2009_final_soln

Tuesday, May 06, 2008
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Robotics II Final - Spring 2009

True / False Questions (14 pts total. 2 pts per question)

① Assume $C = \mathbb{R}^2$ and C_{obs} is a polygon. Vertical decomposition of C_{free} will yield a simplicial complex covering C_{free}.

F There will usually be non-triangular regions.

② C-space of a rigid body free to move in space ($\mathbb{R}^3$) is not $SE(3) = \mathbb{R}^3 \times SO(3)$.

F It is SE(3).

③ Some constraints of a Linear Complementarity problem are not linear in the unknowns.

T The $x^T z = 0$ constraints are bilinear.

④ A^* search with cost-to-come function equal to zero, is equivalent to best-first search.

T (Technically they are different, since best_first's cost-to-go fcn need not be optimistic.)

⑤ Semi-Algebraic sets are composed of a

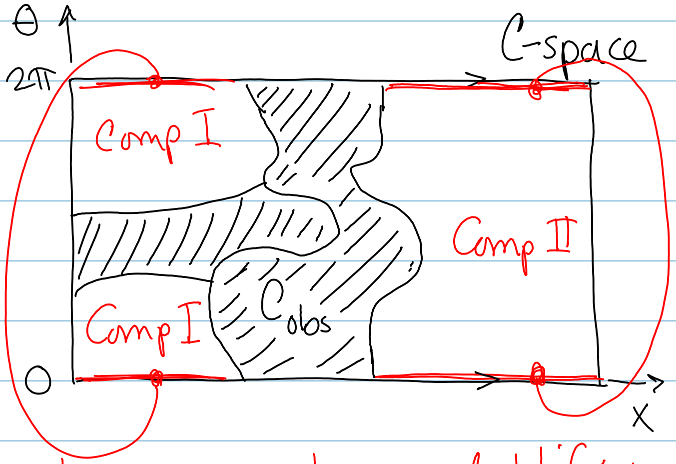
T finite # of unions & intersections of polynomial inequalities.

⑥ Randomized potential field methods are particularly nice to use for a wide range of problems, because they do not require parameter tuning.

F There are a number of parameters to tune.

⑦ The C-space at the right is $I^1 \times S^1$ ($S^1 = [0, 2\pi] \backslash \sim$).

T C_{free} has two components.



These segments are identified

Short Answer Questions (21 pts total. 3 pts per question)

① (a) In words, what is the configuration space of a system of bodies?

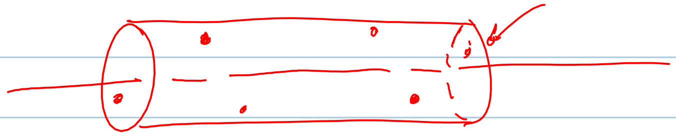
The space of all configs. Every config of the system maps to a point in C-space. Every point in C-space map to a config in W. The mapping is 1-1 & onto.

(b) Assuming no obstacles, what does a curve in C-space represent in the world or physical space of the robot.

A continuous motion of the system

② Someone claims to have a form closure grasp of a cylinder using 7 contacts. Is there a way to think about this problem such that the claim makes sense? Explain.

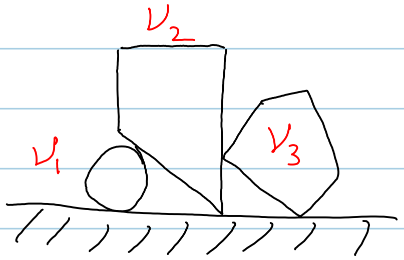
Yes. You can form close 5 dof (excluding rotation about the cylinder's axis. It is reasonable to ignore rotation since the geom. is independent of it anyway.



Alternatively, if the person meant "frictional form closure," the statement could be correct.

Second alternative: the cylinder is not exactly a cylinder.

③ For the planar multibody system shown below, what are the unknown vectors and their sizes when applying the standard LCP (Stewart-Trinkle) time-stepper? (There are 5 unilateral contacts)



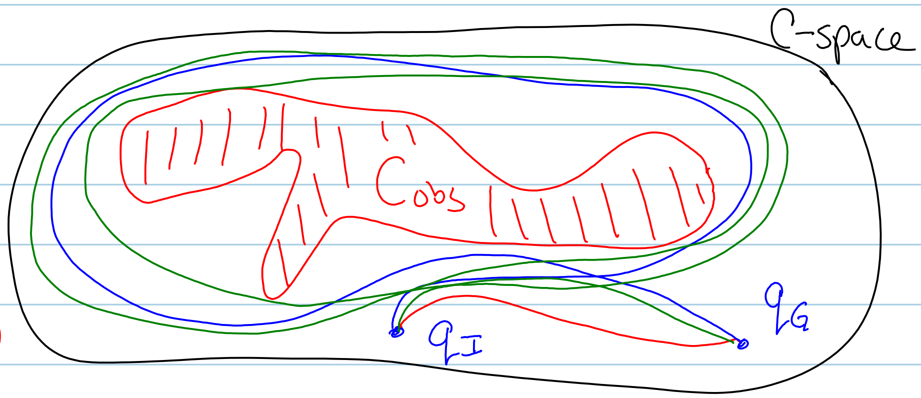
$$\left.\begin{array}{lll} \nu & \text{is} & 9\times 1 \\ p_n & \text{is} & 5\times 1 \\ p_f & \text{is} & 10\times 1 \\ s & \text{is} & 5\times 1 \end{array}\right\} \Rightarrow$$

LCP size is 20

mLCP size is 29

either answer is ok.

④ In the 2D C-space shown, sketch solutions from at least three different homotopy classes.

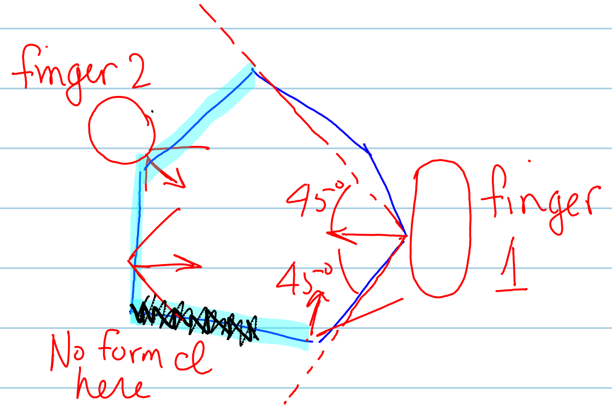


red - does not circle C_{obs}

green - circles C_{obs} 2 times

blue - circles C_{obs} 1 time

⑤ For the two-finger grasp of the object below, determine an approx range of placements of finger 2 (finger 1 remains fixed), such that the grasp has frictional form closure.

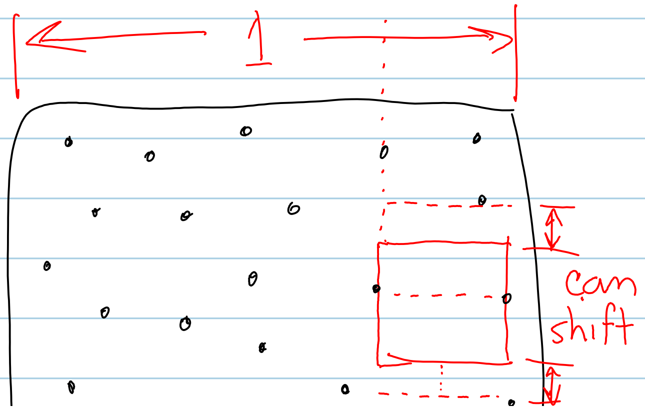


Assume $\mu = 1.0$

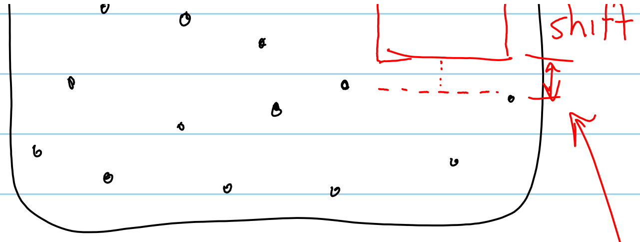
It appears as though both cones can "see" each other as long as finger 2 is on one of the three highlighted edges, excluding the crossed-out section.

Note that finger 2 can also be in contact with the vertices at the min and max y-coord and still the grasp could be form-closed.

⑥ For the samples shown in the unit "square" on the right, what is the approximate



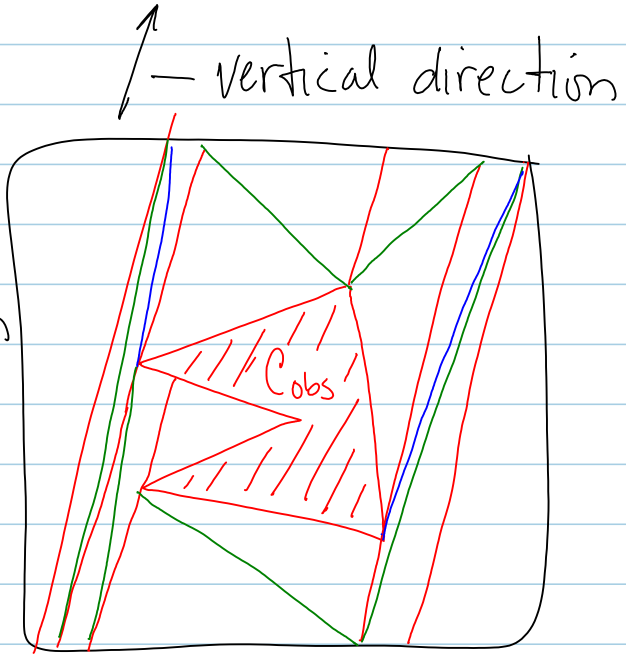
square on the right, what is the approximate dispersion corresponding to the L_∞ norm?



L_∞ norm dispersion corresponds to largest empty square in the space. Notice box is not unique

By my crude "eye-balling" it appears that $\delta = \frac{1}{8} = \frac{1}{2}$ box width.

⑦ (a) For the C-space shown, apply the vertical cell decomposition method,



(b) Add edges so that the decomposition of C_{free} is a simplicial complex.

red verticals give vertical decomp

green lines break cells into triangles

blue lines added to ensure all triangles share each edge w/ only one other triangle.

Analysis Questions (65 pts total)

① Let P_1 and P_2 be convex polygons in a plane. Let n_1 and n_2 be the number of edges of P_1 & P_2, respectively. Assume one polygon is a fixed obstacle and the other is moveable.

(16 pts)

The C-space of the system is $SE(2) = \mathbb{R}^2 \times S^1$

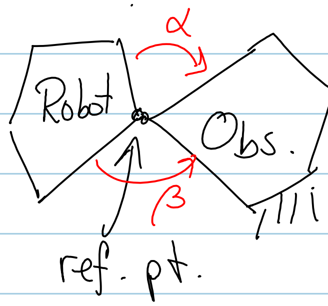
a.) Determine the number of 2-dimensional facets of C_{obs} in $SE(2)$. (Hint: 2-d facets arise from EV and VE contacts)

2D facets arise from VE & VE contacts

Since polygons are convex, # of 2D facets $= \boxed{(n_1 \cdot n_2)2}$

b.) Suppose the reference point on the robot is one of the vertices.

Derive a 1-D edge of C_{obs} corresponding to the ref. pt. in contact with a vertex

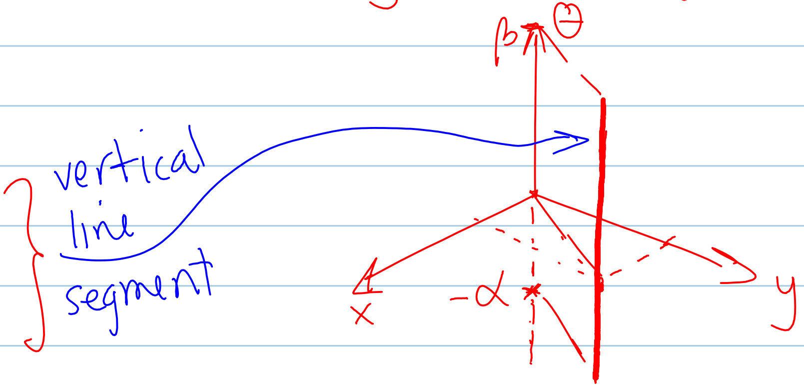


of the obstacle

In this case the ref pt. will not move in (x,y). Robot will only rotate about its ref. point. Assuming $\theta = 0$ in config shown, the edge of C_{obs} is shown in the fig to the right below.

C_{obs} edge is:

$x = \text{const}$
$y = \text{const}$
$-\alpha \leq \theta \leq \beta$



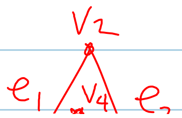
e.) Suppose one of the polygons is nonconvex quadrilateral shown here

Determine lower and upper bounds on the # of 2-D facets of C_{obs}.

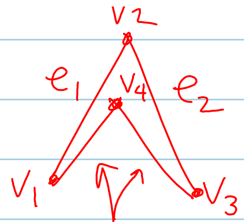
Lower bound comes from inability of the convex polygon to touch the vertex & edges in the local non-convexity of the non-convex quadrilateral.

Suppose the convex polygon has n_1 edges.

The lower bound on # of



The lower bound on # of 2D facets of C_{obs} is :



$$\underset{(EV)}{3n_1} + \underset{(VE)}{2n_1} = 5n_1$$

we assume no vertex of other polygon can touch the inner edges.

Also assume no edge of other polygon can touch v4

For the upper bound, we relax the first assumption but retain the second since it will always apply :

$$\underset{(EV)}{3n_1} + \underset{(VE)}{4n_{11}} = 7n_1$$

② Let X be a space and let $x, x', x'' \in X$ be points.

16 pts Show that $\rho(x,x')$ defined below is or is not a metrix

$$\rho(x,x') = (\Delta x)^2 + \Delta x$$

$$\text{where } \Delta x = x - x'$$

Must satisfy 4 properties.

Fails symmetry since

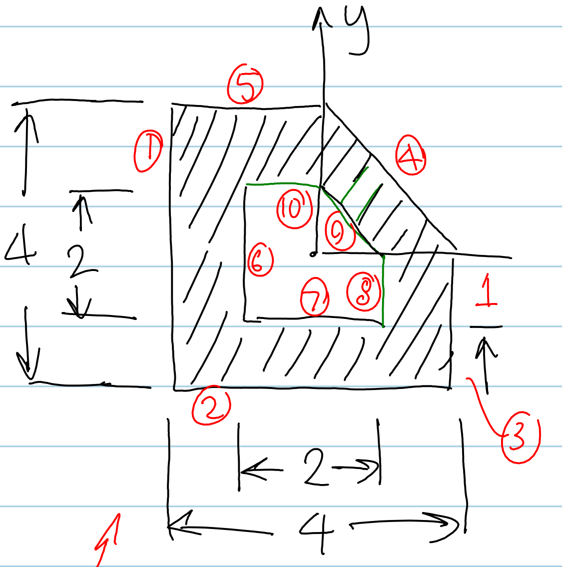
$$\rho(x,x') = \Delta x^2 + \Delta x \leftarrow$$

$$\rho(x,x') = \Delta x^2 + \Delta x$$
$$\rho(x',x) = \Delta x^2 - \Delta x$$
these are not equal

q.e.d.

③ Derive primitives from linear inequalities and combine them with intersections and unions to represent the shaded area.

(16 pts)



10 edges labeled w/ circled numbers

① $P_1: x+2 \geq 0$
② $P_2: y+2 \geq 0$
③ $P_3: -x+2 \geq 0$
④ $P_4: -x-y+2 \geq 0$
⑤ $P_5: -y+2 \geq 0$

→ Inside outer perimeter is given by:

$$P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5 = R_1$$

⑥ $P_6: x+1 \geq 0$
⑦ $P_7: y+1 \geq 0$
⑧ $P_8: -x+1 \geq 0$
⑨ $P_9: -x-y+1 \geq 0$
⑩ $P_{10}: -y+1 \geq 0$

→ Inside inner perimeter is given by

$$P_6 \wedge P_7 \wedge P_8 \wedge P_9 \wedge P_{10} = R_2$$

Represent inside outer perimeter and outside inner perimeter by $R_1 \wedge \overline{R_2}$

How do we represent $\overline{R_2}$? $-P_i$ is formed by changing sense of inequality & removing equality part.

Let $\overline{P_i}$ be formed by reversing inequality and keeping the equality!

Then the shaded region, including the boundaries is given by:

$$R_1 \wedge \overline{R_2}$$

where $\overline{R_2} = \overline{P_6} \vee \overline{P_7} \vee \overline{P_8} \vee \overline{P_9} \vee \overline{P_{10}}$

④ For the LCP (M, b), with $M = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ and

(17 pts) $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, determine the values of b for which the LCP has no solution.

$$0 \leq \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \perp \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq 0$$

$$0 \leq x_2 + b_1 \perp x_1 \geq 0$$

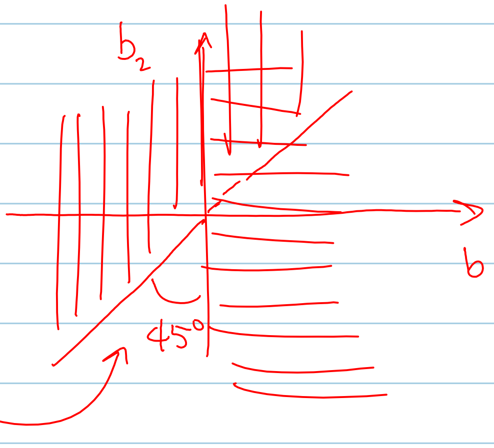
$$0 \leq 2x_1 + x_2 + b_2 \perp x_2 \geq 0$$

Case 1: $z_1 = z_2 = 0 \Rightarrow x_1, x_2 \geq 0$

$$\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \geq 0$$

$$\frac{1}{2}(b_1 - b_2) \geq 0 \Rightarrow \boxed{b_1 \geq b_2}$$

$$\frac{1}{\cancel{2}}(-\cancel{2}b_1) \geq 0 \Rightarrow \boxed{b_1 \leq 0}$$



Valid for case 1.

Case 2: $z_1 = x_2 = 0 \Rightarrow z_2, x_1 \geq 0$

$$z_1 = \underset{0}{\cancel{x_2}} + b_1 = 0 \Rightarrow \boxed{b_1 = 0}$$

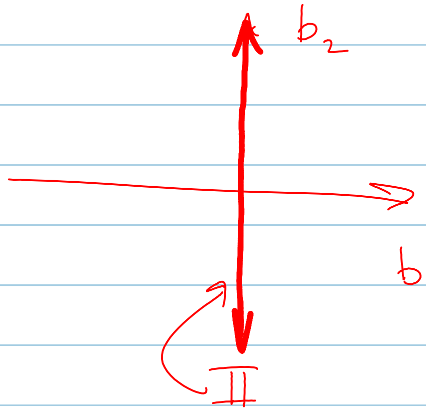
$$z_2 \geq 0 \Rightarrow 2x_1 + \underset{0}{\cancel{x_2}} + b_2 \geq 0 \Rightarrow b_2 \geq -2x_1$$

x_1 is only constrained by $x_1 \geq 0$

∴ we can alway find a solution (x_1, x_2) for any b_2

i.e. for any $b_2 \in \mathbb{R}^1$, x_1 can be chosen $\ni$ $b_2 + 2x_1 \geq 0$.

$$\therefore \boxed{b_2 \in \mathbb{R}^1}$$



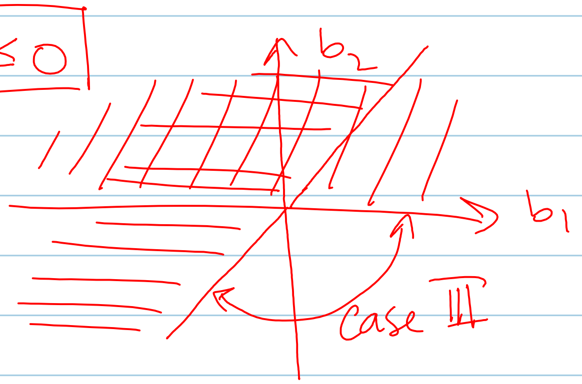
Case 3: $z_2 = x_1 = 0 \Rightarrow z_1, x_2 \geq 0$

$z_2 = 0 \Rightarrow 2\underset{0}{\cancel{x_1}} + x_2 + b_2 = 0 \Rightarrow \underbrace{x_2 = -b_2}$

sub.

$z_1 \geq 0 \Rightarrow x_2 + b_1 \geq 0 \Rightarrow -b_2 + b_1 \geq 0 \Rightarrow \boxed{b_1 \geq b_2}$

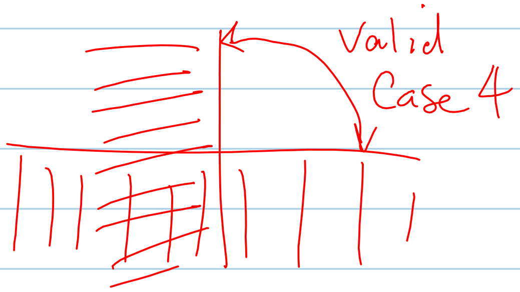
$x_2 \geq 0 \Rightarrow -b_2 \geq 0 \Rightarrow \boxed{b_2 \leq 0}$



Case 4: $x_1 = x_2 = 0 \Rightarrow z_1, z_2 \geq 0$

$x_1 = 0 \Rightarrow z_1 = \underset{0}{\cancel{x_1}} + b_1 \geq 0 \Rightarrow \boxed{b_1 \geq 0}$

$x_2 = 0 \Rightarrow z_2 = 2\underset{0}{\cancel{x_1}} + \underset{0}{\cancel{x_2}} + b_2 \geq 0 \Rightarrow \boxed{b_2 \geq 0}$



Combining results, we get