A little review

Form closure grasp

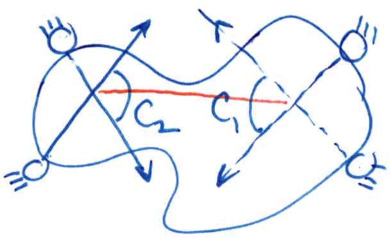
- Most secure kind of grasp. It does not rely on friction! Only geometry matters.

- Need at least $(n_\nu+1)$ contact points, where n_ν is the # of degrees of freedom of the object without contacts AND $\text{rank}(G_n) = n_\nu$.

- Tests

Binary: Object in SE(2).

cones can "see" each other.



Binary: Object in SE(3).

+ columns of wrench matrix G_n must positively span $\mathbb{R}^6$ (the wrench space)

+ equivalently the ~~column~~ convex hull of the columns of G_n must strictly include the origin.

Tests:

Quantitative: Solve the following Linear Program

$$\boxed{\begin{aligned} &\max_{\lambda_n, d} \quad d \\ &\text{subject to:} \quad G_n \lambda_n = 0 \\ &\qquad I\lambda_n - \mathbb{1} d \geq 0 \\ &\qquad d \geq 0 \\ &\qquad -\mathbb{1}^T \lambda_n \geq n_c \end{aligned}}$$

$d \in \mathbb{R}$

$\mathbb{1}^T = [1\ 1\ 1 \cdots 1]^T$ $(n_c \times 1)$

(equivalent to $\mathbb{1}^T \lambda_n \leq n_c$)

Solution d^*, λ_n^*. d^* is a quantitative measure of form closure.

$d^* = 0 \Rightarrow$ not form closed.

$d^* > 0 \Rightarrow$ form closed.

Next, incorporate friction so fewer contacts are needed.

But also make sure hand kinematics allows squeezing so friction can be leveraged.

<u>Grasp Properties</u>, then <u>Force Closure</u>.

Planar Simplifications

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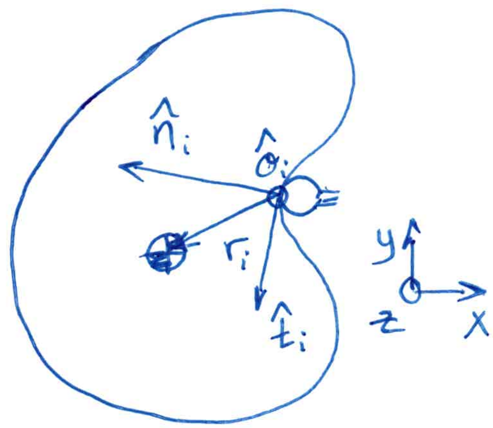
J & G contain cross products.

When the cross products are of vectors in a plane, they reduce/simplify.

$$G_{in} = \begin{bmatrix} \hat{n}_i \\ r_i \times \hat{n}_i \end{bmatrix}$$



<u>In 3D:</u>

0

$$r_i \times \hat{n}_i = \begin{bmatrix} r_y \cancel{n_z} - \cancel{r_z} n_y \\ \cancel{r_z} n_x - r_x \cancel{n_z} \\ r_x n_y - r_y n_x \end{bmatrix}$$

<u>In 2D:</u>

place at $\hat{n}_i, \hat{t}_i$ in the (x,y)-plane.

We care about motion only in the plane.

$$G_{in}^T v = v_{i,obj} = \begin{bmatrix} v_{in} \\ v_{it} \\ \cancel{v_{io}} \\ \cancel{\omega_{in}} \\ \cancel{\omega_{it}} \\ \omega_{io} \end{bmatrix}$$

out of plane translation

out of plane rotation

Example

Planar construction of $\tilde{G}_i$ & G_{in}

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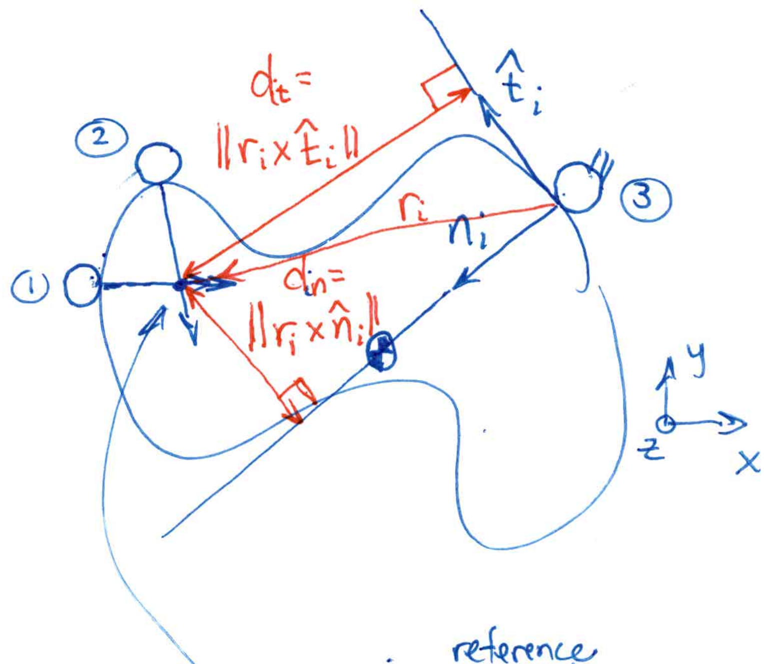
- To construct G you don't need to use the center of gravity of the object (but you can).

- For grasp equilibrium & closure analysis, you may use any point (that is convenient)



Convenient reference point to construct G_n

3D: $G_{in}^T = \left[\hat{n}_i^T \; (r_i \times \hat{n}_i)^T \right]_{(1\times 6)}$

2D: $G_{in}^T = \left[n_{ix} \;\; n_{iy} \;\; -d_{in} \right]$

$\hat{n}_i$ passes on left of reference pt

$G_{it}^T = \left[t_{ix} \;\; t_{iy} \;\; +d_{it} \right]$

$\hat{t}_i$ passes on right of ref point.

Construct G^T for contacts 1 & 2 PwoF and 3 HF

$$G^T = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & -1 & 0 \\ \hline -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & d_{it} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -d_{in} \end{bmatrix}$$

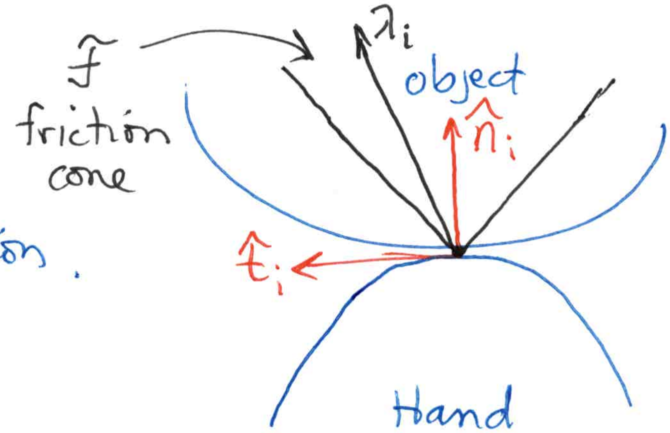
Force Closure & Frictional ~~&~~ Form Closure

Force closure adds:

- friction to reduce # of contacts needed
- kinematic structure of hand to ensure squeezing is possible (so friction is leveraged).

Planar friction model (dry friction, a.k.a Coulomb friction)

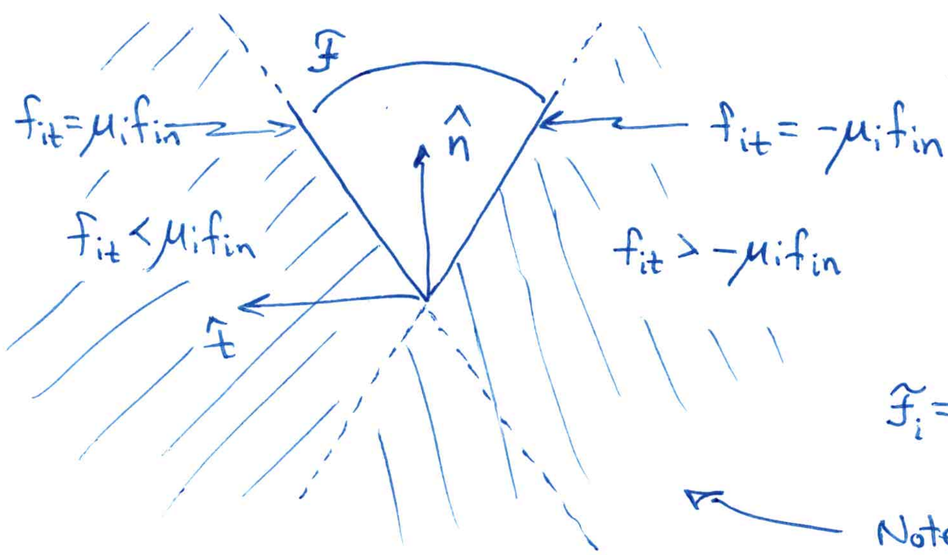
$$\lambda_i = \begin{bmatrix} f_{in} \\ f_{it} \end{bmatrix} \in \mathcal{F}$$



Let μ_i be coeff of friction.

λ_i must satisfy:

$$-\mu_i f_{in} \le f_{it} \le \mu_i f_{in}$$



Friction cone def.

$\lambda_i = (f_{in}, f_{it})$

$$\mathcal{F}_i = \{(f_{in}\, f_{it}) \mid -\mu_i f_{in} \le f_{it} \le \mu_i f_{in},\ \mu_i \ge 0\}$$

Note: $f_{in} \ge 0$ is implied

Other model features:

- If λ_i is strictly inside $\mathcal{F}$ cone, then contact sticks.

Desired in grasping.

- If sliding is occurring (ie $v_{i,t}$ or $v_{i,o}$ nonzero) then λ_i is on the bndry of $\mathcal{F}$.

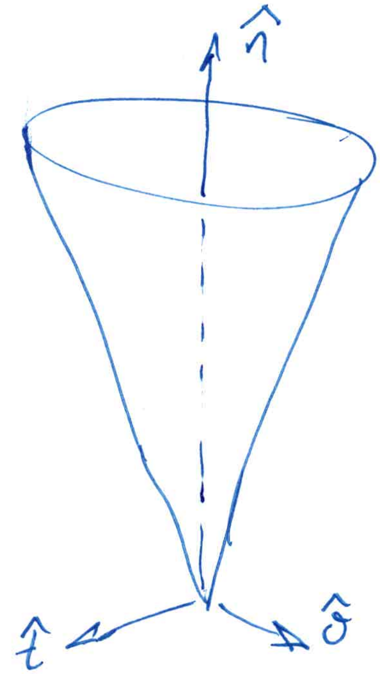
Spatial case

$$\lambda_i = [f_{in} \; f_{it} \; f_{io}]^T$$

$$\mathcal{F}_i = \{(f_{in} \; f_{it} \; f_{io}) \mid \mu^2 f_{in}^2 - f_{it}^2 - f_{io}^2 \geq 0, \ldots$$

$$\ldots \underbrace{f_{in} \geq 0}, \mu_i \geq 0\}$$

needed in 3D.



Frictional Form Closure

Everything done in form closure applies, but we we replace normals with friction cone generators.

<u>Def</u>: A grasp has frictional form closure iff

1.) $\text{rank}(G) = n_\nu$

2.) $\exists\ \lambda$ such that $G\lambda = 0$, $\lambda \in \text{Int}(\mathcal{F}(\mu))$

where $\lambda = [\lambda_1\ \lambda_2\ \cdots\ \lambda_{n_c}]^T$ $\quad \lambda_i = [f_{in}\ f_{it}\ f_{io}]^T$

$$\mathcal{F}(\mu) = \mathcal{F}_1(\mu_1) \cup \mathcal{F}_2(\mu_2) \cup \cdots \cup \mathcal{F}_{n_c}(\mu_{n_c})$$

Note: if $\mu_i \rightarrow 0\ \forall\ i$, then frictional form closure is equivalent to form closure.

Planar case

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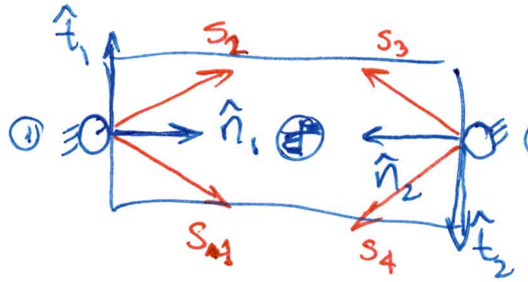
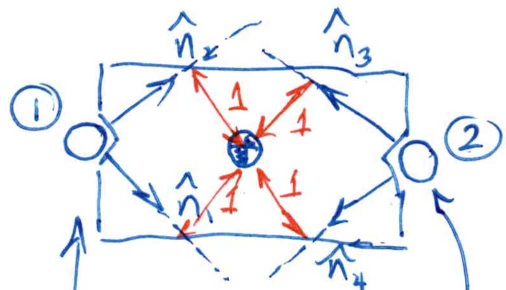
All contacts are HFs, SFs make no sense.

$\therefore \ \forall i, \ \lambda_i = [f_{in} \ f_{it}]^T$.



$\Longleftrightarrow$



2 contacts

2 contacts

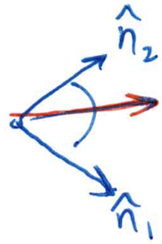
$$G^T = \begin{bmatrix} G_{1n}^T \\ G_{1t}^T \\ G_{2n}^T \\ G_{2t}^T \end{bmatrix} = \begin{bmatrix} s_{1x} & s_{1y} & 1 \\ s_{2x} & s_{2y} & -1 \\ s_{3x} & s_{3y} & 1 \\ s_{4x} & s_{4y} & -1 \end{bmatrix}$$

$$G_n^T = \begin{bmatrix} n_{1x} & n_{1y} & +1 \\ n_{2x} & n_{2y} & -1 \\ n_{3x} & n_{3y} & +1 \\ n_{4x} & n_{4y} & -1 \end{bmatrix}$$

Possible forces at contact 1

$\boxed{\mu=0}$ $\quad f_{1n}\hat{n}_1 + f_{2n}\hat{n}_2$

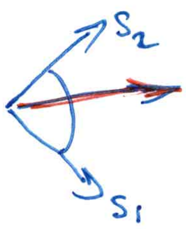
$f_{1n}, f_{2n} \geq 0$



same set of forces can be resisted

$\boxed{HF}$ $\quad \sigma_1 s_1 + \sigma_2 s_2$

$\sigma_1, \sigma_2 \geq 0$



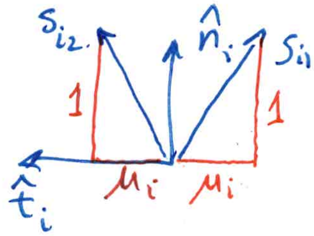
$\boxed{\text{rank}(G_n) = 3}$

(can only be < 3 if all 4 normals intersect at a common point)

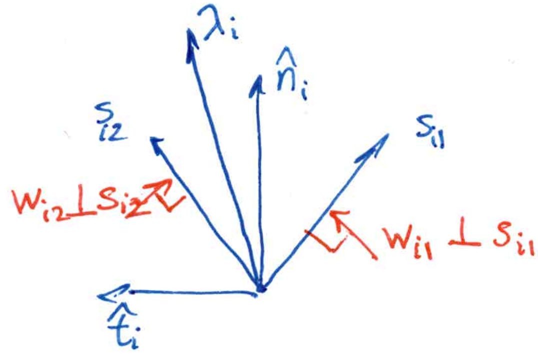
s_1 & s_2 are called friction cone generators.

Generator representation of contact force

$$\left.\begin{array}{l} S\sigma \\ \sigma \geq 0 \end{array}\right\} - \qquad \lambda \in \mathcal{F}$$



The "face form" representation is more convenient



$\lambda_i \in \mathcal{F}$ iff

$$\boxed{\begin{array}{l} w_{i1}^T \lambda_i \geq 0 \\ w_{i2}^T \lambda_i \geq 0 \end{array}}$$

&

$$w_{i1} \perp s_{i1} \Rightarrow \qquad s_{i1} = \begin{bmatrix} 1 \\ -\mu_i \end{bmatrix} \qquad w_{i1} = \begin{bmatrix} \mu_i \\ 1 \end{bmatrix}$$

$$s_{i2} = \begin{bmatrix} 1 \\ \mu_i \end{bmatrix} \qquad w_{i2} = \begin{bmatrix} \mu_i \\ -1 \end{bmatrix}$$

So. $$S_i = \begin{bmatrix} 1 & 1 \\ -\mu_i & \mu_i \end{bmatrix}$$

$$F_i = \frac{\begin{bmatrix} \mu_i & 1 \\ \mu_i & -1 \end{bmatrix}}{\cancel{\sqrt{1+\mu_i^2}}}$$

normalization optional

$$\boxed{\begin{array}{ll} \text{Finally} & F_i \lambda_i \geq 0 \quad \forall\, i \\ \text{means} & \lambda_i \in \mathcal{F}(\mu_i) \quad \forall\, i. \end{array}}$$

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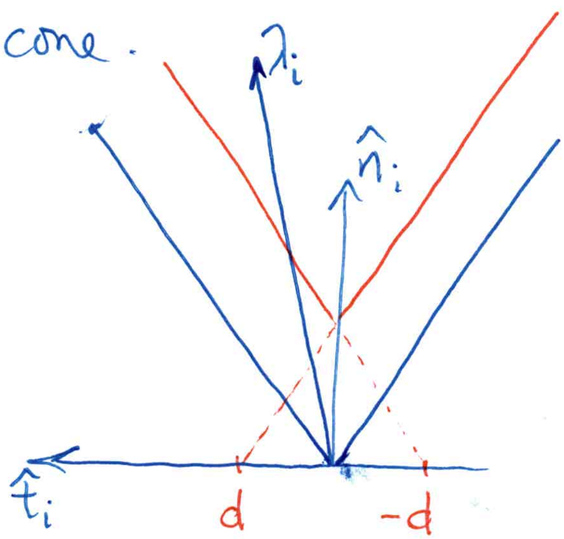
Recall that for form closure, we require $G_n \lambda_n = 0$ with $\lambda_n > 0$.

To accomplish this we enforced $f_{in} - d \geq 0$

$d > 0$

thus forcing $f_{in} \geq d$.

For frictional form closure, we require λ_i to be internal to the friction cone.



We can use a similar trick

$$\left.\begin{aligned} F_i \lambda_i - \mathbb{1} d &\geq 0 \\ d &> 0 \end{aligned}\right\}$$

$d>0$ makes a cone strictly inside the friction cone,

so $$\left.\begin{aligned} F_i \lambda_i - \mathbb{1} d &\geq 0 \\ d &\geq 0 \end{aligned}\right\} \rightarrow \lambda_i \in \text{Interior}(\mathcal{F}(\mu_i))$$

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To apply to all contacts, make one large inequality system

$$F\lambda - \mathbb{1}d \geq 0$$
$$d \geq 0$$

$$F = \begin{bmatrix} F_1 & & & & \\ & F_2 & & 0 & \\ & & F_3 & & \\ & 0 & & \ddots & \\ & & & & F_{n_c} \end{bmatrix}_{(2n_c \times 2n_c)} \leftarrow \text{Blockdiagonal matrix}$$

Frictional Form Closure Test

1) If $\text{rank}(G) = n_\nu$, continue

2) Solve LP2.

Remember: $\lambda_i = \begin{bmatrix} f_{in} \\ f_{it} \\ f_{io} \\ m_{ia} \end{bmatrix}$ for SF

$$\begin{aligned} &\underset{\lambda, d}{\text{maximize}} \quad d \\ &\text{subject to:} \quad G\lambda = 0 \\ &\qquad F\lambda - \mathbb{1}d \geq 0 \\ &\qquad d \geq 0 \\ &\qquad \sum_{i=1}^{n_c} f_{in} \leq n_c \end{aligned}$$

$G\lambda = 0$ ← ensure ~~gra~~ equilibrium of grasped object.

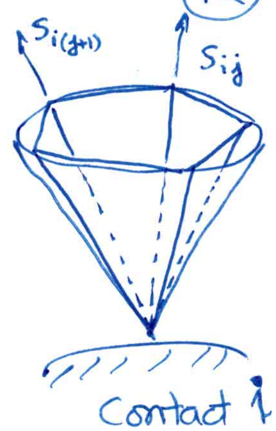
$F\lambda - \mathbb{1}d \geq 0$, $d \geq 0$ ← make sure $\lambda_i \in \text{Int}(\mathcal{F}(\mu_i))$ $\forall i$

$\sum_{i=1}^{n_c} f_{in} \leq n_c$ ← make sure d cannot grow to ∞.

If $d^* > 0$, frict. form closure exists!

What changes in 3D frictional form closure?

$$G_i^T = \begin{bmatrix} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{o}_i^T & (r_i \times \hat{o}_i)^T \\ 0 & \hat{n}_i^T \end{bmatrix} \begin{array}{l} \left.\vphantom{\begin{array}{c} a \\ a \\ a \end{array}}\right\} \text{HF} \\ \vphantom{a} \end{array} \hspace{-2em} \left.\vphantom{\begin{array}{c} a \\ a \\ a \\ a \end{array}}\right\} \text{SF}$$



Approx $\mathcal{F}_i$ with n_f ~~edges~~ generators $s_{i,j}$.

normal to facet, $w_{ij} = s_{ij} \times s_{i(j+1)}$

$$F_i = \underbrace{\left[\begin{array}{c|c} \overbrace{\hat{w}_{i1}^T}^{\text{HF}} & 0 \\ \hat{w}_{i2}^T & 0 \\ \hat{w}_{i3}^T & 0 \\ \vdots & \vdots \\ \hat{w}_{in_f}^T & 0 \\ \hline \mu_\tau \; 0 \; 0 & 1 \\ \mu_\tau \; 0 \; 0 & -1 \end{array}\right]}_{\text{SF}}$$

HF SF

Note: for SF contacts, a second coeff. of friction, μ_τ, for torsional friction.

Use the G_i^T above in $G^T = \begin{bmatrix} G_1^T \\ G_2^T \\ \vdots \\ G_{n_c}^T \end{bmatrix}$ & $F = \text{blockdiag}(F_1, F_2, \ldots, F_{n_c})$

The LP stays the same.