^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0 \quad \leftarrow \left\{\begin{array}{l}\sum_{i=1}^{n_c} \kappa_i \text{ equations}\\ \text{in } n_q + n_\nu \text{ unknowns.}\end{array}\right.$$

where $J = H\tilde{J}$ & $G^T = H\tilde{G}^T$

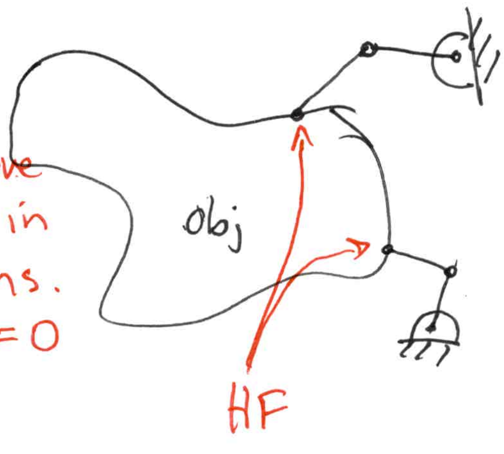
Planar Example.

Physical interpretation:

$$\begin{bmatrix}J & -G^T\end{bmatrix}\begin{bmatrix}\dot{q}\\ \nu\end{bmatrix} = 0 \quad \leftarrow \text{set } \dot{q} = 0$$

then we have 4 equations in 3 unknowns. $\Rightarrow \nu = 0$

HF, HF in planar grasp



Constraints enforce sticking point contact. Object ~~will~~ cannot slip out, because we are not yet ~~[illegible]~~ using a reasonable contact model w/ friction.

$$\begin{bmatrix} J & -G^T \end{bmatrix}\begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$ ← 4 equation ← 7 variables (3 variables if $\dot{q}=0$)

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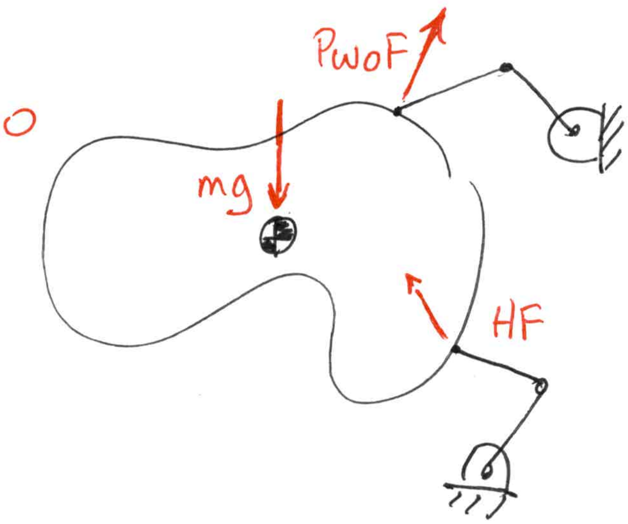
Suppose we replace one HF contact model with PwoF ?



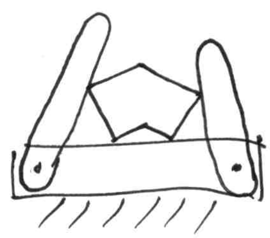
Same result. if $\dot{q}=0 \rightarrow \nu=0$

Enforcing HF removed 2 dof from object.

Enforcing PwoF removes 1 dof; none left.

However, for the grasp to be sustained, the force applied to object by PwoF is pulling, but frictionless contacts can't pull.

Planar grasp w/ 4 PwoF contacts.



The 4 contacts remove 4 dofs so the object can't move ?

$$\begin{bmatrix} J & -G^T \end{bmatrix}\begin{bmatrix} \dot{q} \\ \nu \end{bmatrix} = 0$$ ← 4 equations in 5 variables

1 dof left !

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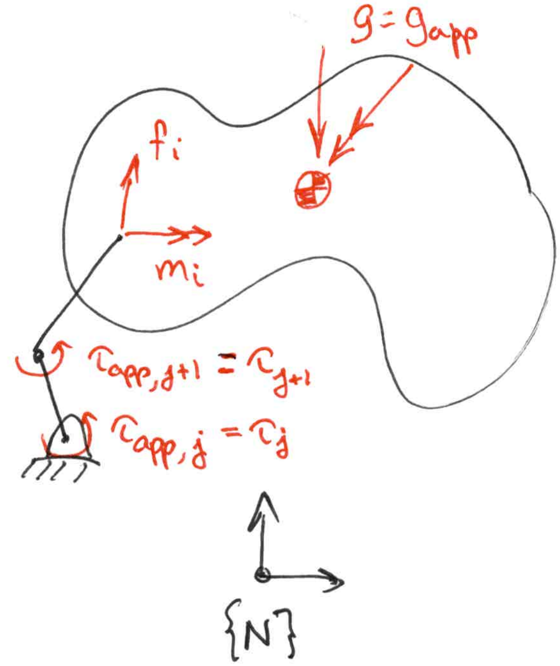
Hand-Object Equilibrium



Contact forces & moments are dual to translational and rotational velocities.

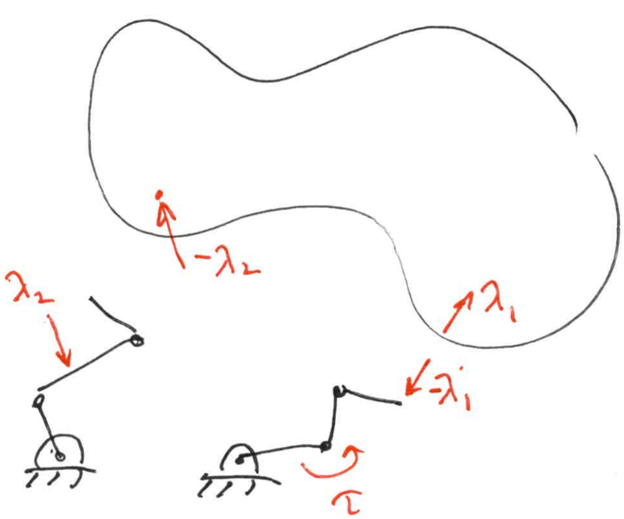
$$\tilde{\lambda}_i = g_i^{c_i} = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \\ m_{in} \\ m_{it} \\ m_{io} \end{bmatrix}$$ a.k.a. wrench

$$\lambda_i = H_i \tilde{\lambda}_i$$ ← transmit forces corresponding to constraints on relative twists

Equilibrium

Σ forces = 0

Σ moments = 0



Hand Equilibrium

$$J^T \lambda = \tau$$

Object equilibrium

$$-G^T \lambda = g$$

$$\boxed{\begin{bmatrix} J^T \\ -G \end{bmatrix} \lambda = \begin{bmatrix} \tau \\ g \end{bmatrix}}$$

eq (38.21)

g is wrench applied to object by environment; not hand.

torques & forces applied by the hand's actuators

where $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \vdots \\ \lambda_{n_c} \end{bmatrix}$

PwoF:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \lambda_i = f_{in}$$

← normal component of contact force

HF:

$$\begin{bmatrix} I_{3\times3} & O_{3\times3} \end{bmatrix} \lambda_i = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \end{bmatrix}$$

← components of sliding friction

SF:

$$\begin{bmatrix} I_{4\times4} & O_{4\times2} \end{bmatrix} \lambda_i = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \\ m_{in} \end{bmatrix}$$

← moment about contact normal.

Form Closure

Lock fingers and use PwoF contact model.

Ask: Can the object move?

If yes, it is not form closed

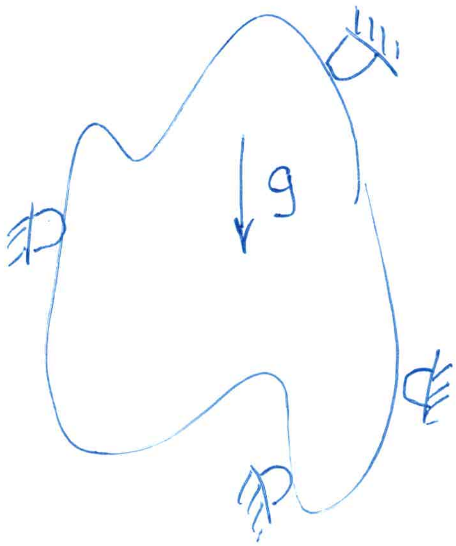
If no, it is form closed and the grasp is said to be a form closure grasp.

Column captures rotation info

Recall:

These rows only needed for 3D grasps

$$\tilde{G}_i^T = \begin{bmatrix} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{o}_i^T & (r_i \times \hat{o}_i)^T \\ 0_{1\times3} & \hat{n}_i^T \\ 0_{1\times3} & \hat{t}_i^T \\ 0_{1\times3} & \hat{o}_i^T \end{bmatrix}$$



H_i for PwoF $= [1\ 0\ 0\ \ 0\ 0\ 0]$

$$H_i \tilde{G}_i^T = G_i^T = [\hat{n}_i^T \ (r_i \times \hat{n}_i)^T] \quad \forall\, i = 1, \dots, n_c$$

$$G_n^T = \begin{bmatrix} \hat{n}_1^T & (r_1 \times \hat{n}_1)^T \\ n_2^T & (r_2 \times \hat{n}_2)^T \\ \vdots & \\ \hat{n}_{n_c}^T & (r_{n_c} \times \hat{n}_{n_c})^T \end{bmatrix}$$

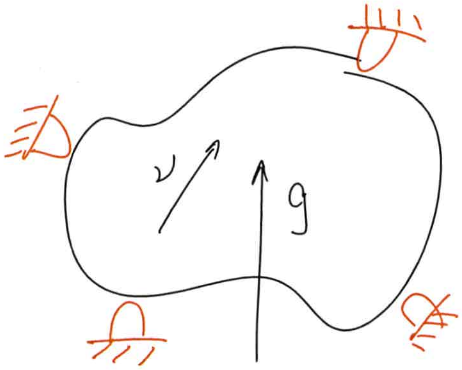
← one row for each (frictionless) contact

Form Closure: First-Order

Thursday, January 31, 2008
9:12 AM

Definition 1: a grasp has <u>form closure</u> iff it cannot be moved when the fingers are locked

$$\boxed{G_n^T \nu \geq 0 \Rightarrow \nu = 0}$$



All $H_i = [1\,0\,0\,0\,0\,0]$

$\Downarrow$

$$G_n^T = \begin{bmatrix} \hat{n}_1^T & (r_1 \times \hat{n}_1)^T \\ \hat{n}_2^T & (r_2 \times \hat{n}_2)^T \\ \vdots & \\ \hat{n}_{n_c}^T & (r_{n_c} \times \hat{n}_{n_c})^T \end{bmatrix}$$

Definition 2: a grasp has <u>form closure</u> iff it is possible for frictionless fingers to generate any wrench.

$$\boxed{\left.\begin{aligned} G_n \lambda_n &= g \\ \lambda_n &\geq 0 \end{aligned}\right\} \forall g \in \mathbb{R}^{n_\nu}} \equiv \boxed{\exists\, \lambda_n > 0 \ni G\lambda_n = 0}$$

~~Will use~~ this for computation

$$\lambda_n = \begin{bmatrix} f_{1n} \\ f_{2n} \\ \vdots \\ f_{n_c n} \end{bmatrix}$$

Necessary condition $\text{rank}(G_n) = n_\nu$!

Necessary condition $n_c \geq n_\nu + 1$

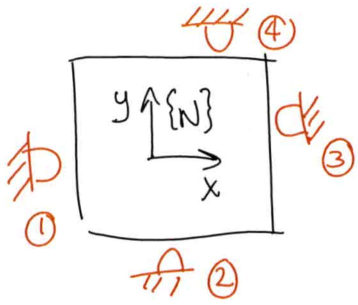
$\therefore$ Need at least $\boxed{7}$ contacts to form close a $\boxed{3D}$ object.

Need at least $\boxed{4}$ contacts to form close a $\boxed{2D}$ object.

Example: Object in $\mathbb{R}^2$

$$\nu = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \Leftarrow \left.\text{assume translation only}\right\} \nu \in \mathbb{R}^2$$

$\dot{\theta} = 0$



$$G_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

<u>Twist Test</u>

$$G_n^T \nu \geq 0 \Rightarrow \left.\begin{aligned} \dot{x} &\geq 0 \\ \dot{y} &\geq 0 \\ -\dot{x} &\geq 0 \\ -\dot{y} &\geq 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \dot{x} &= 0 \\ \dot{y} &= 0 \end{aligned} \Rightarrow \boxed{\text{Form Closure exists!}}$$

$$G_n^T v \geq 0 \Rightarrow \left.\begin{array}{l} \dot{x} \geq 0 \\ \dot{y} \geq 0 \\ -\dot{x} \geq 0 \\ -\dot{y} \geq 0 \end{array}\right\} \Rightarrow \begin{array}{l} \dot{x} = 0 \\ \dot{y} = 0 \end{array} \Rightarrow \boxed{\text{Form Closure exists!}}$$

<u>Wrench Test</u>

$$G_n \lambda_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{1n} \\ f_{2n} \\ f_{3n} \\ f_{4n} \end{bmatrix} = \begin{bmatrix} f_{1n} - f_{3n} \\ f_{2n} - f_{4n} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = -g$$

applied moments are ignored (assumed to be resisted since object can't rotate).

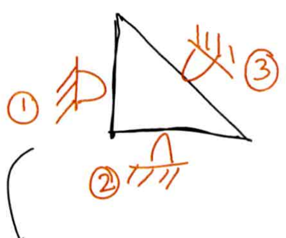
$$f_{1n}, f_{2n}, f_{3n}, f_{4n} \geq 0$$

Note that given any g, we can find $\lambda_n \ni G_n \lambda_n = -g$!

$\boxed{\therefore \text{ Form closure exists.}}$

Other interpretations:

Assume Triangle can only translate in the plane.

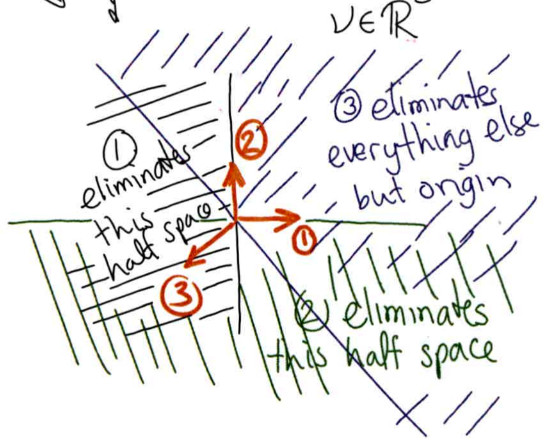


?

<u>Velocity constraints</u>

Each row of $G_n v \geq 0$ eliminates a half-space in the twist space

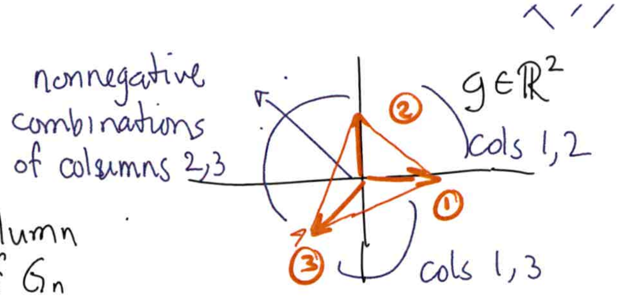
Form closure means that no non-trivial twists are possible!



<u>Wrench generation</u>

Non negative span of

columns of G_n
must equal $\mathbb{R}^{n_\nu}$.



Example:

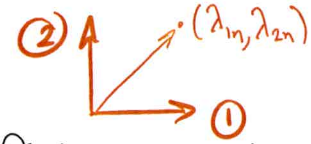
positive span of 1 column
of G_n

$\underline{G_{1n}\lambda_{1n},\ \lambda_{1n} \geq 0}$

this is a ray labeled ①

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \lambda_{1n},\ \lambda_{1n} \geq 0$

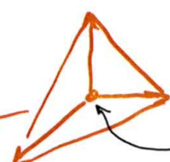
Positive span of 2 columns



$$\left.\begin{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} \\ \lambda_{1n}, \lambda_{2n} \geq 0 \end{matrix}\right\} = \text{any point in first quadrant of wrench space}$$

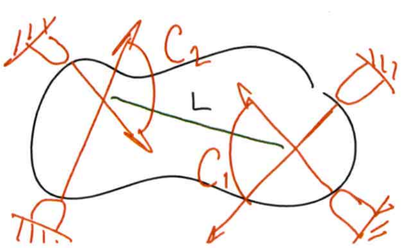
So the positive span includes the two rays
and everything between them!

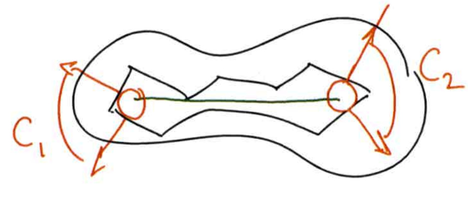
Geometric interpretation
in Wrench Space



Convex hull of columns of G_n strictly includes origin.

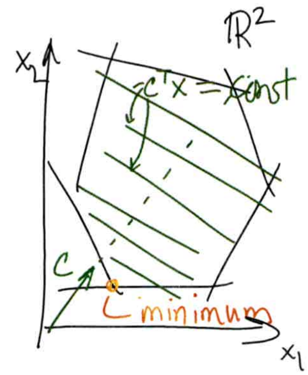
Geometric interpretation
in work space (planar only)





If $\exists$ cones, C_1 & C_2, formed with pairs of normals, and line segment L connecting the cone apexes $\ni L$ lies entirely in $C_1 \cap C_2$ or $-C_1 \cap -C_2$, then the grasp has form closure.

Computation Tests for Form Closure.



Linear program

Matlab has good solver.

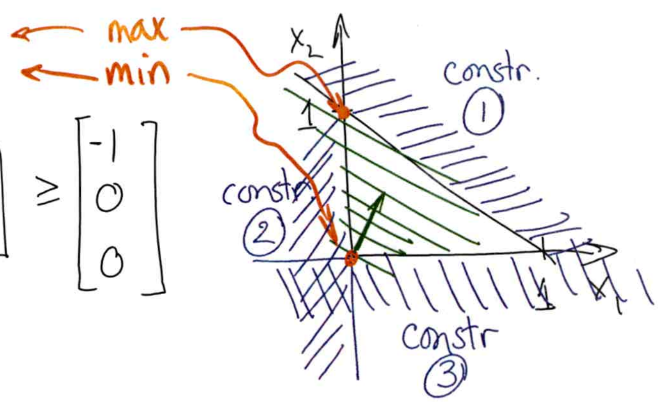
$$\min_{x} \; c^T x$$

$$\text{s.t.} \quad Ax \geq b$$

Example:

$$\text{Min} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{s.t} \quad \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$



Equivalent Definition: Form closure requires $\text{rank}(G) = n_v$ and the existence of $\lambda_n > 0 \ni G_n \lambda_n = 0$

Second part

We must satisfy $G_n \lambda_n = 0$

$\lambda_n > 0$

How do we "encourage" λ_n to be positive?

$$G_n \lambda_n = 0$$

"slack variable"

$$\lambda_n - d \geq 0 \quad \longleftarrow \quad \lambda_n \geq 0$$

$$d \geq 0$$

Maximize the slack variable, d.

L.P.

$$\max_{\lambda_n, d} \; d$$

$\longleftarrow$ d is a measure of how far the grasp is from losing form closure

$$\text{s.t.: } G_n \lambda_n = 0$$

$$I \lambda_n - \mathbb{1} d \geq 0$$

$$d \geq 0$$

$$\mathbb{1}^T \lambda_n \leq n_c$$

$\longleftarrow$ prevent unboundedness

Test for form closure.

1 $\text{Rank}(G_n) = n_\nu$

If not, stop. Form closure does not exist.

If yes, continue

2 Compute solution to LP1

If $d^* > 0$, then form closure exists,

Simple example – bead on wire

$$G_n = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$n_\nu = 1 \qquad \text{rank}(G_n) = 1$$

$$\lambda_n = \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} f_{1n} \\ f_{2n} \end{bmatrix}$$

$$\max_{f_{1n}, f_{2n}, d} \; d$$

$$\text{s.t.} \quad f_{1n} - f_{2n} = 0$$

$$f_{1n} - d \geq 0$$

Feasible Solutions

$f_{2n} - d \geq 0$

$d \geq 0$

$f_{1n} + f_{2n} \leq 2$

$f_{1n} = f_{2n}$	domain of d
0	0
0.5	[0, 0.5]
1	[0, 1.0]
~~1.1~~	~~[0, 1.1]~~

d^*

Without last constraint, f_{1n} could increase without bound – infinitely tight squeezing. Form closure requires ability to squeeze, which is evident without letting $d \to \infty$.

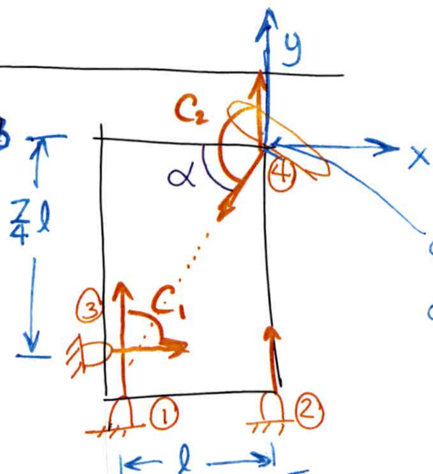
Without the bound ($\mathbf{1}^T \lambda_n \leq n_c$), d^* for all form closure grasps would be ∞ and then d^* could not be used to compare different form closure grasps.

$\mathbb{R}^1$ wrench space

convex hull of cols of G_n contain origin

Planar Example

Figure 38.18

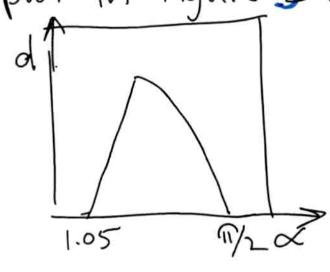


Choose origin at 4th contact point

Form closure exists for $1.052 < \alpha < \pi/2$

No form closure if $\alpha = \pi/2$

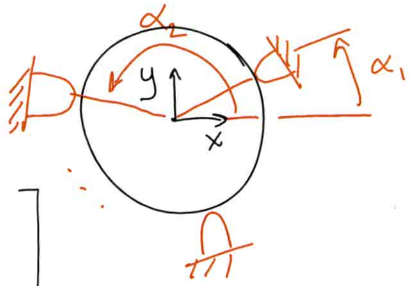
See plot in Figure 38.19



$$G_n^T = \begin{bmatrix} 0 & 1 & -l \\ 0 & 1 & 0 \\ 1 & 0 & \frac{7}{4}l \\ -c_\alpha & -s_\alpha & 0 \end{bmatrix}$$

Partial Form Closure – can't prevent all twists, but can prevent a subspace of all twists.

Example: Disk $\longrightarrow$



$$G_n = \begin{bmatrix} -\cos(\alpha_1) & -\cos(\alpha_2) & \cdots \\ -\sin(\alpha_1) & -\sin(\alpha_2) & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

$\max(\mathrm{Rank}(G_n)) = 2 \implies$ greatest number of d.o.f. constrained is 2.

For this problem, as long as the angle between each pair of contacts is less than π, the disk is form closed in the (v_x, v_y) subspace of v.

Can't form close ω_z dimension with any # of contacts.

why doesn't this satisfy form closure?

$$\max_{\lambda_{in}} d$$

$G_n \lambda_n = 0$

$\lambda_n - \mathbb{1}d \geq 0$

$d \geq 0$

$\mathbb{1}^T \lambda_n \leq n_c$

$$\underset{(n_v \times n_c)}{G_n} = \begin{bmatrix} & & & & \\ & & & & \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \vdots \\ \lambda_{in} \\ \vdots \end{bmatrix}$$

Because $\mathrm{Rank}(G_n) < n_v$!