Grasp Properties

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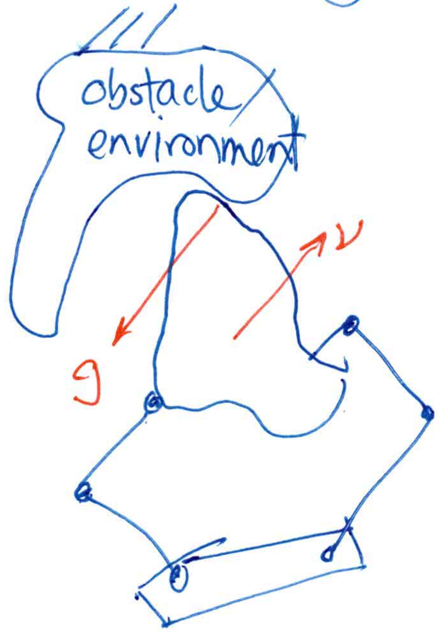


Determine the controllable twists & wrenches of a given grasp.

Usually a grasp should be able to move object with any chosen twist and control the net wrench applied by the fingers/palm through the hand contacts.

Force closure requires:

- control all object twists
- control all object wrenches
- control all <u>internal</u> wrenches

squeezing forces

Linear algebra provides a way to answer these questions using G and J.

Basic concepts from Linear Algebra

Let A be an $m \times n$ matrix, i.e. $A \in \mathbb{R}^{m \times n}$.

$\mathcal{R}(A) \triangleq$ the range (a.k.a. $\boxed{\text{column space}}$) of A
is the space of all linear combinations
of the columns of A.

Let $x \in \mathbb{R}^n$, then $Ax \in \mathcal{R}(A)$

Let $y = Ax$, then

$$\boxed{\mathcal{R}(A) = \{ y \mid y = Ax,\ x \in \mathbb{R}^n \}}$$

$\mathcal{R}(A^T) \triangleq \boxed{\text{row space}}$ of A, is the space of
all linear combinations of the rows of A

Let $y \in \mathbb{R}^m$

$$\boxed{\mathcal{R}(A^T) = \{ x \mid x = A^T y,\ y \in \mathbb{R}^m \}}$$

$\mathcal{N}(A) \triangleq$ the $\boxed{\text{null space}}$ of A is the ~~set~~ space of all $x \in \mathbb{R}^n$ such that $Ax = 0$

$$\boxed{\mathcal{N}(A) = \{x \mid Ax = 0,\ x \in \mathbb{R}^n\}}$$

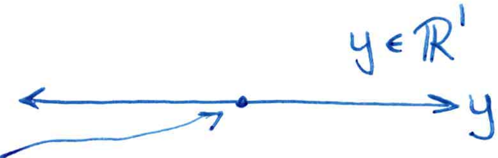
$\mathcal{N}(A^T) \triangleq$ is the left null space of A.

$$\boxed{\mathcal{N}(A^T) = \{y \mid A^T y = 0,\ y \in \mathbb{R}^m\}}$$

Example: $A = [1 \ \ 1]$; $m \times n = 1 \times 2$.

$x \in \mathbb{R}^2$, $y \in \mathbb{R}^1$

Column space $[1 \ \ 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = y$



y can achieve any value. $\therefore$ $\mathcal{R}(A) = \mathbb{R}^1 = y$-axis

Left null space $\begin{bmatrix} 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. $\therefore$ $\mathcal{N}(A^T) = 0$

Note: $\mathcal{R}(A)$ and $\mathcal{N}(A^T)$ are pieces of the space y lives in. They are orthogonal complement of y's space

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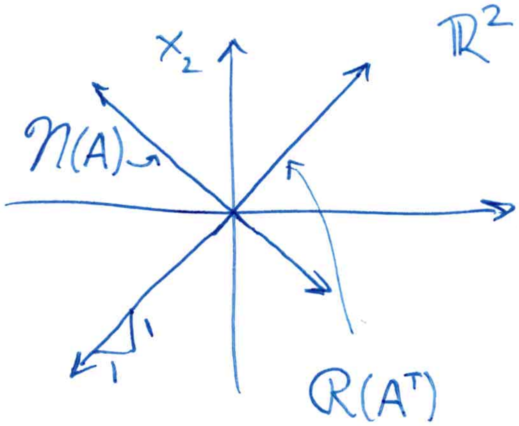
Row space of A is column space of A^T

row space = $\begin{bmatrix}1\\1\end{bmatrix}\alpha$, $\alpha\in\mathbb{R}^1$

just any scalar don't have to call it "y".



null space of A

$$\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=0 \rightarrow x_1+x_2=0 \rightarrow x_1=-x_2$$

Note: every vector in $\mathcal{N}(A)$ $\perp$ to every vector in $\mathcal{R}(A^T)$.

e.g. $\begin{bmatrix}1\\-1\end{bmatrix}\cdot\begin{bmatrix}2\\2\end{bmatrix}=0$

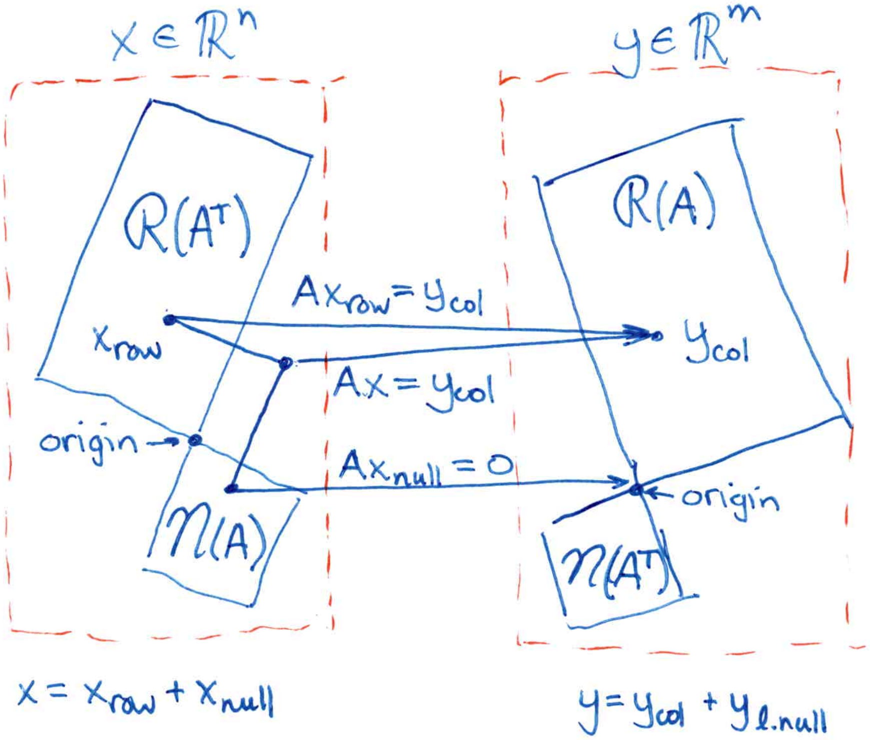
Think of $\mathbb{R}^2$ as $\mathcal{R}(A^T)\times\mathcal{N}(A)$

Cartesian product.

$\mathbb{R}^1$ as $\mathcal{R}(A)\times\mathcal{N}(A^T)$

Four Fundamental Subspaces

$$Ax = y$$



- Every space must have an origin
- A maps $\mathcal{R}(A^T)$ onto $\mathcal{R}(A)$

$$Ax = A(x_{row} + x_{null}) = Ax_{row} + \underbrace{Ax_{null}}_{0} = y_{col}$$

Rank(A) = # of linearly independent columns of A
= # of " " rows of A.

Let $r = \text{rank}(A)$. Then:

$$\text{Dim}(\mathcal{R}(A)) = \text{Dim}(\mathcal{R}(A^T)) = r \leq \min(m,n)$$

$$\text{Dim}(\mathcal{N}(A)) = n - r$$

$$\text{Dim}(\mathcal{N}(A^T)) = m - r$$

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Solve for x given A and y.

$$Ax = y$$

If A^{-1} exists, then $x = A^{-1}y$.

If not there may still be solutions.

$\boxed{x \text{ exists only if } y \in \mathcal{R}(A)}$ In other words,

A map x onto $\mathcal{R}(A)$, so $\exists\, x \ni Ax = y$ if $y \in \mathcal{R}(A)$.

If $\mathcal{N}(A) = 0$ and $y \in \mathcal{R}(A)$, then x is unique.

If $\text{Dim}(\mathcal{N}(A)) > 0$ and $y \in \mathcal{R}(A)$, then there is a space of solutions of dimension $n-r$.

General solution: if $y \in \mathcal{R}(A)$

$$\boxed{x = A^{+}y + N(A)\alpha}$$

where $\alpha \in \mathbb{R}^{n-r}$, N is a basis of $\mathcal{N}(A)$, and A^{+} is the pseudo-inverse of A.

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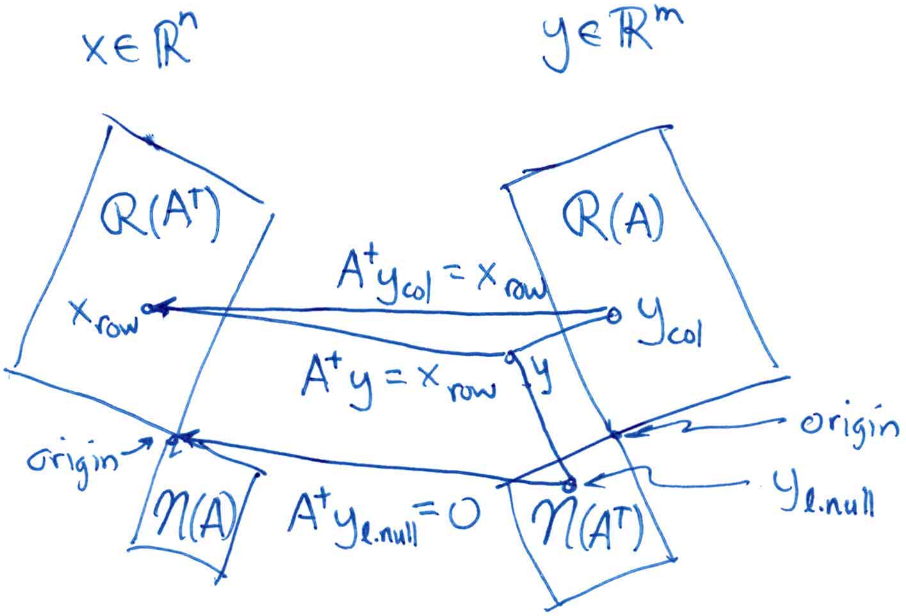

The mappings A and A^+ are 1-1 and onto.

Note that $\mathcal{R}(A^T) = \mathcal{R}(A^+)$ and $\mathcal{N}(A^T) = \mathcal{N}(A^+)$.

Matlab functions:

determinant ----- det(A)

rank ---------- rank(A)

$\mathcal{N}(A)$ -------- null(A)

A^+ --------- pinv(A)

Forward wrench & twist mappings

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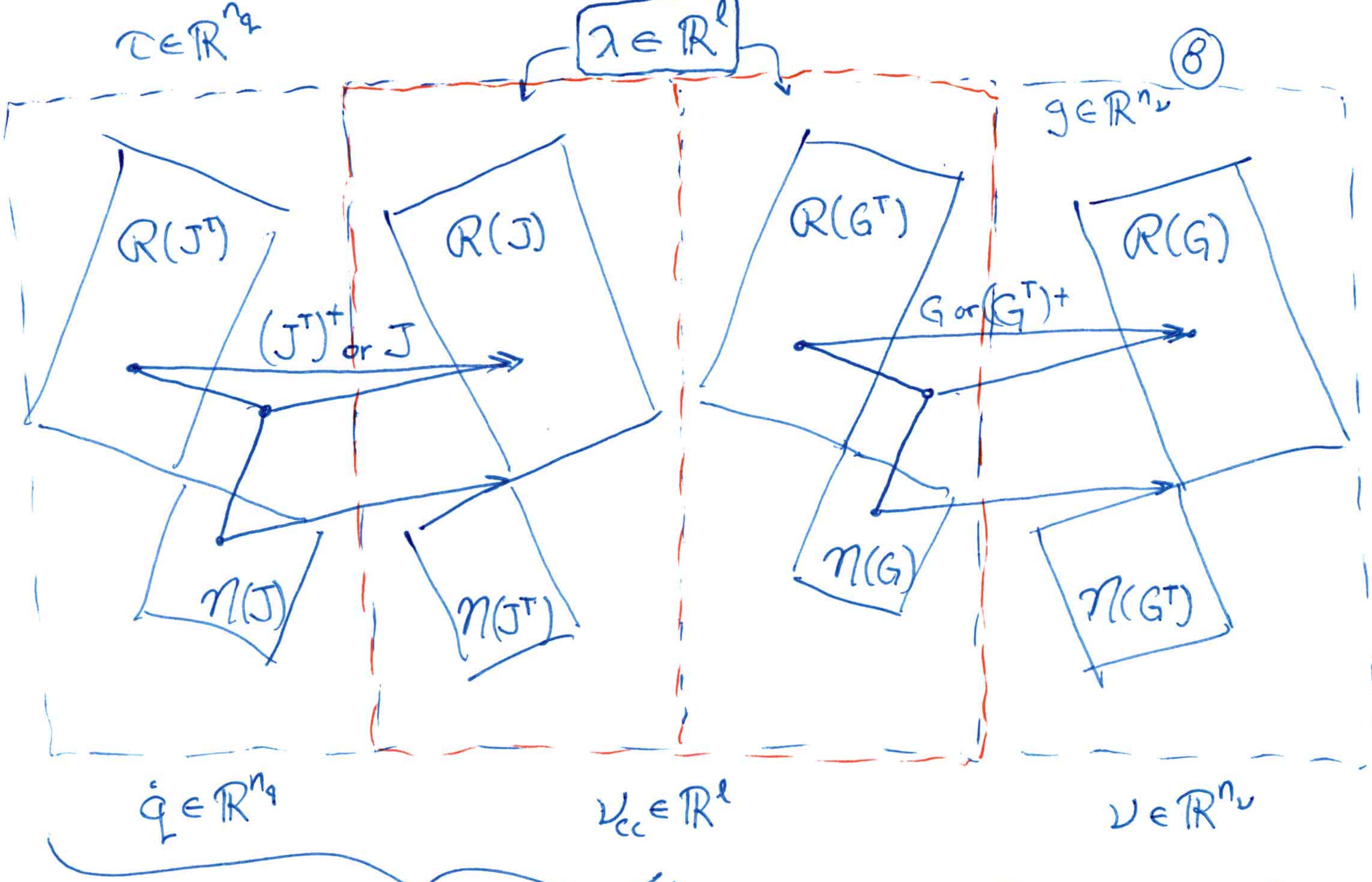

WRENCH

$$(J^T)^+ \tau = \lambda \longrightarrow G\lambda = -g$$

$$\boxed{G(J^T)^+ \tau = -g}$$

TWIST

$$J\dot{q} = \nu_{cc} \longrightarrow (G^T)^+ \nu_{cc} = \nu$$

$$\boxed{(G^T)^+ J \dot{q} = \nu}$$

Internal Wrenches & Twists

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internal joint torques ($\tau \in \mathcal{N}(J)$ do not change contact wrenches, but they do redistribute the joint torques

internal grasp forces ($\lambda \in \mathcal{N}(G)$ do not change the net wrench applied to the object, but they change the squeezing forces.

internal hand velocities ($\mathring{q} \in \mathcal{N}(J)$ do not cause motions at the hand contacts in the directions constrained by the contact models.

internal object velocities ($\nu_{cc} \in \mathcal{N}(G)$ do not cause object motion, but do maintain contact constraints

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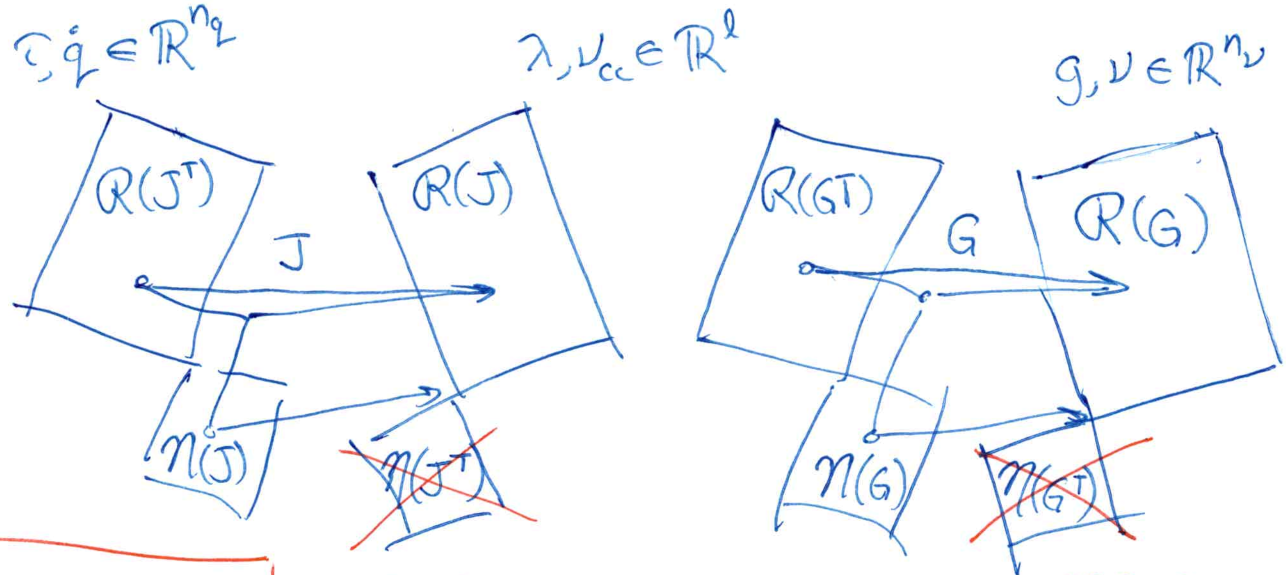
Requirements for Dexterous Manipulation (as shown on old Salisbury video)

1. • All object twists & wrenches possible. $\boxed{\text{rank}(G) = n_\nu}$
2. • All object twists & wrenches controllable. $\boxed{\text{rank}(GJ) = n_\nu}$
   (by choice of τ and $\dot{q}$)
3. • All internal wrenches controllable. (by choice of τ). $\boxed{\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0}$

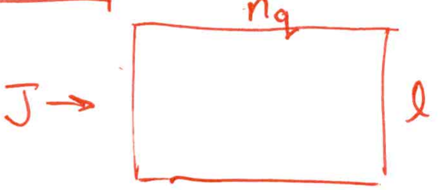


Recall:
$l = n_{PwoF} + 3n_{HF} + 4n_{SF}$

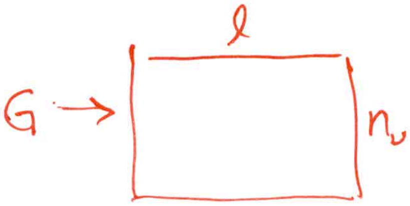
$\mathcal{N}(J^T) = 0$
by 2 & 3

$\mathcal{N}(G^T) = 0$
by 1.

$= \text{rank}(G) = n_\nu$



$n_q \geq l$



$l \geq n_\nu$

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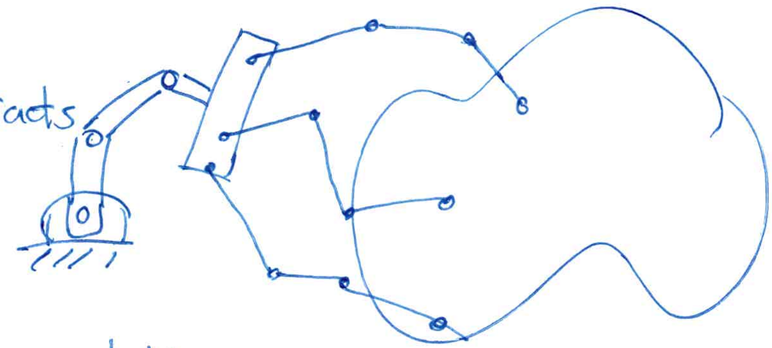
Salisbury Hand Designed for Dexterous Manip (11)

$n_q = 9$

$l = 9 \leftarrow$ 3HF contacts

$n_v = 6$



The intuitive interpretation –

3 non-sliding contacts give a triangle in the object. Move vertices of triangle to move object.

to move each triangle vertex arbitrarily, each finger has three joints.

if finger locations somewhat oppose each other, then hand can squeeze as hard as necessary along the three edges of the triangle

J is 9×9 and G is 6×9

For 3-finger tip grasp, typically $\mathrm{rank}(J) = 9$, $\mathrm{rank}(GJ) = 6$, $\mathrm{rank}(G) = 6$ and $\mathcal{N}(J^T) = 0$ $\therefore$ $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$.

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Force Closure Test

A grasp has force closure iff it has frictional form closure and all internal forces are controllable.

Test

1. Construct G

   If rank(G) $= n_\nu$ ($n_\nu = \#$ of rows of G), continue

   else stop. No force closure.

2. ~~Construct J~~ Solve LP2 for d^*

   (check that origin in inside convex hull of positive span of columns of G)

   Must use $G^T = \begin{bmatrix} s_1^T & (r_1 \times s_1)^T \\ s_2^T & (r_2 \times s_2)^T \\ \vdots & \vdots \\ s_n^T & (r_n \times s_n)^T \end{bmatrix}^T$ where s_i is edge of friction cone.

   If $d^* > 0$, frict form cl. exists, continue

   else $d^* = 0$, stop. No force closure.

3. Construct J & Solve LP3 for d^*

   (check that all internal forces are controllable)

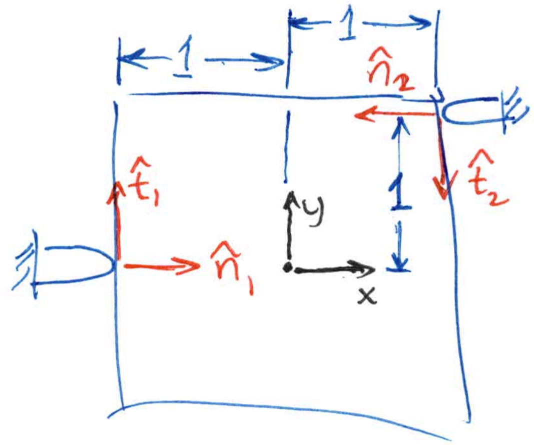
   If $d^* = 0$, Force closure exists

   else $d^* > 0$, Force closure does not exist.

Planar Example.

Assume 2 HF contacts

$$G = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

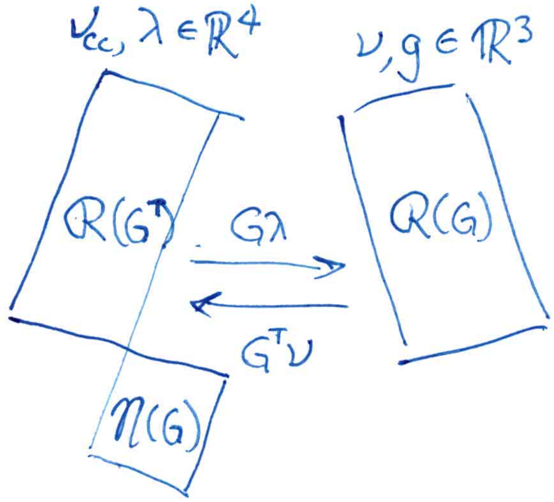


$\text{rank}(G) = 3 = n_\nu$

determinant of first 3 columns is $\neq 0$

$\therefore\ \mathcal{R}(G) = \mathbb{R}^3 = \mathcal{R}(G^T)$

and $\mathcal{N}(G) = \mathbb{R}^1$, $\mathcal{N}(G^T) = \mathbb{R}^0 = 0$

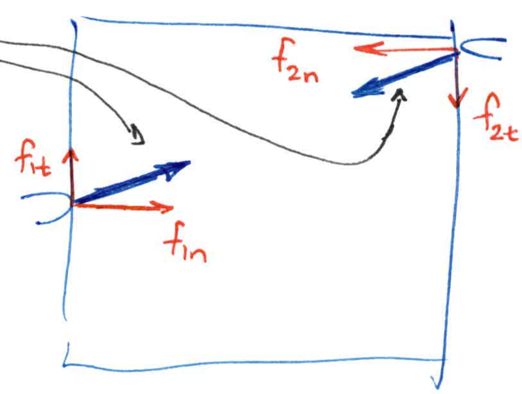


$$\boxed{G\lambda = -g \rightarrow \lambda = -G^{+}g + N\alpha}$$

From Matlab,

$\mathcal{N}(G)$ is represented by

$$N = \begin{bmatrix} 0.6325 \\ 0.3163 \\ 0.6325 \\ 0.3163 \end{bmatrix} \leftarrow \text{basis vector}$$



$$\mathcal{N}(G) = \{\lambda \mid \lambda = N\alpha,\ \alpha \in \mathbb{R}\}$$

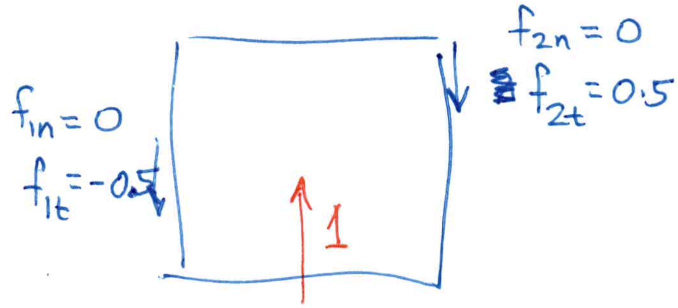
Complete solution

$$\lambda = -G^+ g + N\alpha$$

From Matlab

$$\lambda = -\begin{bmatrix} 0.6 & 0 & 0.2 \\ -0.2 & 0.5 & -0.4 \\ -0.4 & 0 & 0.2 \\ -0.2 & -0.5 & -0.4 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0.6 \\ 0.3 \\ 0.6 \\ 0.3 \end{bmatrix} \alpha = \begin{bmatrix} f_{1n} \\ f_{1t} \\ f_{2n} \\ f_{2t} \end{bmatrix}$$

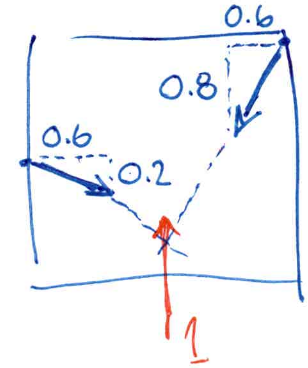
Cases: Let $g = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\alpha = 0$



Forces balance, so object is in equilibrium, but no squeezing requires infinite friction, $\mu \to \infty$

If Fingers could squeeze, that would help.

Let $\alpha = 1 \rightarrow \lambda = \begin{bmatrix} 0.6 \\ -0.2 \\ 0.6 \\ 0.8 \end{bmatrix}$

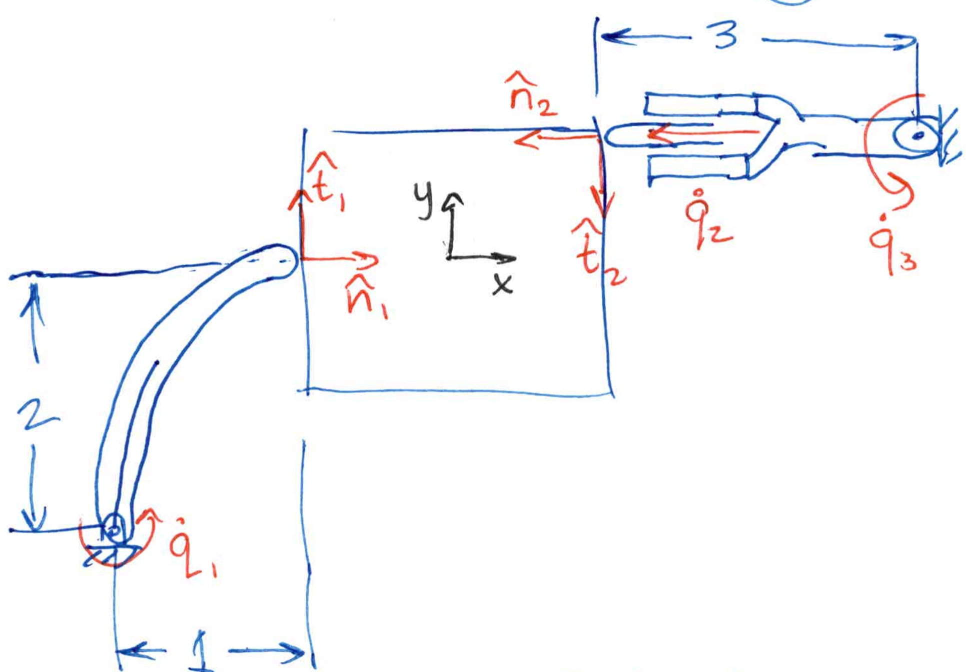


The 3 forces intersect at a point.

Equilibrium still satisfied, but $\mu \geq 4/3$ required.

Now include finger mechanisms

Recall, $J\dot{q} = \nu_{cc}$, contact twists in contact frames.



Determine J by inspection

$$
\underbrace{
\begin{array}{ccc}
\dot{q}_1 & \dot{q}_2 & \dot{q}_3 \\
\left[\begin{array}{c|cc}
-2 & 0 & 0 \\
1 & 0 & 0 \\
\hline
0 & 1 & 0 \\
0 & 0 & 3
\end{array}\right.
& & \left.\right]
\end{array}
}_{J}
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}
=
\begin{bmatrix} \nu_{1n} \\ \nu_{1t} \\ \hdashline \nu_{2n} \\ \nu_{2t} \end{bmatrix}
$$

$\text{Rank}(J) = 3$

(rows 2-4 form a diagonal matrix)

$\mathcal{N}(J) = 0 = \mathbb{R}^0$

$\mathcal{R}(J) = \mathcal{R}(J^T) = \mathbb{R}^3$

$\mathcal{N}(J^T) = \mathbb{R}^1$

From Matlab

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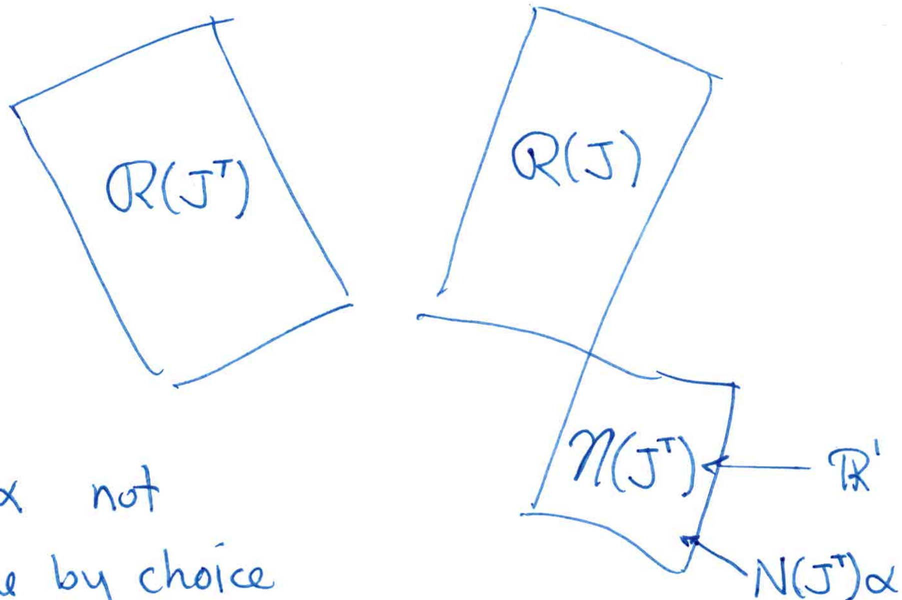
$$J^{+} = \left[\begin{array}{cc|cc} -0.4 & 0.2 & 0 & 0 \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & 1.0 & 0 \\ & & 0 & 1/3 \end{array}\right] \qquad N(J^{T}) = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

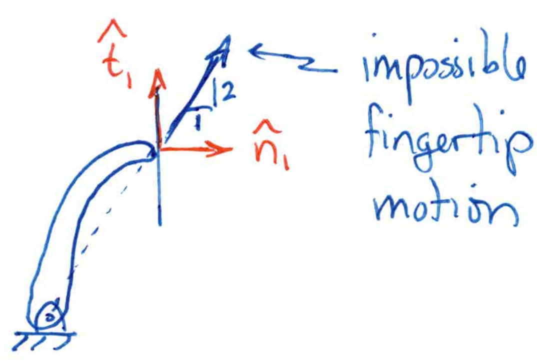
$\dot{q} \in \mathbb{R}^3$ $\qquad\qquad$ $\nu \in \mathbb{R}^4$



$\therefore$ $N(J^T)\alpha$ not achievable by choice of $\dot{q}$

IN Matlab. Solve:

$$J\dot{q} = \nu_{cc} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$



yields $\dot{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ← That's the best we can do to generate $v_{1n} = 1$, $v_{1t} = 2$

What about force closure?

We know $\text{Rank}(G) = n_v = 3$

$\text{Rank}(J) = 3$

but force closure requires $\text{Rank}(GJ) = n_v$

Matlab check – yes, $\text{Rank}(GJ) = n_v = 3$

Next test: $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$?

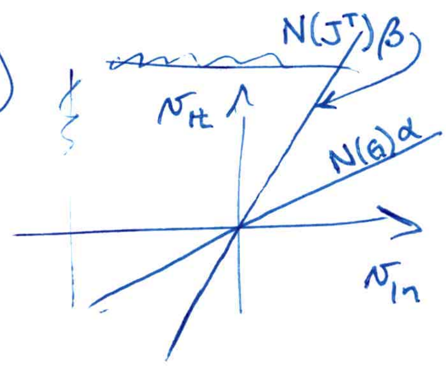
$$N(G) = \begin{bmatrix}2\\1\\2\\1\end{bmatrix} \qquad N(J^T) = \begin{bmatrix}1\\2\\0\\0\end{bmatrix}$$

Does a solution to

$$\begin{bmatrix}2\\1\\2\\1\end{bmatrix}\alpha + \begin{bmatrix}1\\2\\0\\0\end{bmatrix}\beta = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

(rows 1–2) $\Rightarrow \beta = 0$

(rows 3–4) $\Rightarrow \alpha = 0$



exist with α or $\beta \neq 0$?

So, $\operatorname{rank}(G) = n_\nu$

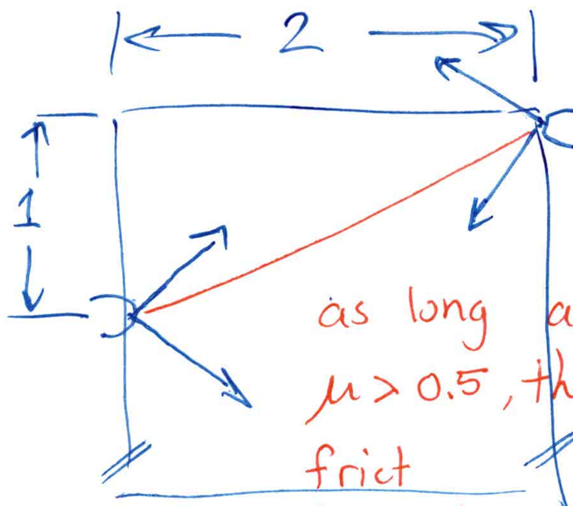
$\operatorname{rank}(GJ) = n_\nu$

$\& \quad \mathcal{N}(G) \cap \mathcal{N}(J^T) = 0 \Big\} \leftarrow$ In general, to test this solve LP3.

Last thing to check for force closure is frictional form closure.

In general, solve LP2 from Grasp Chapter.

If $d^* > 0$, then friction form closure exists.



as long as $\mu > 0.5$, then frict form cl exists

In this simple planar problem, choose μ & draw friction cones. Do they see each other?

LP3:

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$$\max_{\lambda, d} \quad d$$

subject to: $G\lambda = 0$ } Check for control of
$J^T\lambda = 0$ } all internal forces

$E\lambda - \mathbb{1} d \geq 0$ } make sure $\lambda \neq 0$

$d \geq 0$

$e\lambda \leq n_c$ } keep $d < \infty$

Note:

$d^* = 0 \implies \mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$

$d^* = 0 \implies$ success.

$d^* > 0 \longrightarrow$ internal forces not all controllable.