LaValle_Ch4-1 The Configuration Space (C-space)

Thursday, March 19, 2009
11:11 AM

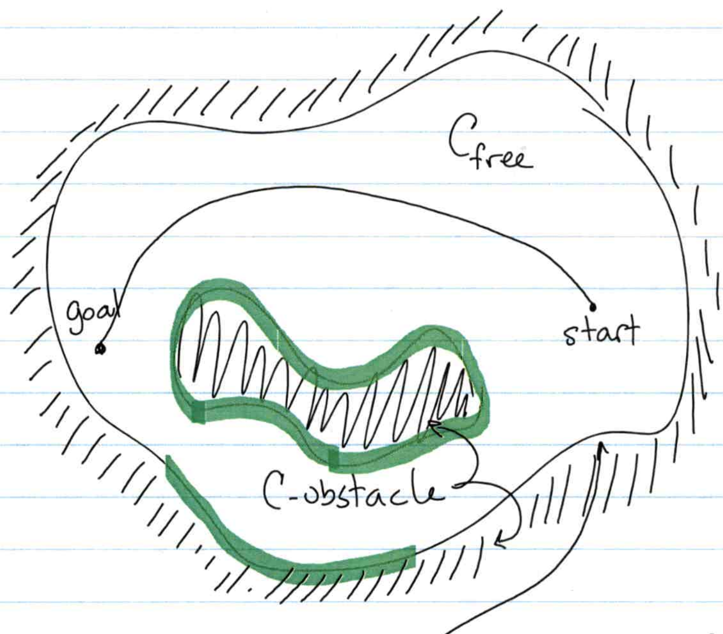
Goal of Ch4 - completely characterize C-space and understand ~~characterize~~ its structure.

C-space is a (non-unique) space in which the robot is represented as a point

C-space is the set of points corresponding to every possible configuration of the robot, even those requiring overlap of bodies.

Robot motions correspond to continuous paths in

$C_{obstacle}$ = set of configs. corresponding to overlap



C_{free} = set of configs. corresp. to no overlap or contact

$C_{contact}$ = set of configs where one or more objects touch but don't overlap.

Let C-space be denoted by $\mathcal{C}$. Then

$$\mathcal{C} = \mathcal{C}_{obs} \cup \mathcal{C}_{cont} \cup \mathcal{C}_{free}$$

where $\mathcal{C}_{obs}$, $\mathcal{C}_{free}$, $\mathcal{C}_{cont}$ are mutually exclusive.

$\mathcal{C}_{free} = \mathcal{C} \setminus (\mathcal{C}_{obs} \cup \mathcal{C}_{contact})$

set element removal or subtraction

All motion planning problems can be transformed into finding a continuous path in $(\mathcal{C}_{free} \cup \mathcal{C}_{contact})$ connecting the starting and goal configs.

If we can approximate $\mathcal{C}$ as a discrete set of points with paths between them, then we can use discrete search methods!

For noncontact problems, it's usually best if the path is smooth (easier on actuators and structure of system).

Manipulation planning problems require a portion of the path to be in $\mathcal{C}_{cont}$.

Although at micro scale you can push things around w/ photons

When dynamics is important planning may need to be done in state space X.

$$X = \mathcal{C} \times T$$

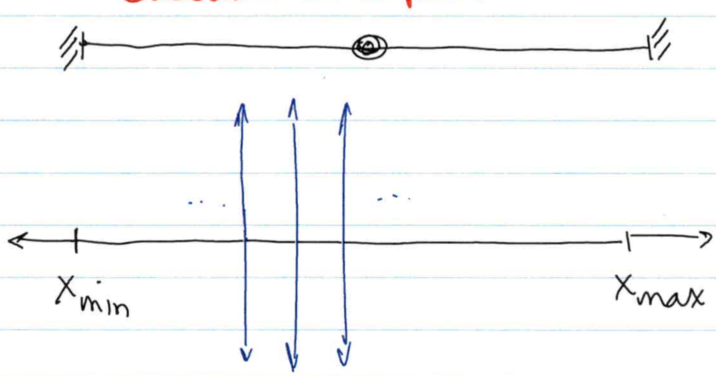
$\times$ — Cartesian product

T — space of velocities, a.k.a, the tangent space.

At every point in $\mathcal{C}$, T is a "fiber" that is a space of velocities.

Bead on wire

Duckiebot on a path



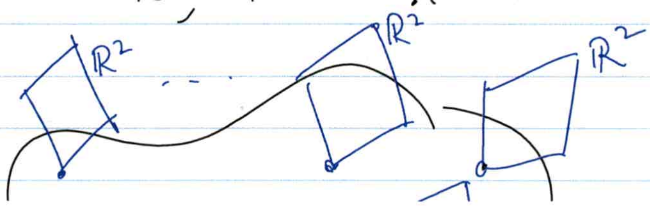
$X = I^1 \times \mathbb{R}^1$

possible velocities at each position

if velocities are bounded,

then $X = I^1 \times I^1 = D^2 =$ the disk in $\mathbb{R}^2$

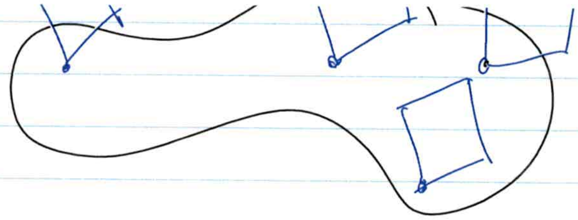
If $\mathcal{C} \subset \mathbb{R}^2$, $T \subset \mathbb{R}^2$, then $X \subset \mathbb{R}^4$



Example: Duckiebot on a segment of a path. Choose position & speed.

(x, y) (v, ω)

Better to use $SE(2) \times \mathbb{R}^2$



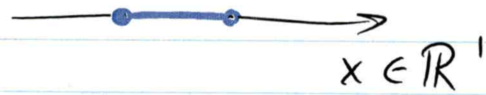
Definitions (in loose terms) :

Closed set – all bndry points are in the set

Open set – all bndry points are not in the set

In $\mathbb{R}^1$

Open set



closed set

Half open set

Disc in $\mathbb{R}^2$:

$D_c = \{(x,y) \mid x^2+y^2 \leq 1\}$ ← closed disc

$D_o = \{(x,y) \mid x^2+y^2 < 1\}$ ← open disc

$\partial D_c = \{(x,y) \mid x^2+y^2 = 1\} = \partial D_o$

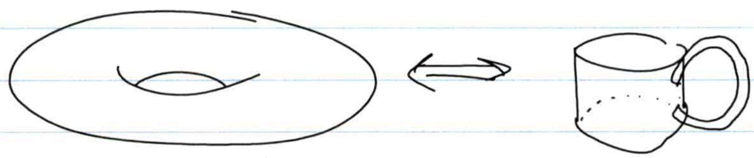
A bit of topology

Homeomorphic – two shapes are homeomorphic if one can be deformed smoothly into the other. For example, $\cancel{T^2} = S^1 \times S^1 = T^2$

meaning torus not tangent space.

don't create or eliminate holes!

T^2 ~~T^2~~ is the 2D torus, a surf in 3D.

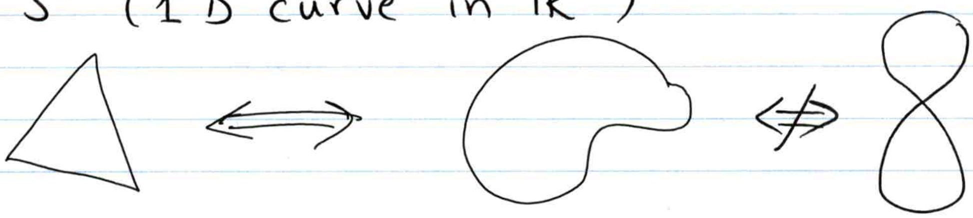


torus mug

both have one hole!

solid sphere $\Longleftrightarrow$ solid cube

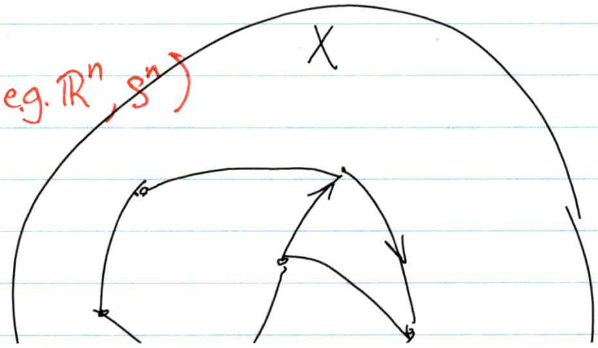
Example: S^1 (1 D curve in $\mathbb{R}^2$)



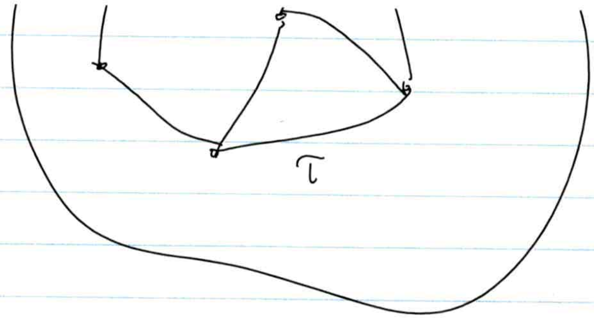
Topological Graphs

(Graphs on topological spaces, e.g. $\mathbb{R}^n$, S^n)

every vertex is in X



every edge
is in X.



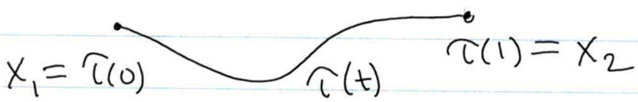
More precisely

every edge maps I^1 onto a curve in X

$$\tau : [0,1] \longrightarrow X$$

or $\tau(t) : [0,1] \longrightarrow X$

(i.e., think of τ as $\tau(t)$, such that $\tau(t) \in X \;\; \forall t$)

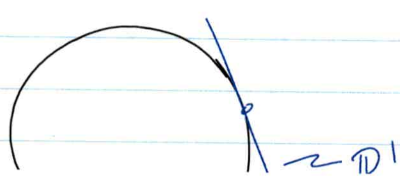


A common definition of manifolds ...

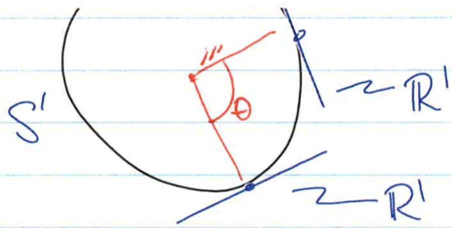
<u>Manifolds</u> – <u>Most C-spaces are manifolds</u>

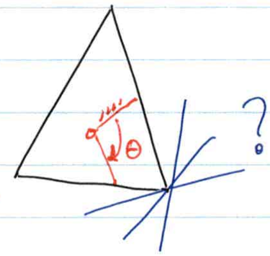
A set M is a manifold if $\forall x \in M$, M is locally Euclidean.

This means there is a well-defined tangent plane at each point of M



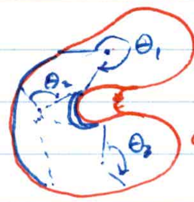
Also given T^2 embedded in $\mathbb{R}^3$, T^2 is a manifold w/ tangent space at each point being $\mathbb{R}^2$.





no single θ works!

Circle is a 1D manifold



use an atlas & charts

Is the triangle a manifold?

You need coord. frame on M for motion planning.

For S', you could use a single parameter

$\Theta \in [0, 2\pi)$

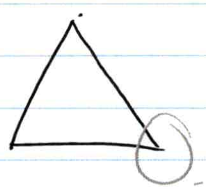
Lavalle's definition from section 4.1.2.

<u>Def:</u>

A topological space $M \subset \mathbb{R}^m$ is a manifold if for every $x \in M$ an open set $O \subset M$ exists $\ni x \in O$, O is homeomorphic to $\mathbb{R}^n$, and <u>n is the same</u> for all $x \in M$.

If not, then we have a stratification

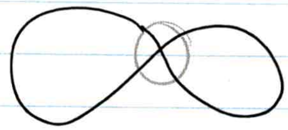
Revisit the triangle:



locall

homeomorphic to $\mathbb{R}^1$. Just straighten

Revisit the figure "8":



not locally homeom. to $\mathbb{R}^1$

2 tangent dirs.

An uncommon uses of "manifold," is found in Bullet:

"contact manifold" – a set of distinct contact points

Sometimes it is more convenient to embed M in a higher-dimensional space. e.g.

S^1 can be embedded in $\mathbb{R}^2$

$S^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\} \Leftarrow$ 2 dof reduced to 1 by 1 eq.

One could also embed in $\mathbb{R}^3$, but then you need 2 eqs. Here's a trivial example:

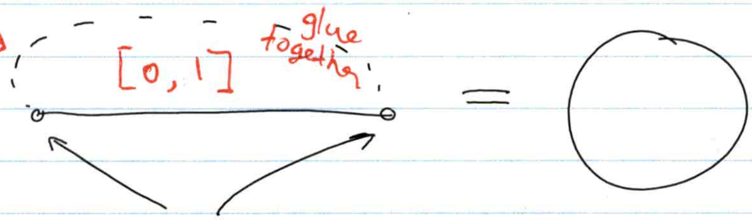
Why do this? You replace θ w/ x & y, but expressions don't have $\sin(\theta)$ or $\cos(\theta)$ any more.

$$S^1 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2=1,\ z=14\}$$

Identifications - must handle these carefully in motion planning.

$S^1 = I^1$ with endpoints "identified"

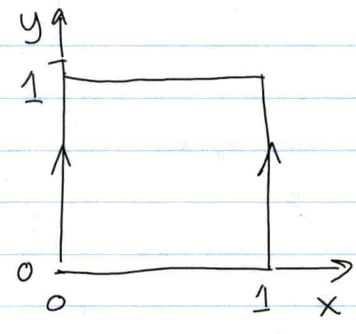
Let $X = I^1$
$X/\sim$ is X redefined by an identification



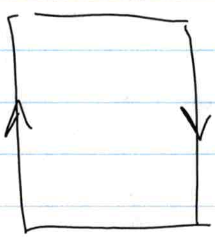
Treat those points as if they were identical.

If we identify 0 & 1, the $X/\sim\ = S^1$.
We say 0 is "equivalent" to 1 and write $0 \sim 1$.

Some well-known identifications of $I^1 \times I^1$

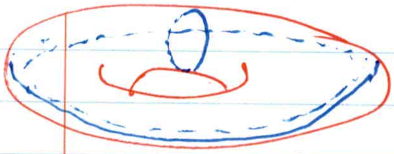


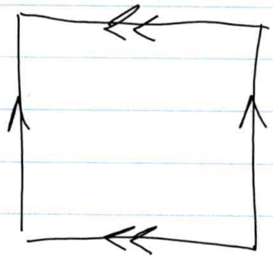
$(0,y) \sim (1,y)$.
Cylinder



$(0,y) \sim (1, 1-y)$
Mobius Strip

SHOW PPT manifolds

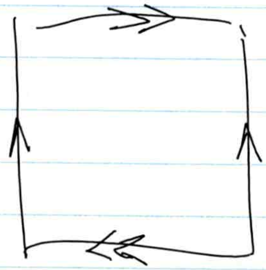




$(0,y) \sim (1,y)$

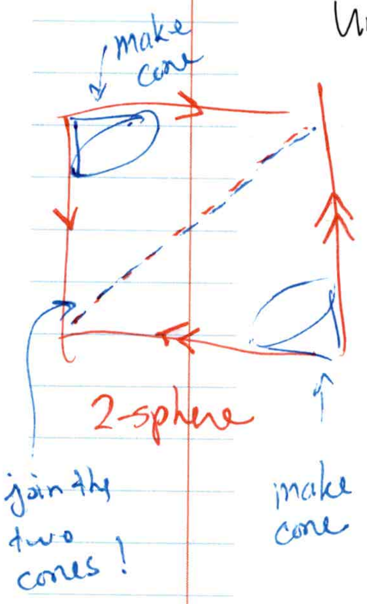
$(x,0) \sim (x,1)$

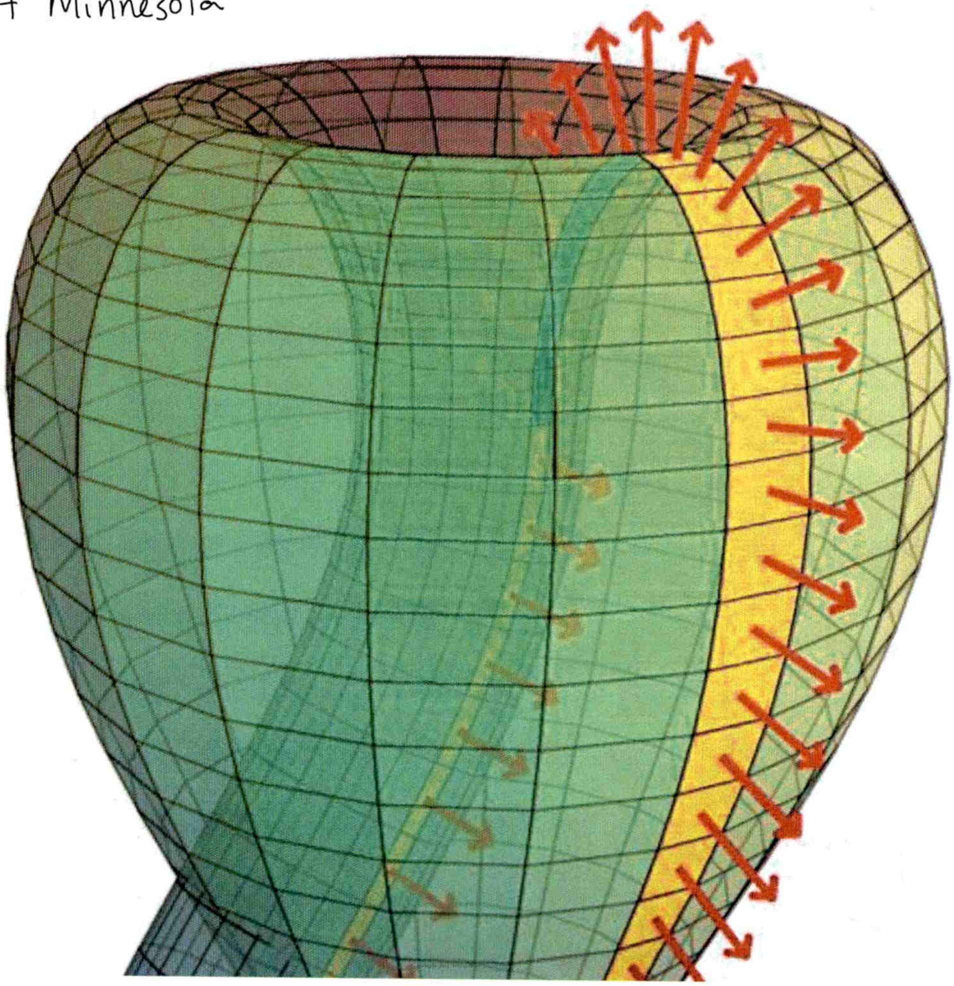
Torus

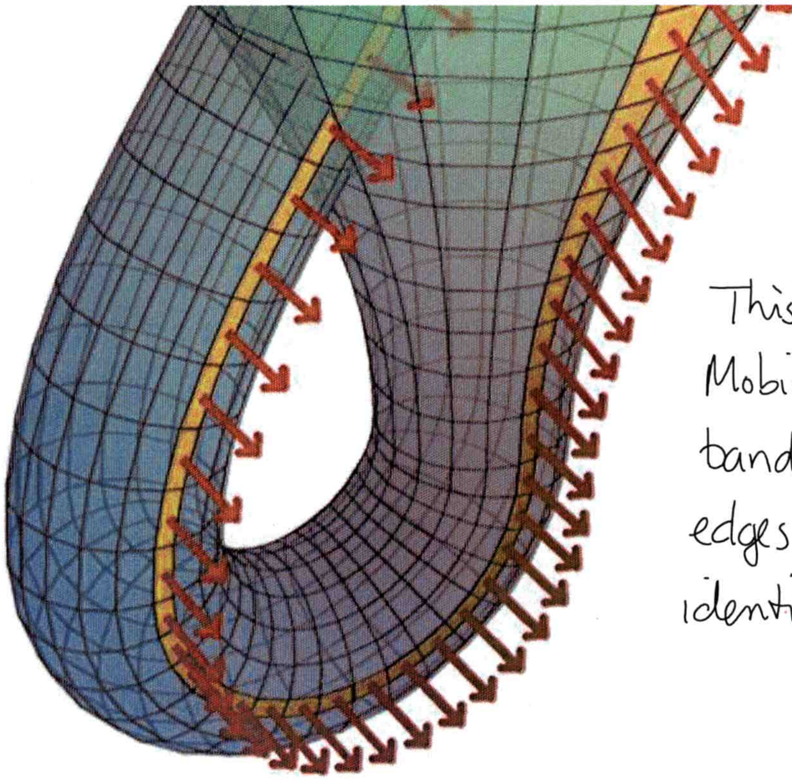


Klein Bottle

From Institute for Mathematics and its Application
Univ of Minnesota





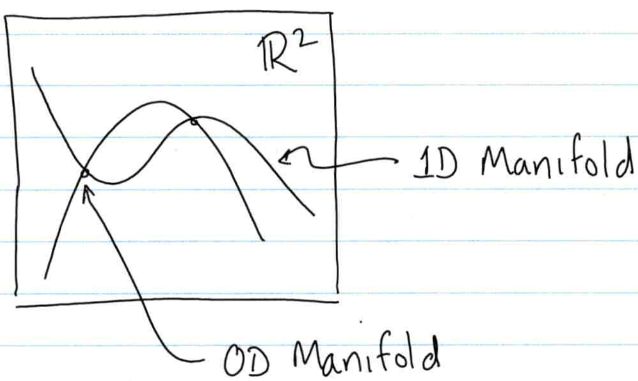


This is a Mobius band with edges identified

Higher-dimensional Manifolds

e.g. $S^n = \{ x \in \mathbb{R}^n \mid \|x\| = 1 \}$ ← (n-1) dimensional

Typically m simultaneous equations in $n > m$ variables yields a manifold



Intersection is a manifold of 2 distinct points (a point is considered a manifold of dimension zero)

Simply connected space

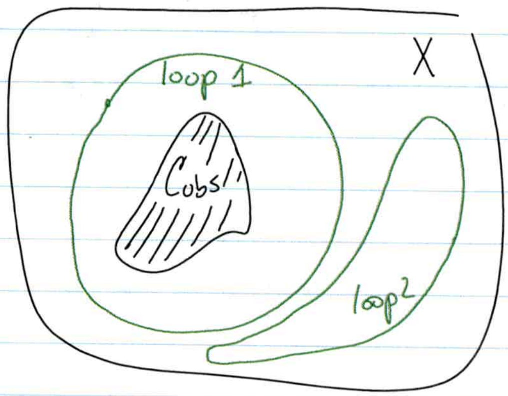
Every loop can be contracted to a point

Loop 1 cannot be contracted

Loop 2 can be

X is not simply connected!

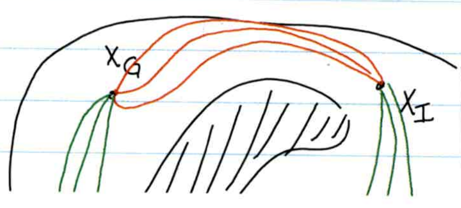
∴ X is multiply connected.



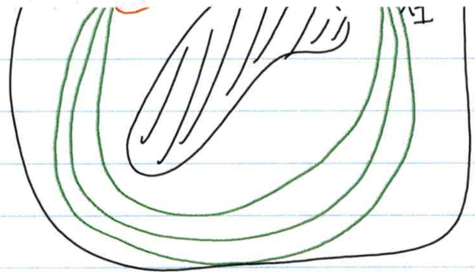
In a simply connected space all paths can be morphed into any other path.

i.e. All paths are homotopic
" " " in the same homotopy class

Green paths cannot be smoothly morphed



Green paths cannot be smoothly morphed into the red paths

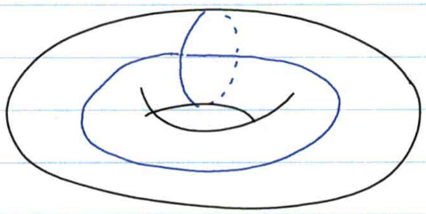


Why do we care? when searching for paths we don't want to limit our search to one homotopy class of paths. We'll get stuck in a local min.

Are there more classes of paths?

what about paths that encircle the obstacle?

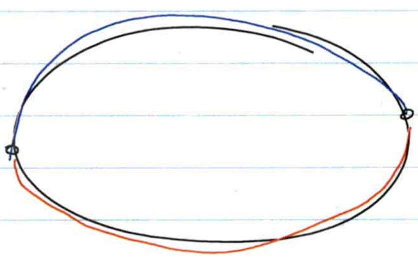
Is the torus simply connected?



No. Not simply connected.

$\therefore$ multiply connected

Is S^1 simply connected?



No!

An example of homotopic paths: parametric parabolas

$$\tau_1(s) = \begin{bmatrix} s - \frac{1}{2} \\ (s-\frac{1}{2})^2 \end{bmatrix}, \qquad \tau_2(s) = \begin{bmatrix} s - \frac{1}{2} \\ \frac{1}{2} - (s-\frac{1}{2})^2 \end{bmatrix},$$

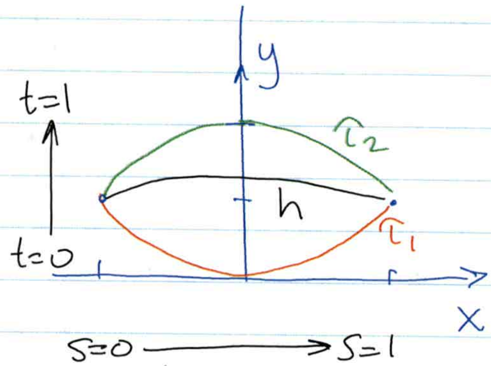
$$0 \le s \le 1$$

t is morphing parameter

$\tau_1(0) = \begin{bmatrix} -1/2 \\ 1/4 \end{bmatrix}$ $\qquad$ $\tau_2(0) = \begin{bmatrix} -1/2 \\ 1/4 \end{bmatrix}$

$\tau_1(1) = \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix}$ $\qquad$ $\tau_2(1) = \begin{bmatrix} 1/2 \\ 1/4 \end{bmatrix}$

$\tau_1(1/2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\qquad$ $\tau_2(\tfrac{1}{2}) = \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$



Define the morphing function, $h(s,t)$

$$h(s,t) = (1-t)\,\tau_1(s) + t\,\tau_2(s) \longleftarrow$$ convex combination of τ_1 & τ_2 parabolas

This function must be continuous in t and s,

i.e.: • $h(\alpha,\beta)$ must exist at every α,β

For each α,β, we must have:

• $\lim_{\alpha,\beta \to s,t} h(s,t)$ must exist at every α,β

• $\lim_{\alpha,\beta \to s,t} h(s,t) = h(\alpha,\beta)$